Victorian Summer School on Ultracold Atoms

Course: Bosonic Josephson Junction

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Outline

1. *Josephson effect*
2. *Double well as a Josephson junction*
3. *Wavefunctions and energy spectrum*
4. *Bloch vector model*
5. *Decay of interference contrast*
6. *Adiabatic splitting of a double well*

The discovery of tunnelling supercurrents*

B. D. Josephson

Cavendish Laboratory, Cambridge, England

Reviews of Modern Physics, Vol. 46, No. 2, April 1974

The events leading to the discovery of tunnelling supercurrents took place while I was working as a research student at the Royal Society Mond Laboratory, Cambridge, under the supervision of Professor Brian Pippard. During my second year as a research student, in

The embarrassing feature of the theory at this point was that the effects predicted were too large! The magni-

Brian Pippard then suggested that I should try to observe tunnelling supercurrents myself, by measuring the characteristics of a junction in a compensated field. The result was negative—a current less than a thousandth of the predicted critical current was sufficient to produce a detectable voltage across the junction. This experiment was at one time to be written up in a chapter of my thesis entitled "Two Unsuccessful Experiments in Electron Tunnelling between Superconductors."

Then a few days later Phil Anderson walked in with an explanation for the missing supercurrents, which was sufficiently convincing for me to decide to go ahead and publish my calculation (Josephson, 1962), although it turned out later not to have been the correct explanation.

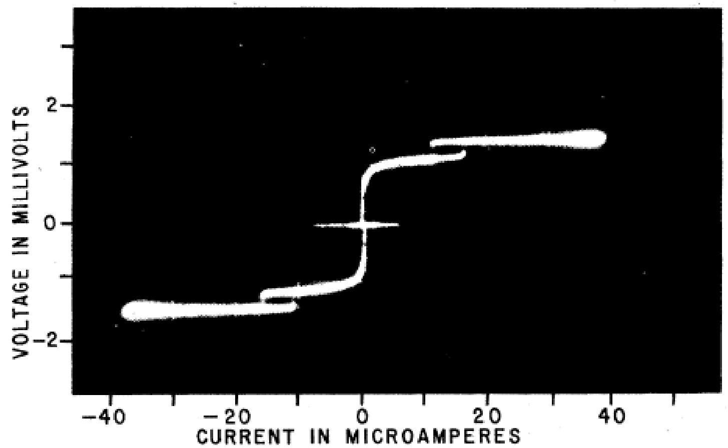
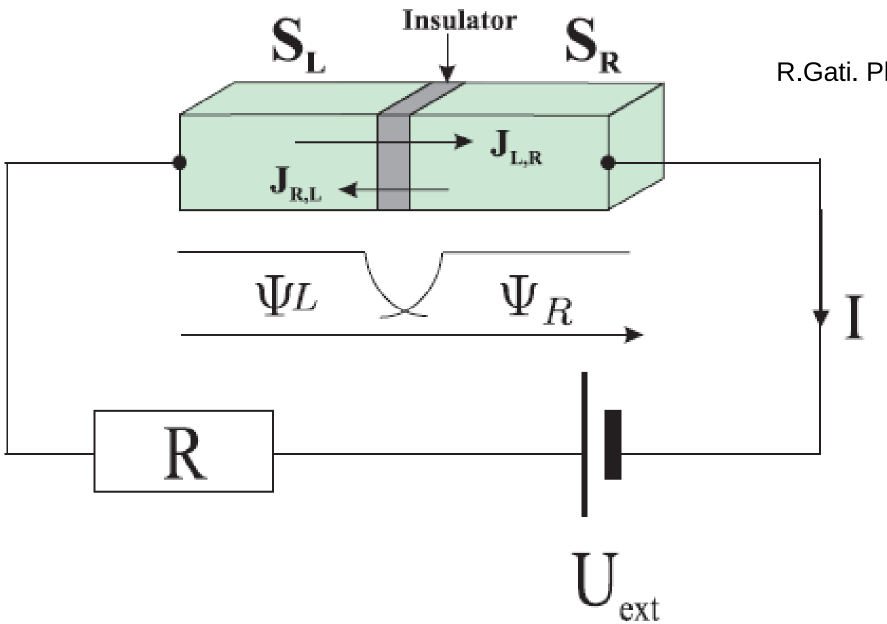


FIG. 3 The first published observation of tunnelling between two evaporated-film superconductors (Smith *et al.*, 1961). A zero-voltage supercurrent is clearly visible. It was not until the experiments of Anderson and Rowell (1963) that such supercurrents could be definitely ascribed to the tunnelling process.

Josephson effect



R.Gati. PhD thesis 2007

Supercurrent $$I = I_c \sin(\Delta\phi)$$

Relative phase $$\Delta\phi = \phi_L - \phi_R$$

$$\frac{\partial(\Delta\phi)}{\partial t} = \frac{2e}{\hbar}V$$

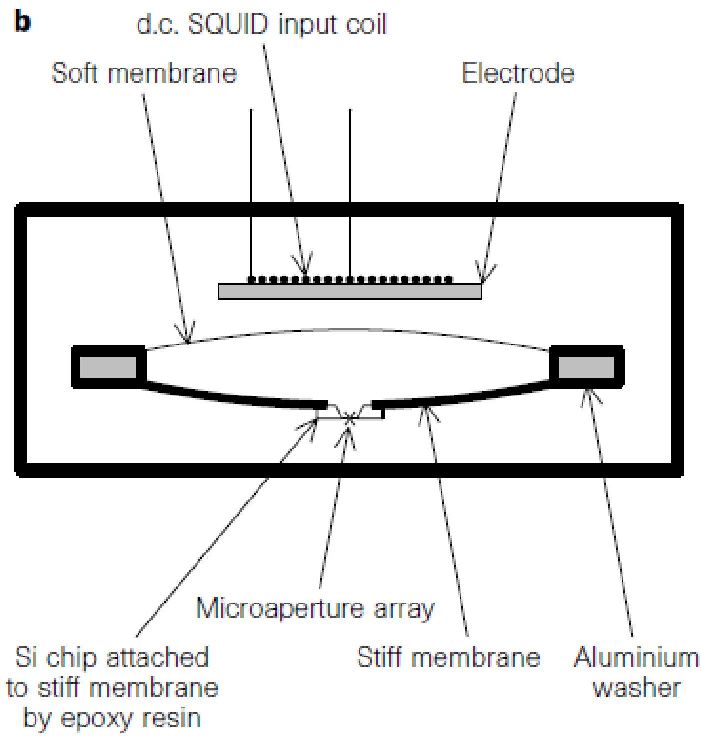
dc Josephson effect: $V = 0$, $\Delta\phi(t) = 0$, $I = I(\Delta\phi)$, $I < I_c$

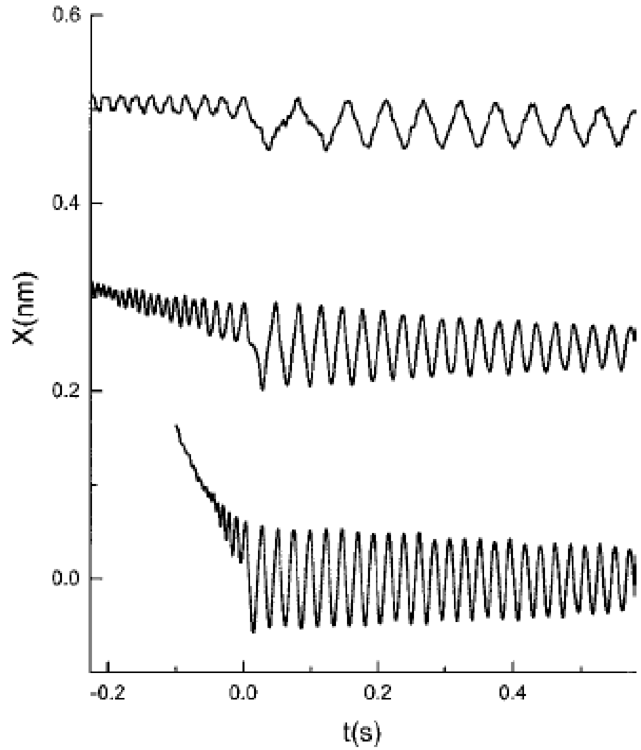
ac Josephson effect (Josephson oscillations): $$I = I_c \sin(\omega t)$$

$$\omega = \frac{\partial(\Delta\phi)}{\partial t} = \frac{2e}{\hbar}V$$

Josephson (1962), Anderson (1964), Feynman (1965):

These equations are applicable for any two coupled, phase-coherent quantum systems.





ac measurements in ^{3}He (Pereverzev et al, 1997)

Oscillatory Exchange of Atoms between Traps Containing Bose Condensates

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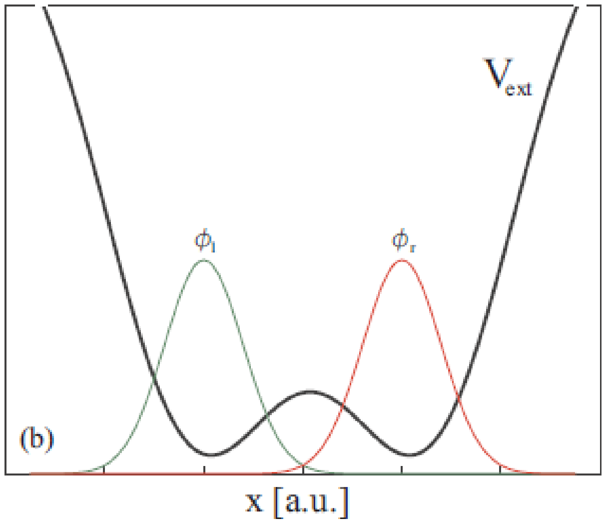
(Received 4 August 1986)

An oscillatory exchange of atoms governed by the phases of the "macroscopic wave functions" between two traps containing Bose condensates, as might be realized with laser cooling and trapping, is predicted. The discussion exploits analogs from lasers and the Josephson junction.



The population of the Left trap

$$n_L = N_L \cos^2 \kappa t + N_R \sin^2 \kappa t + \sqrt{N_L N_R} \sin \Delta\phi \times \sin 2\kappa t$$

Apart from trivial oscillations of the population between the traps if they start out with different numbers of atoms, there appears an interference term which is effective even if $N_l = N_r$. Oscillatory transfer of atoms between the traps under such a condition constitutes a novel macroscopic quantum phenomenon.

PHYSICAL REVIEW A **74**, 023612 (2006)

Asymmetric double-well potential for single-atom interferometry

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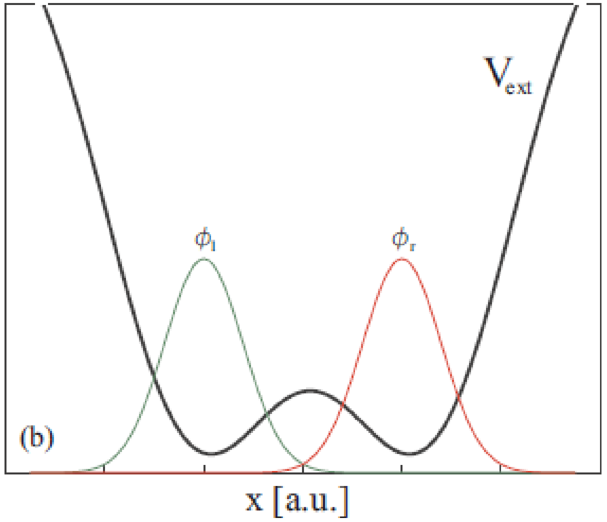
Hamiltonian with asymmetric component

$$\hat{H}(x,t) \;=\; -\frac{\hbar^2}{2m}\nabla^2 + \hat{V}_{tr}(x,t) + \hat{V}_{as}(x)$$

Two-mode approximation

We use the |L⟩, |R⟩ basis vectors



Energy differences

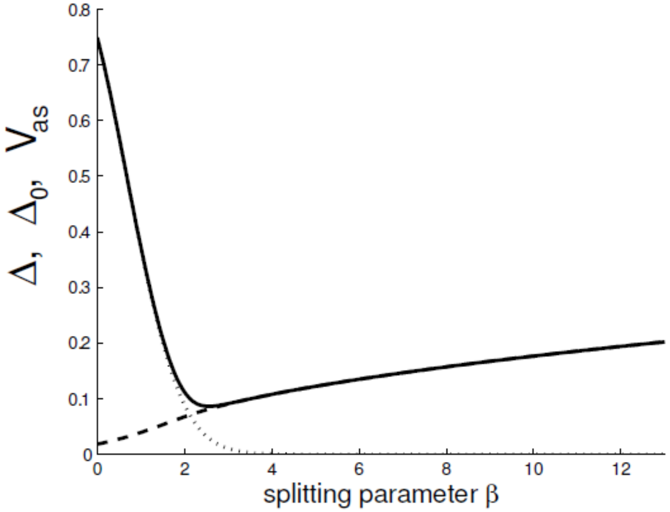


FIG. 1: Energy difference Δ between ground and first excited states (solid line) for $\widehat{V_{as}}$=0.02$\,\widehat{x}$ as a function of the splitting parameter β. Dotted line - energy difference Δ_0 for symmetric Hamiltonian, dashed line - asymmetry quantity V_{as}.

$$\Delta_0 = E_{01} - E_{00}$$

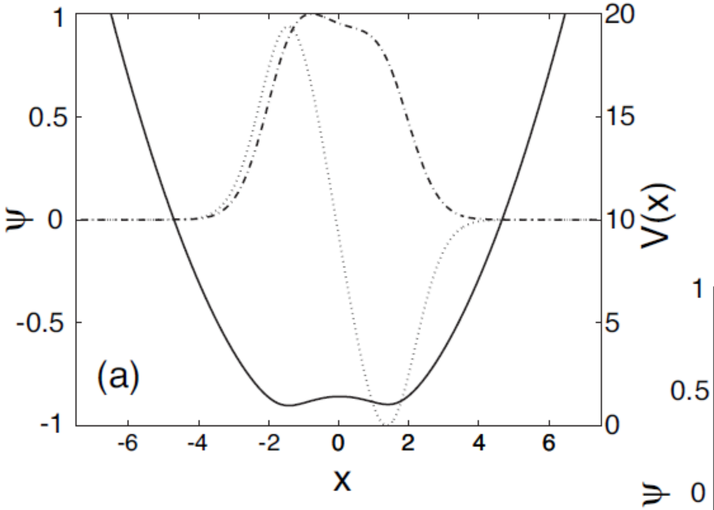
$$V_{as} = \langle R|\hat{V}_{as}|R\rangle - \langle L|\hat{V}_{as}|L\rangle$$

$$\Delta = E_1 - E_0 = \sqrt{\Delta_0^2 + V_{as}^2}$$

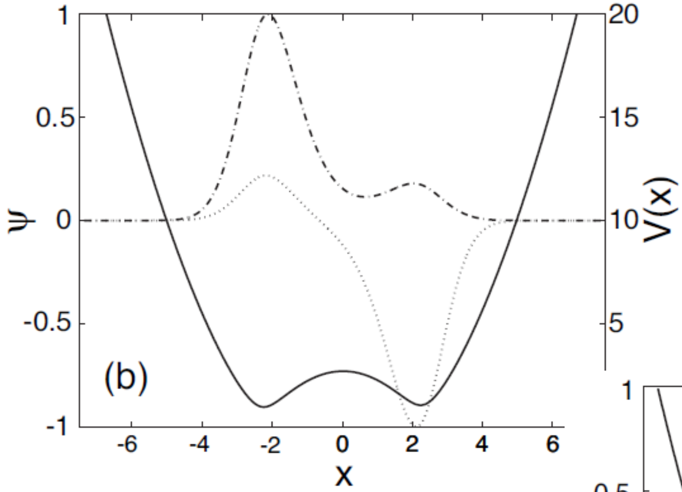
$$|\psi_0\rangle = \sqrt{\frac{1+V_{as}/\Delta}{2}}|L\rangle + \sqrt{\frac{1-V_{as}/\Delta}{2}}|R\rangle$$

$$|\psi_1\rangle = \sqrt{\frac{1+V_{as}/\Delta}{2}}|L\rangle + \sqrt{\frac{1+V_{as}/\Delta}{2}}|R\rangle$$

Ground and excited states wavefunctions

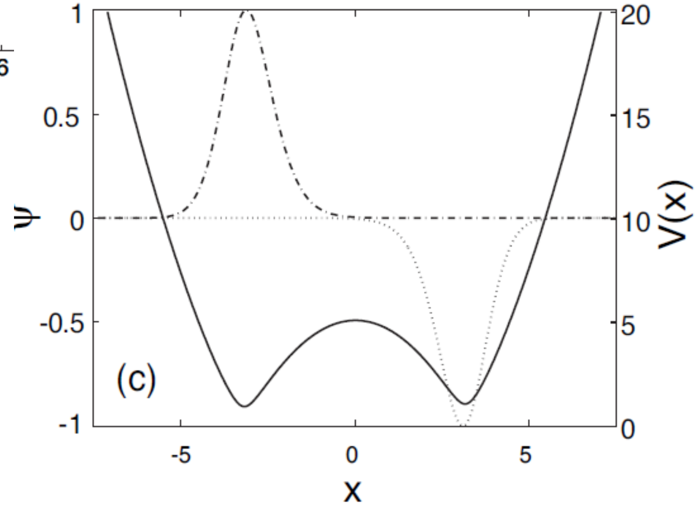


$\beta = 1$

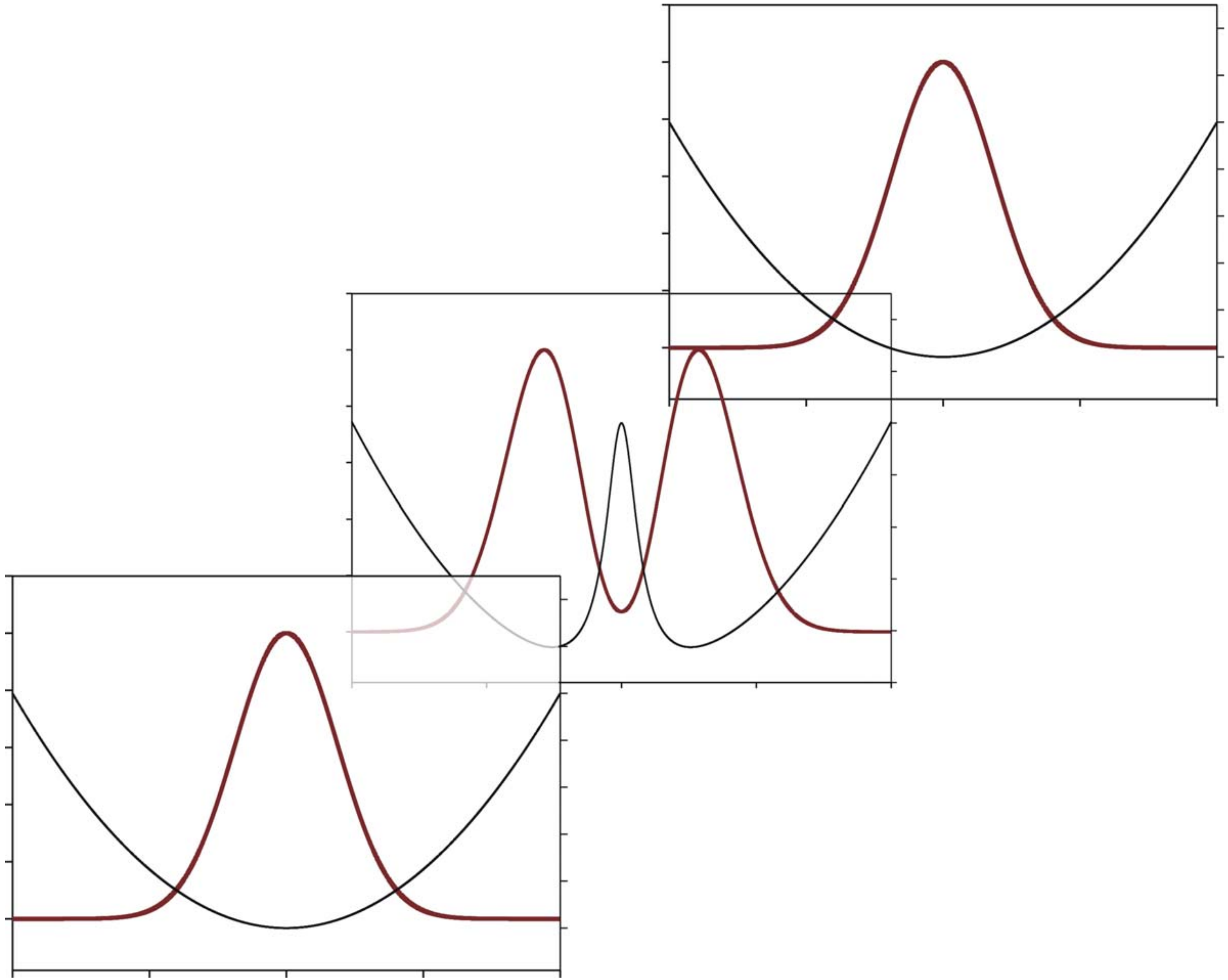


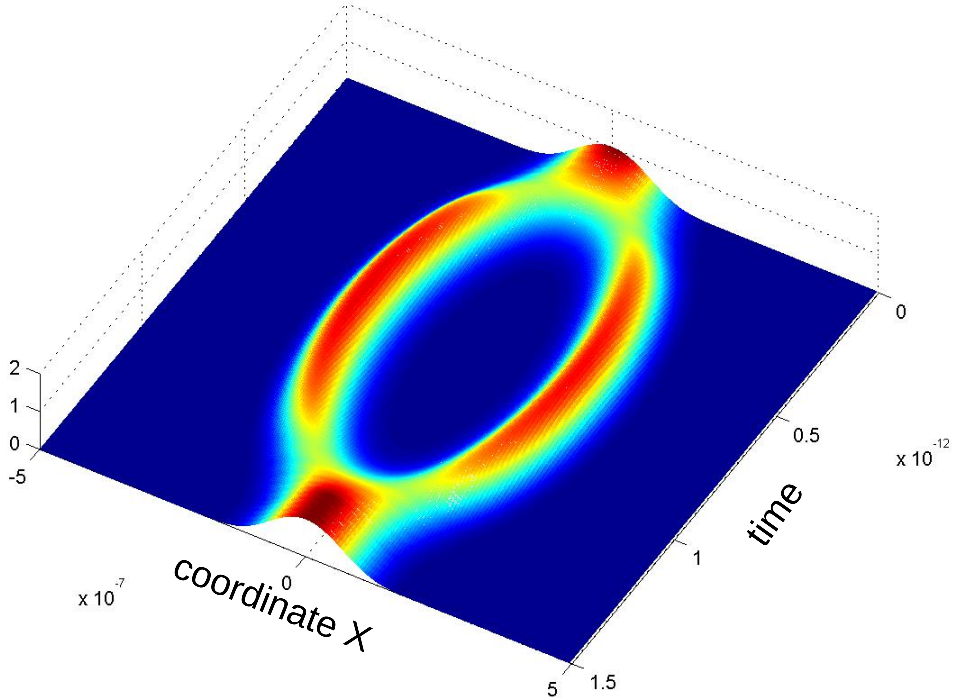
$\beta = 2.5$

$\beta = 5$



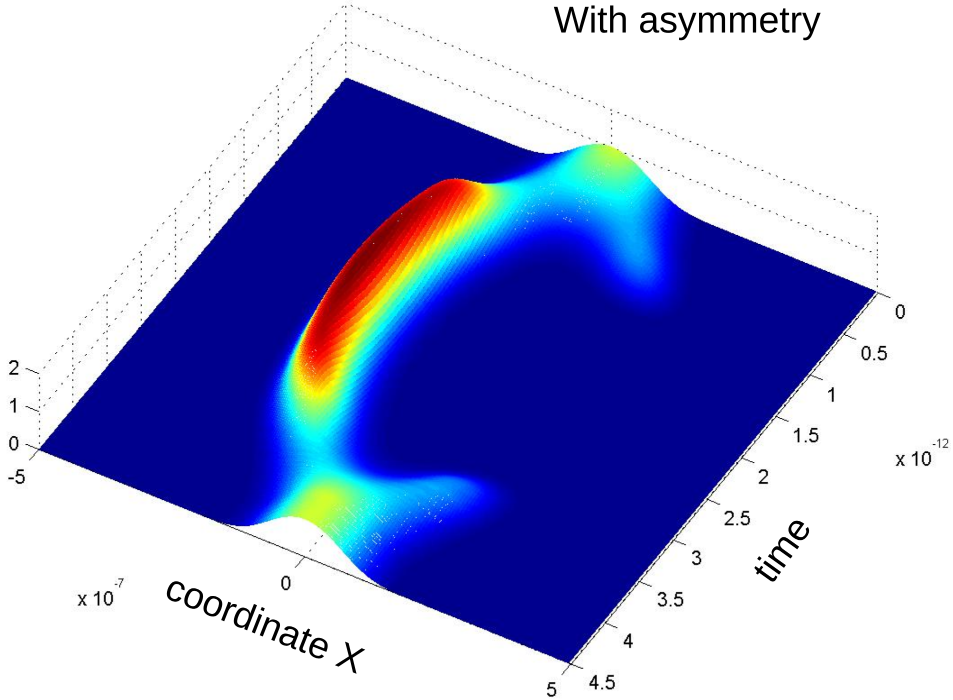
DW interferometer

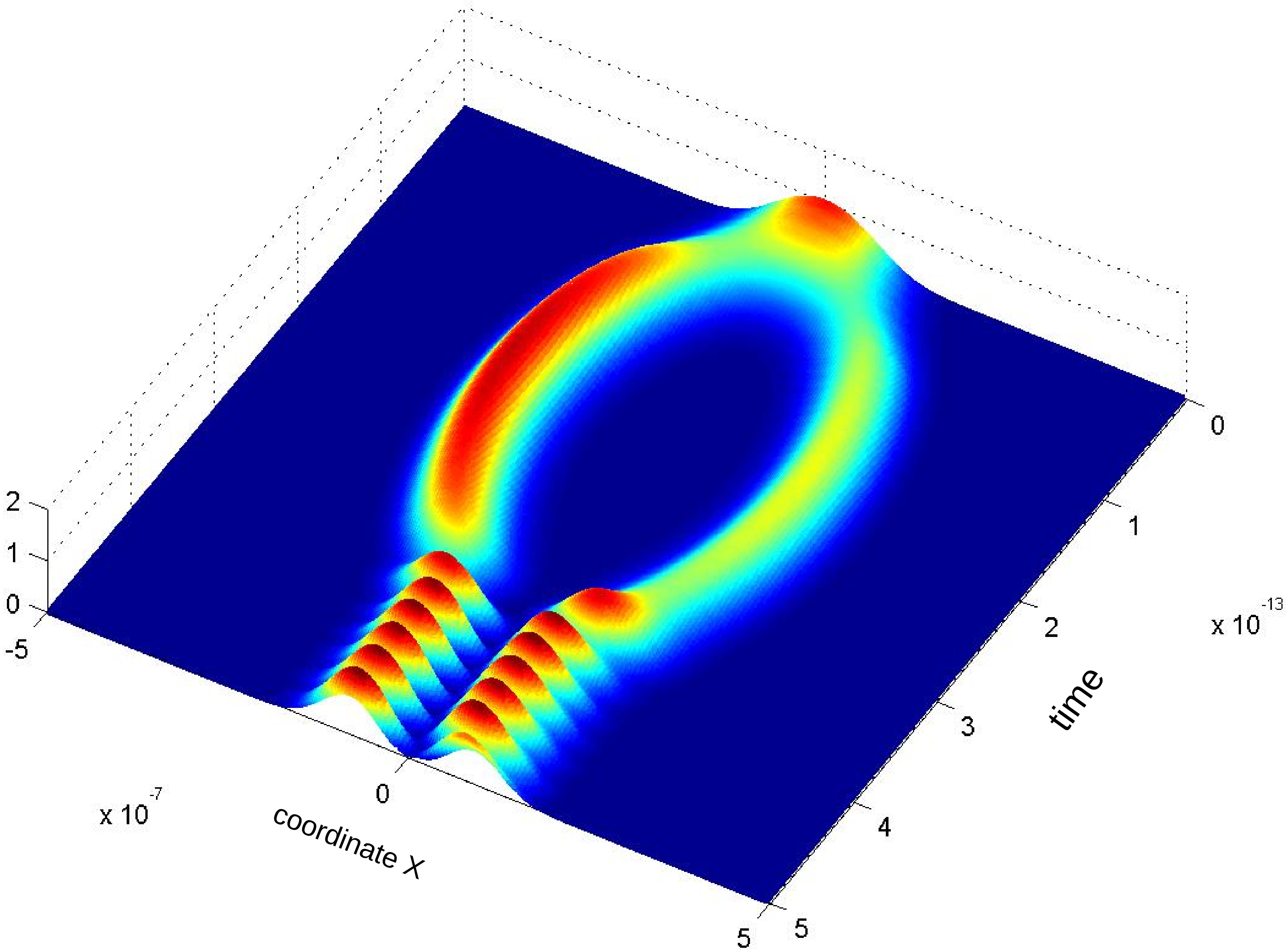




No asymmetry

With asymmetry





2
1
0
-5
x 10^-7
0
coordinate X
5
0
1
2
x 10^-13
time
3
4
5

Bloch vector

$$|\Psi\rangle \;=\; c_L|L\rangle + c_R|R\rangle$$

Spin operators:

$$\begin{aligned}
\hat{\sigma}_x &= |R\rangle\langle L| + |L\rangle\langle R| \\
\hat{\sigma}_y &= \frac{1}{i}(|R\rangle\langle L| - |L\rangle\langle R|) \\
\hat{\sigma}_z &= |R\rangle\langle R| - |L\rangle\langle L|
\end{aligned}$$

Hamiltonian

$$\hat{H} = \frac{1}{2}\left(\Omega_0\hat{1} + \Omega_0\hat{\sigma}_x + \Omega_y\hat{\sigma}_y + \Omega_z\hat{\sigma}_z\right) \qquad \Omega_0 = E_{01} + E_{00}$$

Bloch vector $\boldsymbol{\sigma} = \left(\sigma_x, \sigma_y, \sigma_z\right)$

Torque vector $\boldsymbol{\Omega} = \left(\Omega_x, \Omega_y, \Omega_z\right) = \left(-\Delta_0, 0, V_{as}\right)$

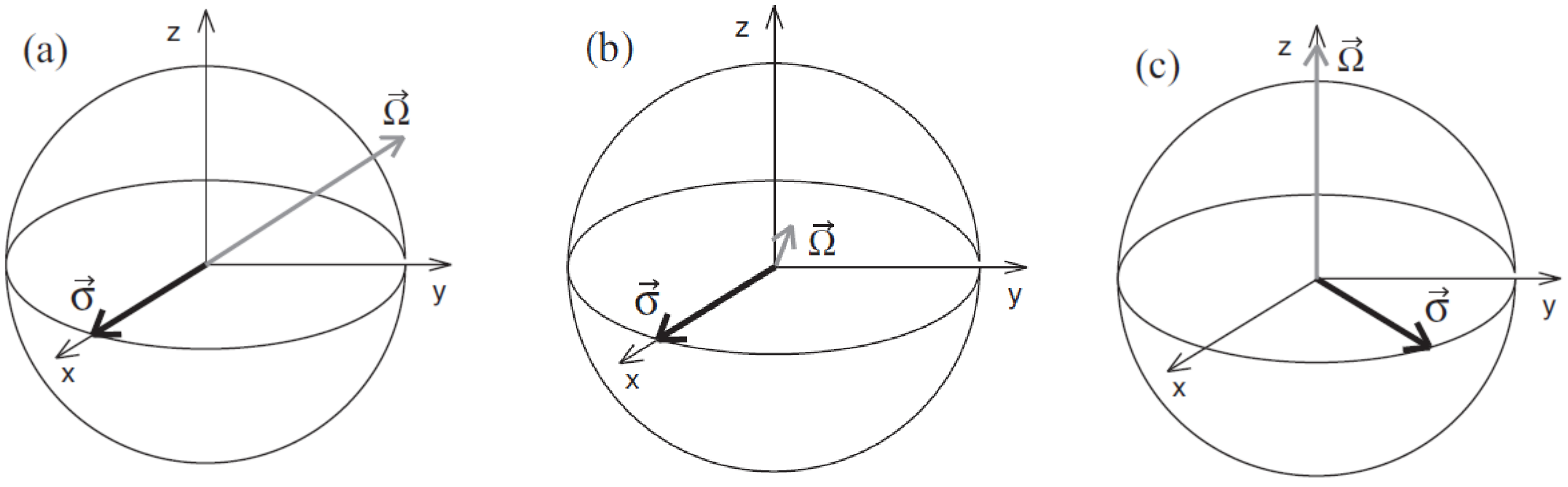
Bloch equations:

$$\begin{aligned}
\frac{\partial\sigma_x}{\partial t} &= -\Omega_z\sigma_y \\
\frac{\partial\sigma_y}{\partial t} &= \Omega_z\sigma_x - \Omega_x\sigma_z \\
\frac{\partial\sigma_z}{\partial t} &= \Omega_x\sigma_y
\end{aligned}$$

$$\frac{\partial\boldsymbol{\sigma}}{\partial t} = \boldsymbol{\Omega}\times\boldsymbol{\sigma}$$

Bloch vector evolution

$$\frac{\partial \boldsymbol{\sigma}}{\partial t} = \boldsymbol{\Omega} \times \boldsymbol{\sigma} \qquad \boldsymbol{\Omega} = \left(-\Delta_0, 0, V_{as}\right)$$



$\boldsymbol{\Omega} = \left(-\Delta_0, 0, 0\right)$ $\qquad$ $\boldsymbol{\Omega} = \left(-\Delta_0, 0, V_{as}\right)$ $\qquad$ $\boldsymbol{\Omega} = \left(0, 0, V_{as}\right)$

Bloch vector evolution

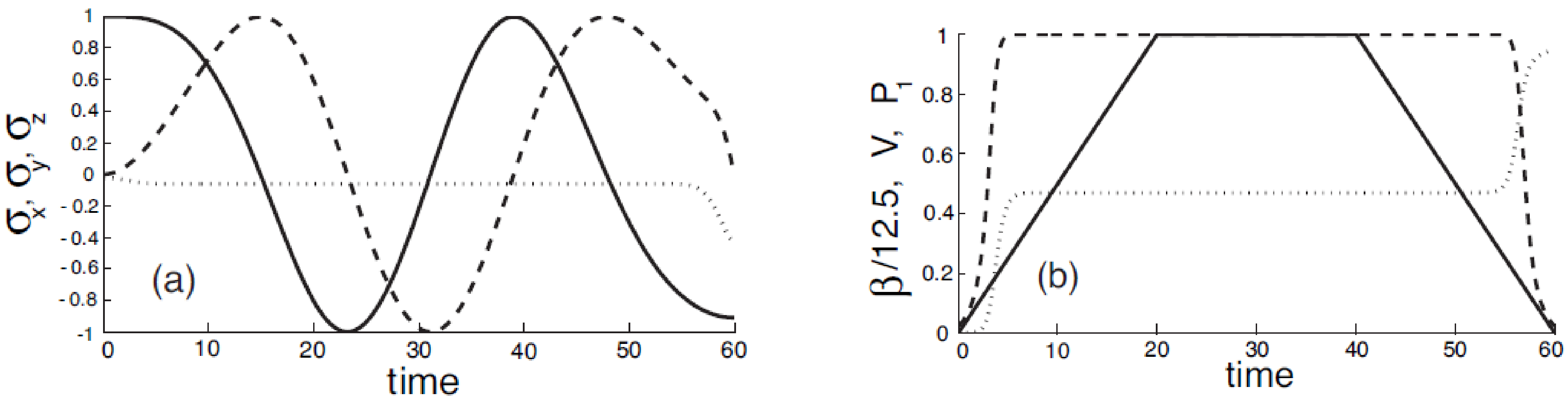


FIG. 4. (a) Time evolution of the Bloch vector components σ_x (solid line), σ_y (dashed line), and σ_z (dotted line) for $T_s = T_h = T_r = 20$ and $\beta_{\max} = 12.5$; (b) time evolution of the first excited state population P_1 (dotted line), the asymmetry parameter V (dashed line), and the splitting parameter β / β_{max} (solid line).

Short and long splitting times

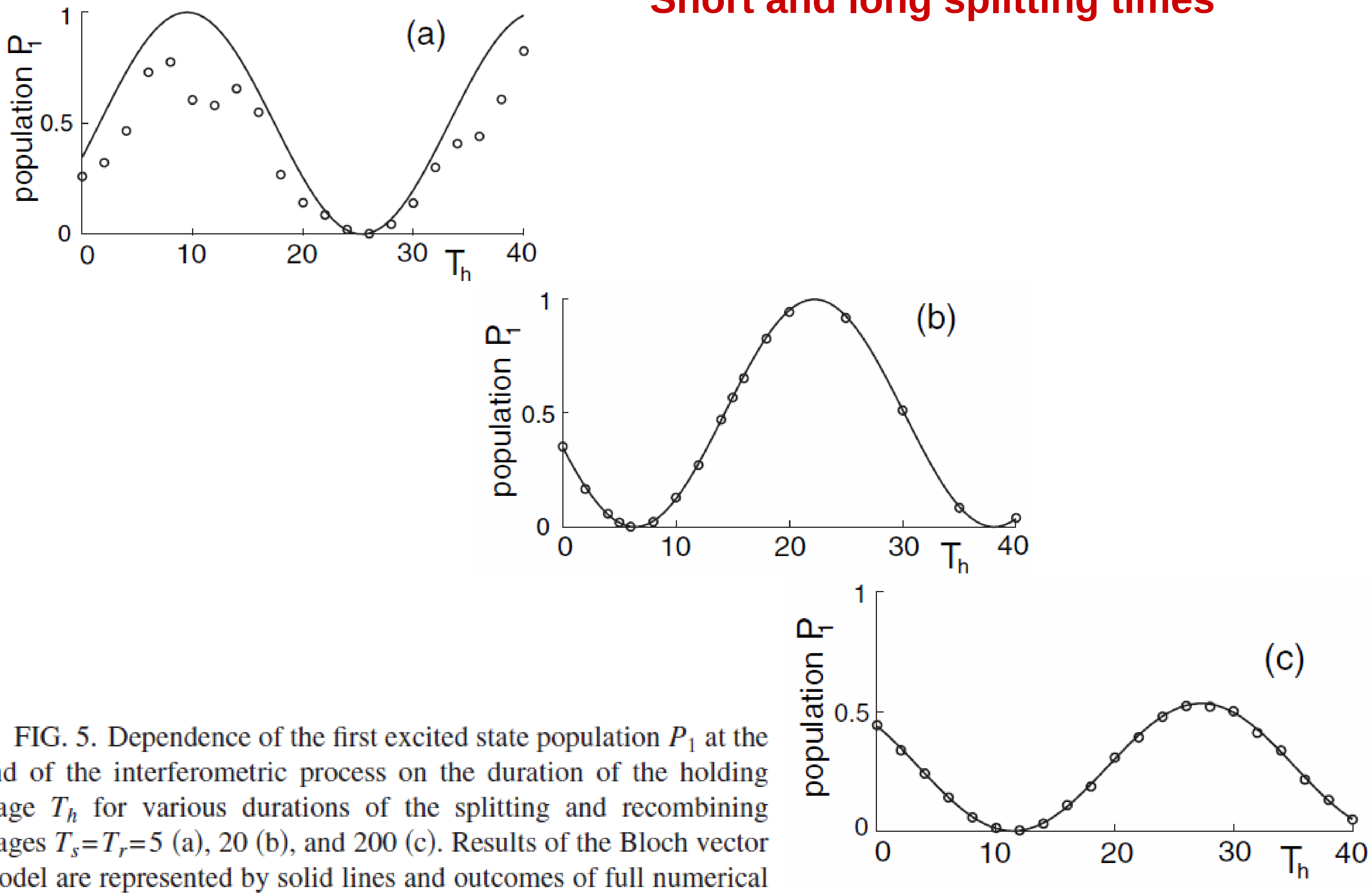


FIG. 5. Dependence of the first excited state population P_1 at the end of the interferometric process on the duration of the holding stage T_h for various durations of the splitting and recombining stages $T_s = T_r = 5$ (a), 20 (b), and 200 (c). Results of the Bloch vector model are represented by solid lines and outcomes of full numerical simulations are presented by circles.

Coherence at short and long splitting times

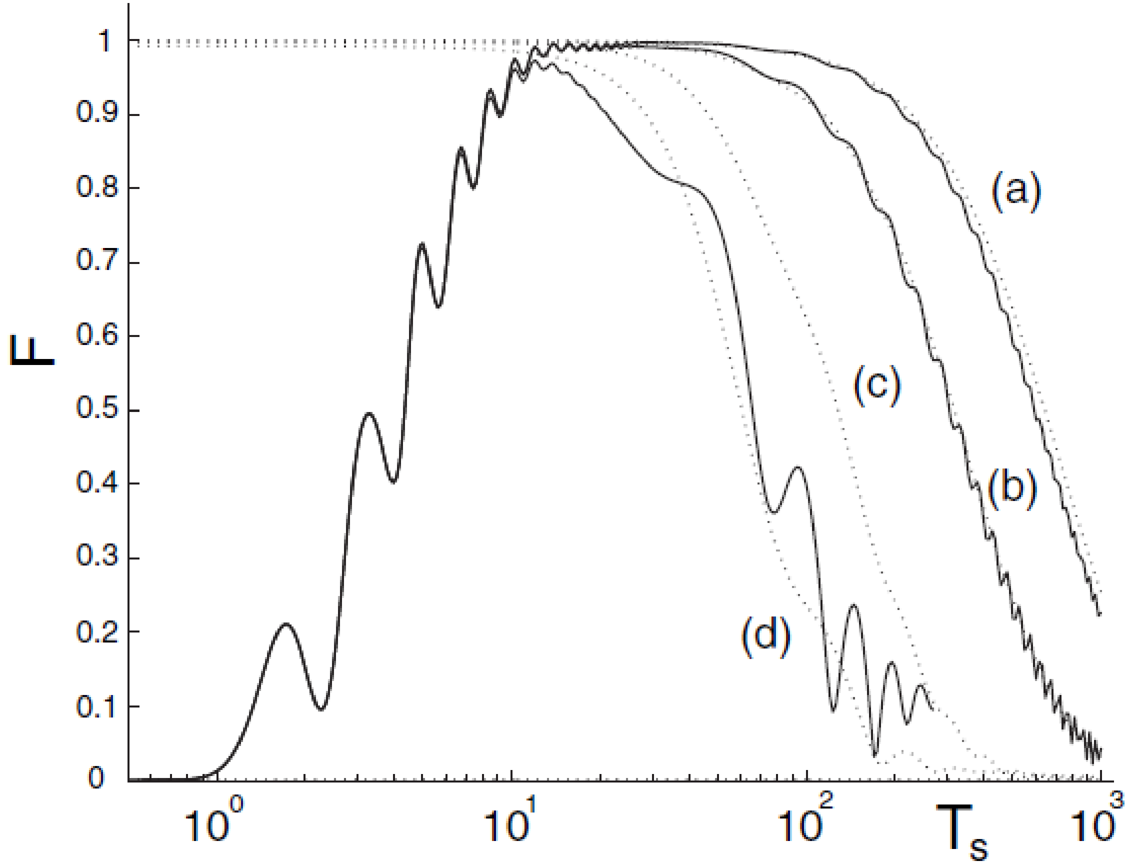


FIG. 6. Dependence of the filling factor F on the duration of the splitting stage T_s for $\beta_{\max}=12.5$ and various values of asymmetry $\widehat{V_{as}}=0.01\hat{x}$ (a), $0.02\hat{x}$ (b), $0.05\hat{x}$ (c), and $0.1\hat{x}$ (d).

Phase-Sensitive Recombination of Two Bose-Einstein Condensates on an Atom Chip

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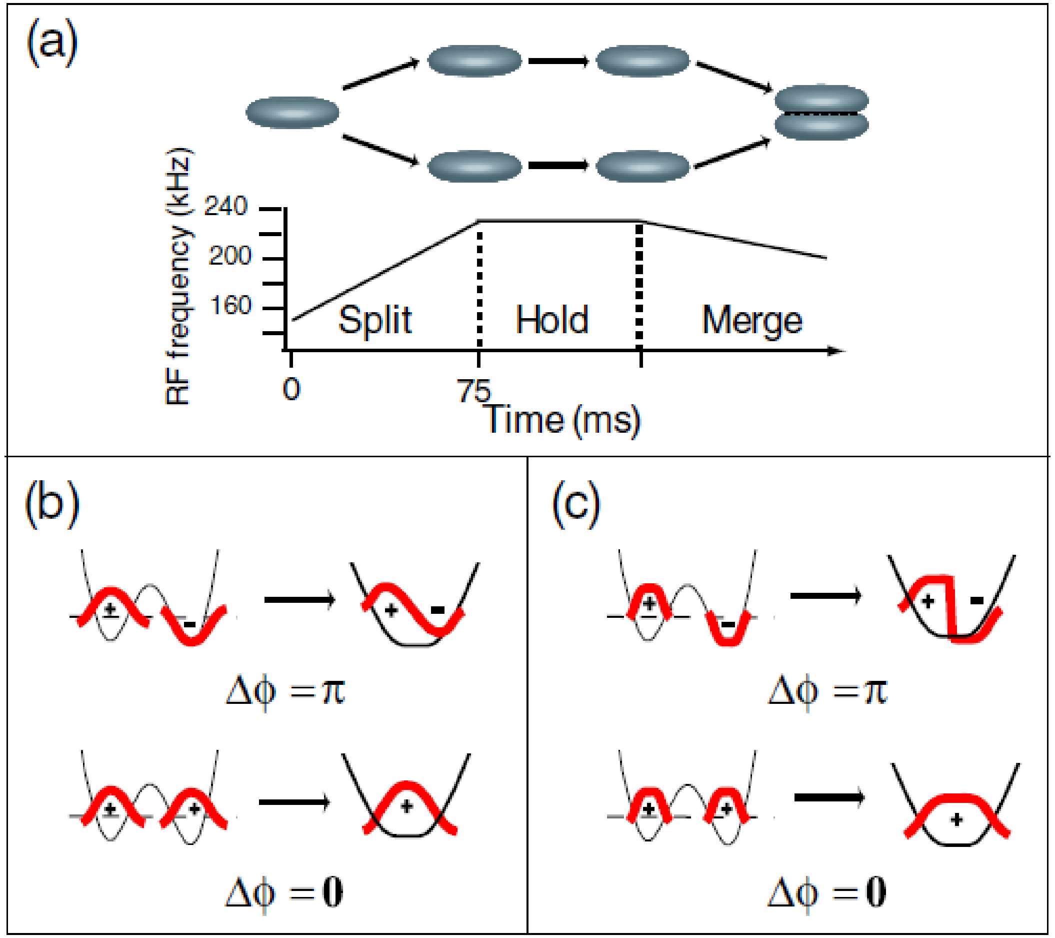


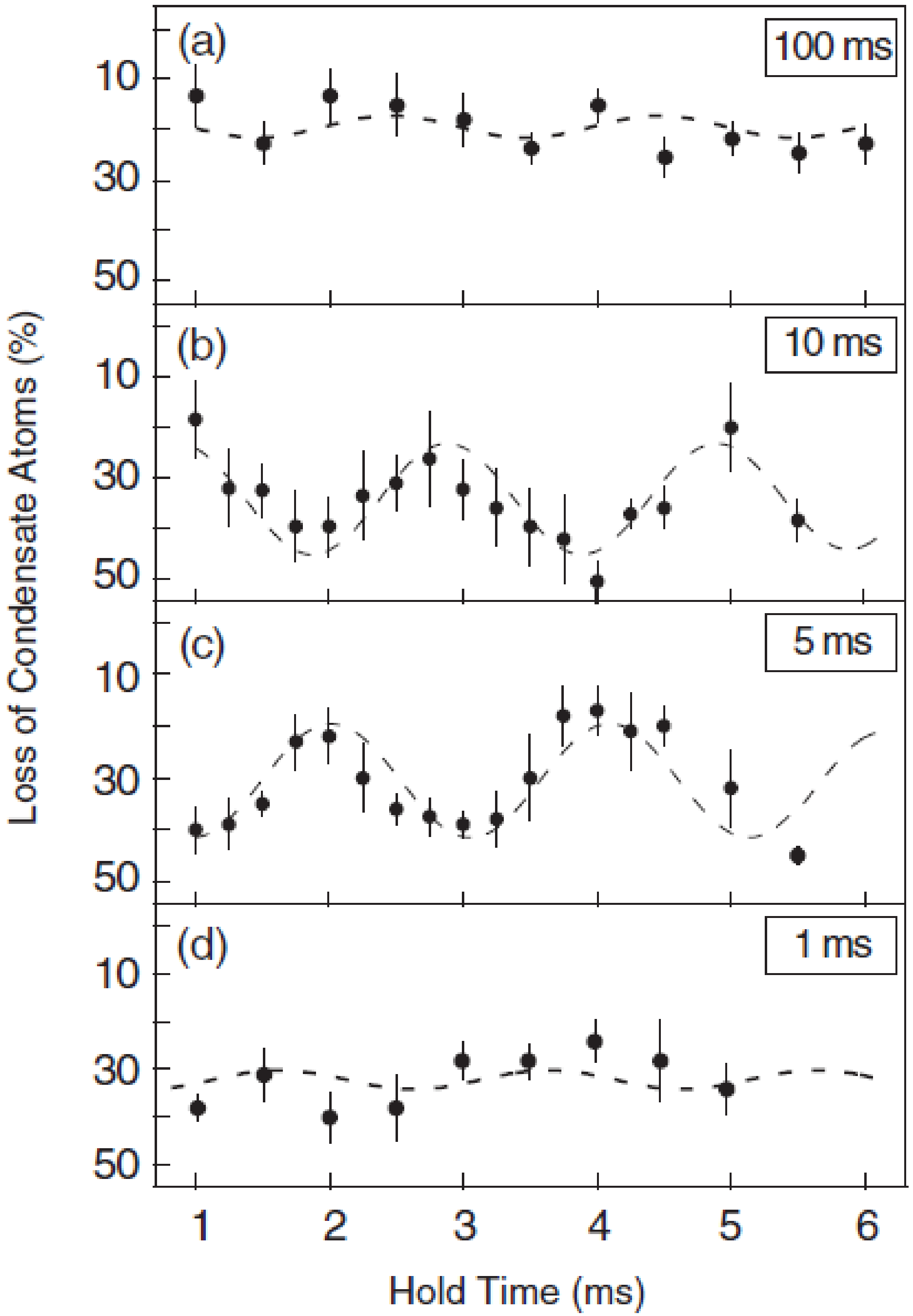


FIG. 3. Oscillations of condensate atom loss after recombination reflecting the coherent phase evolution. The condensate atom loss was monitored during a variable hold time for the two split condensates whose relative phase evolved at ~500 Hz. The merging was done for different values of the recombination time: 100 (a), 10 (b), 5 (c), and 1 ms (d). The dotted lines are sinusoidal curves fitted with fixed frequency ~500 Hz. The reproducible phase shift for the 5 and 10 ms data occurred during the recombination process. The data points represent

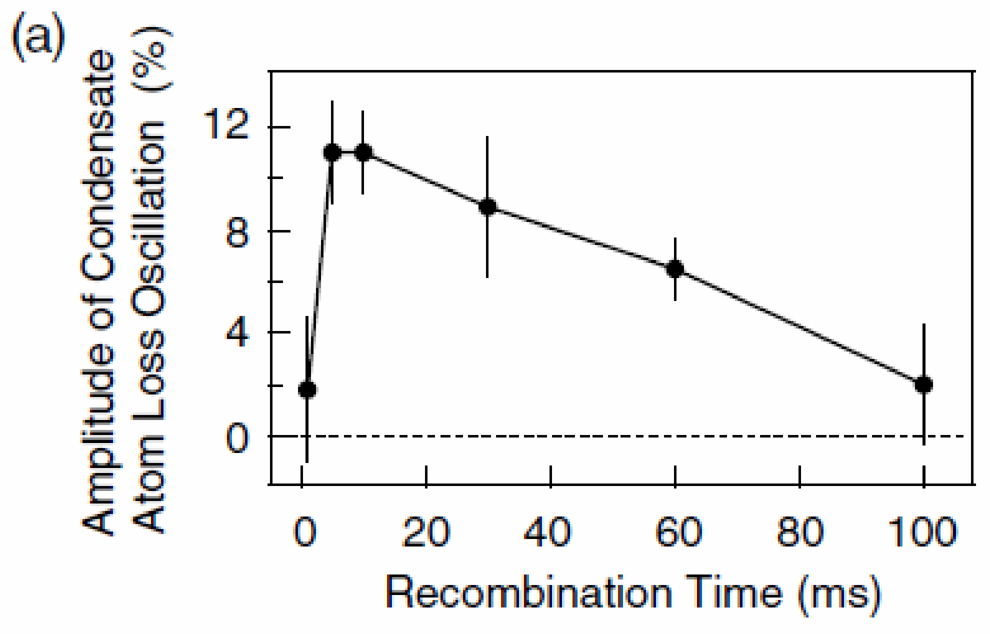
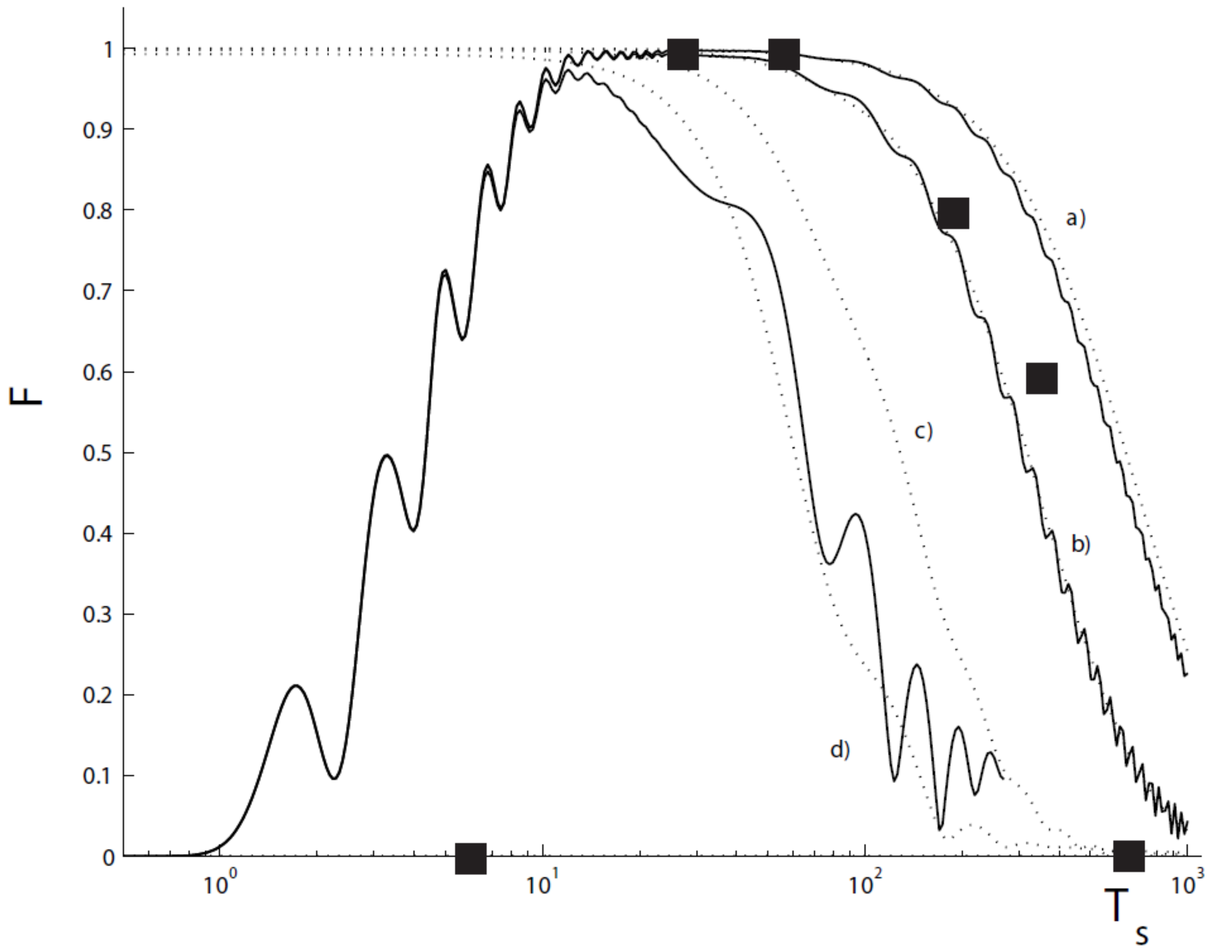


FIG. 4 (color online). Recombination time and atom loss. (a) The amplitude of atom loss oscillations was determined for various recombination times. (b) Assuming that minimum atom

Theory vs experiment



030402 (2007) PHYSICAL REVIEW LETTERS week ending 19 JANUARY 2007

Condensate Splitting in an Asymmetric Double Well for Atom Chip Based Sensors

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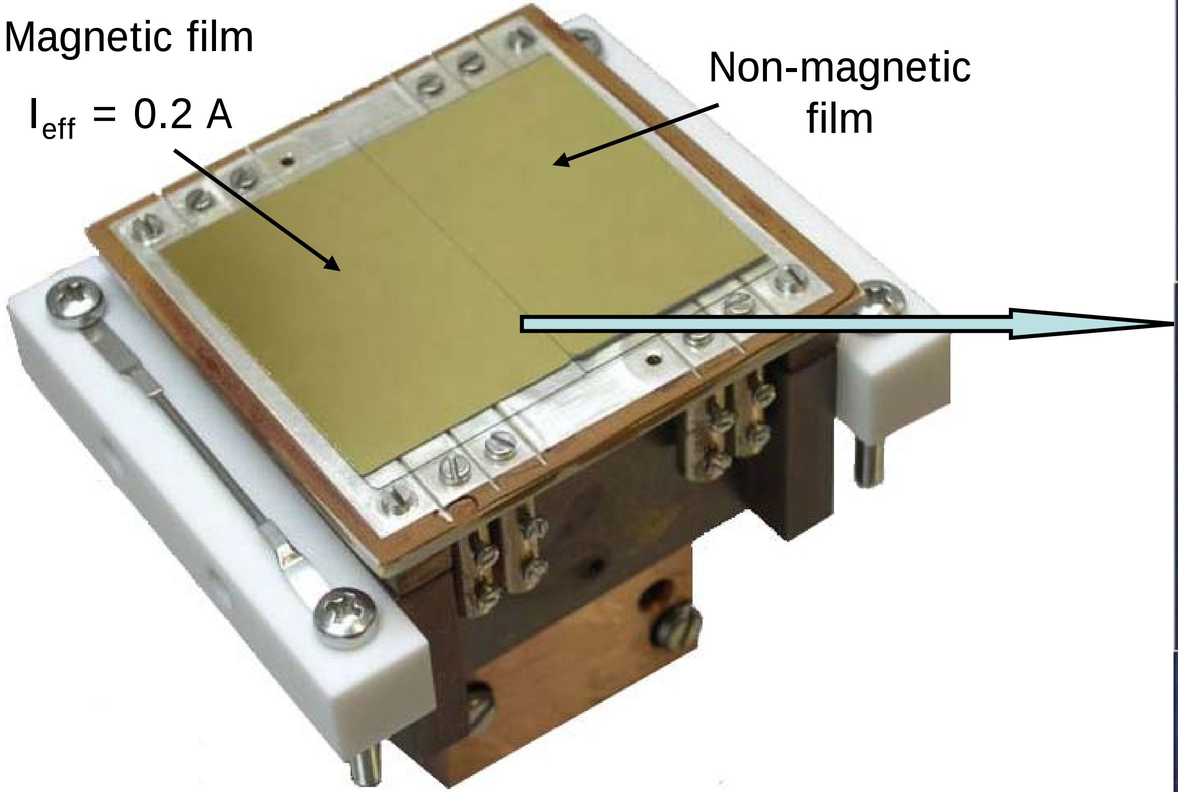
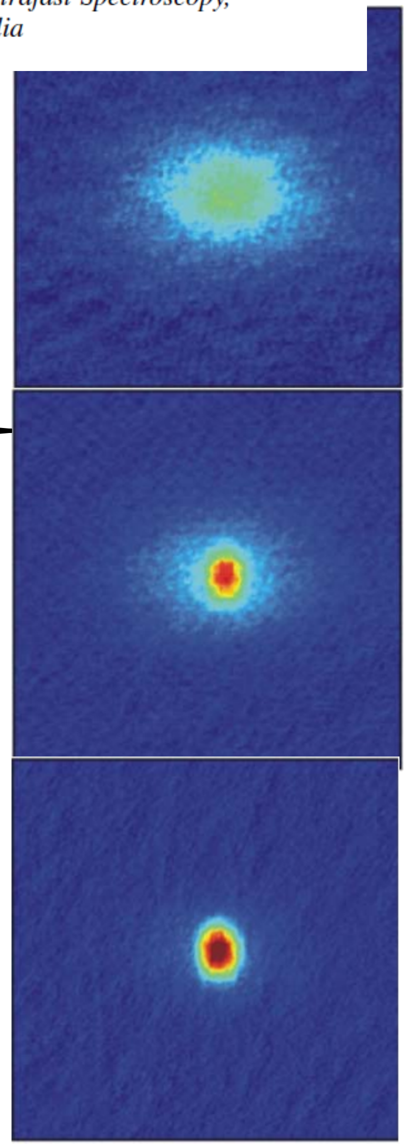
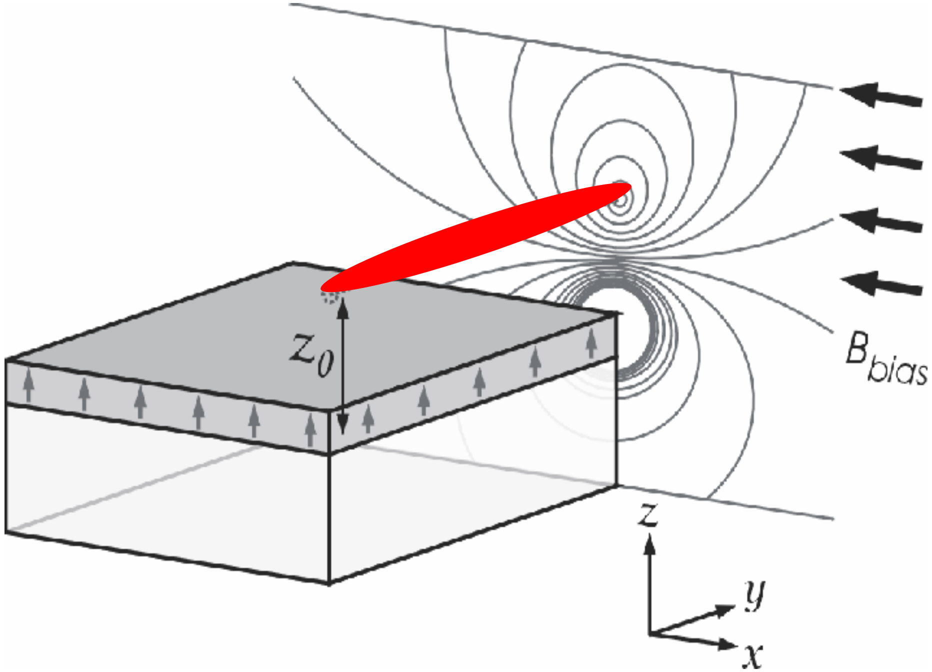
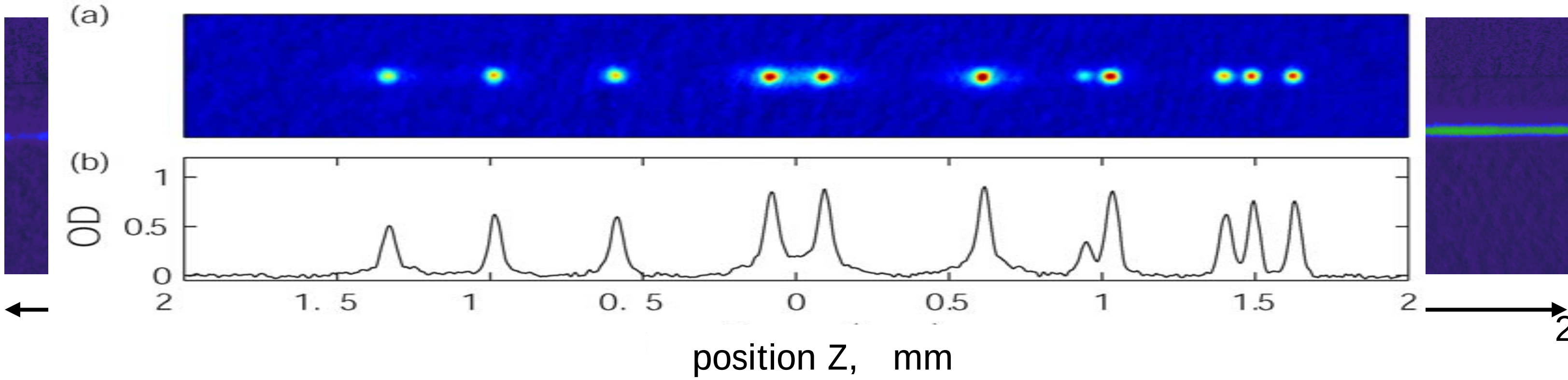




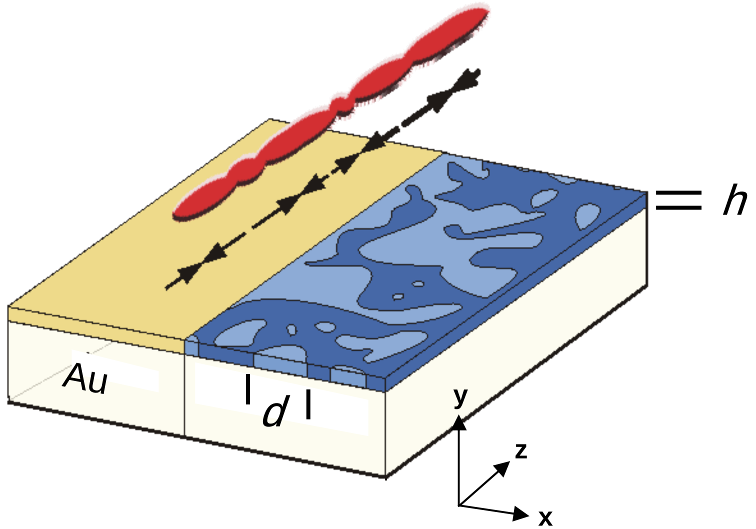
BEC of 2×10^5 ^{87}Rb

Multiple BECs on atom chip





Model of random impurities in magnetization



S. Whitlock et al, Phys. Rev. A **75**, 043602 (2007)

Evaluate some integrals and obtain an expression for the magnetic field *rms* roughness

$$B_{z,rms} = \sqrt{\frac{3}{\pi}} \frac{\mu_0 h \Delta M d}{16 y^2} \sqrt{1 + \frac{15}{8}\alpha - \frac{5}{4}\alpha^3 + \frac{3}{8}\alpha^5},$$

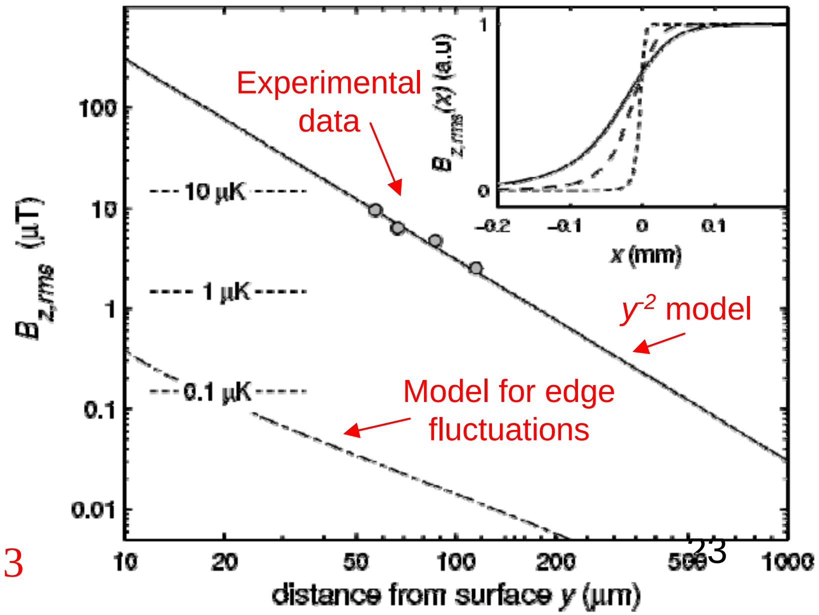
where $\alpha = x/\sqrt{x^2+y^2}$.

and $\Delta M^2 = \left\langle \left(M_y(x,y) - \sqrt{\langle M_y(x,y)^2 \rangle} \right)^2 \right\rangle$

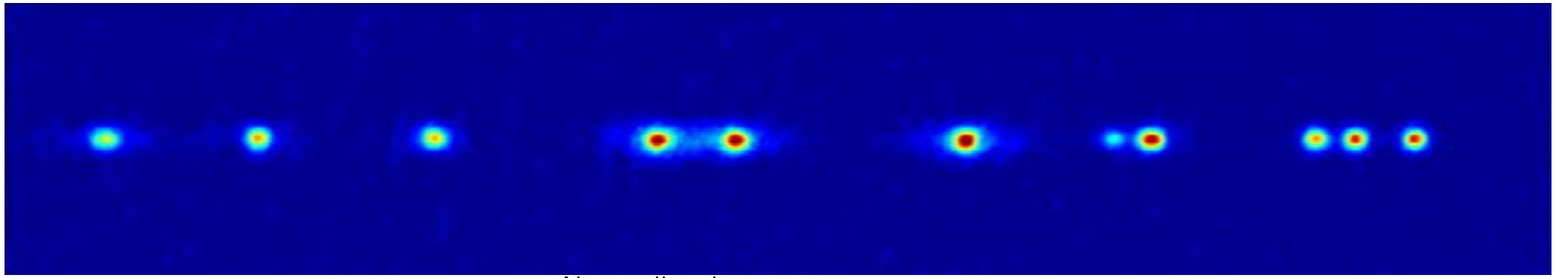
Find for x=0 (at the edge)

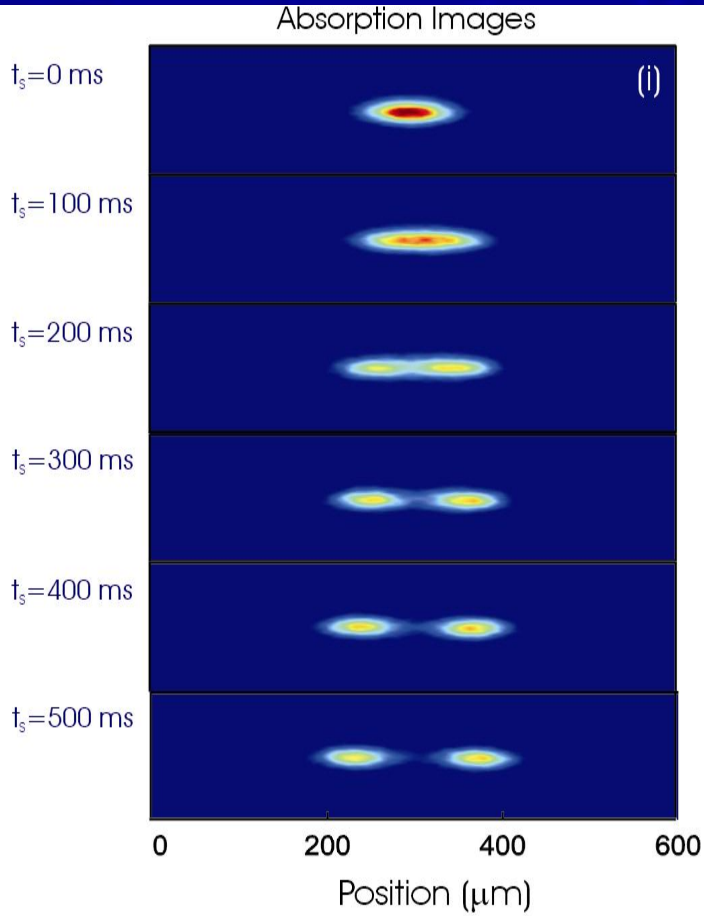
$$B_{z,rms} \propto y^{-2}$$

Experimental result $y^{-1.8\pm0.3}$



BEC in a double-well potential

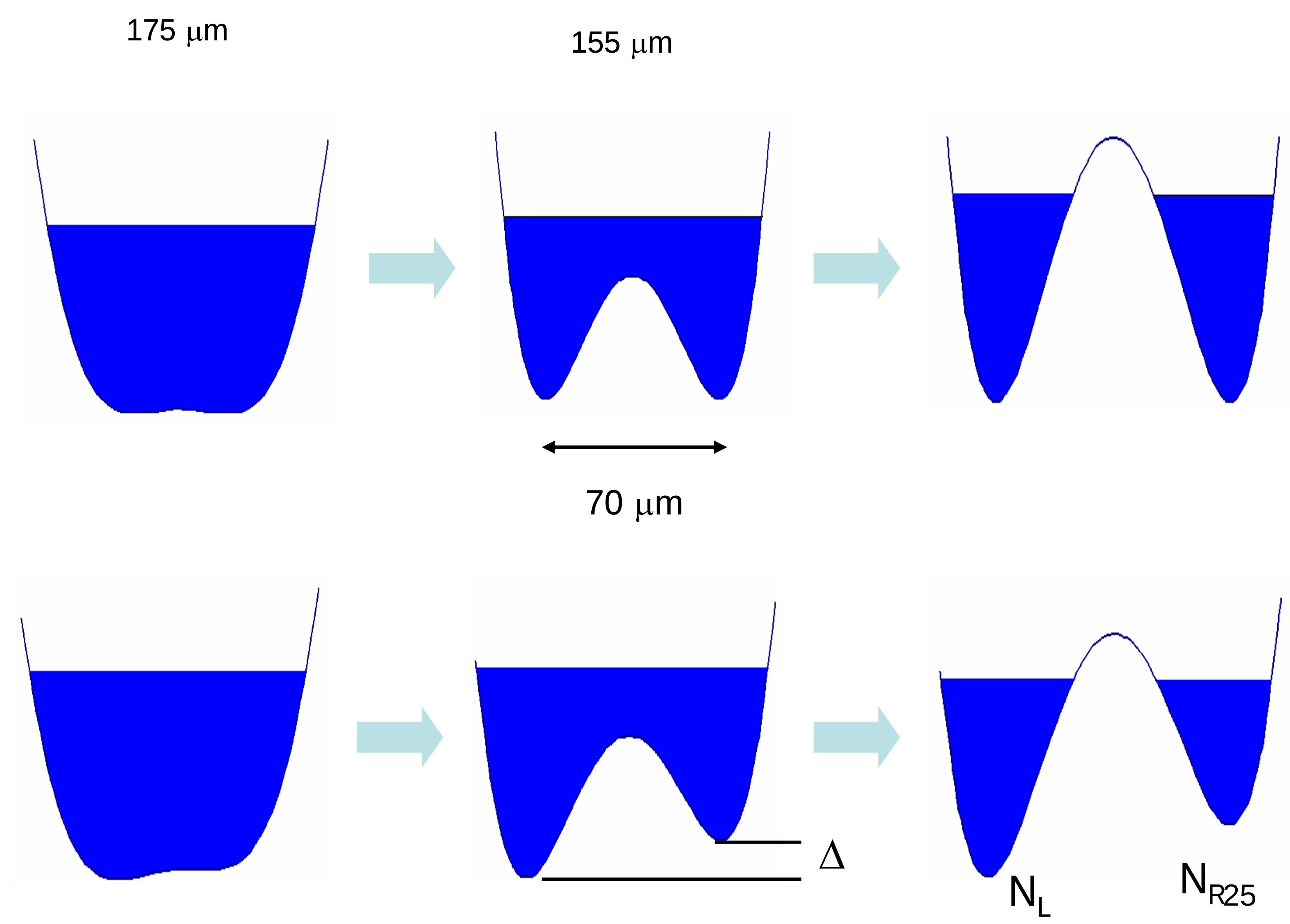




Splitting in a double-well potential

Symmetric double well

Presence of asymmetry



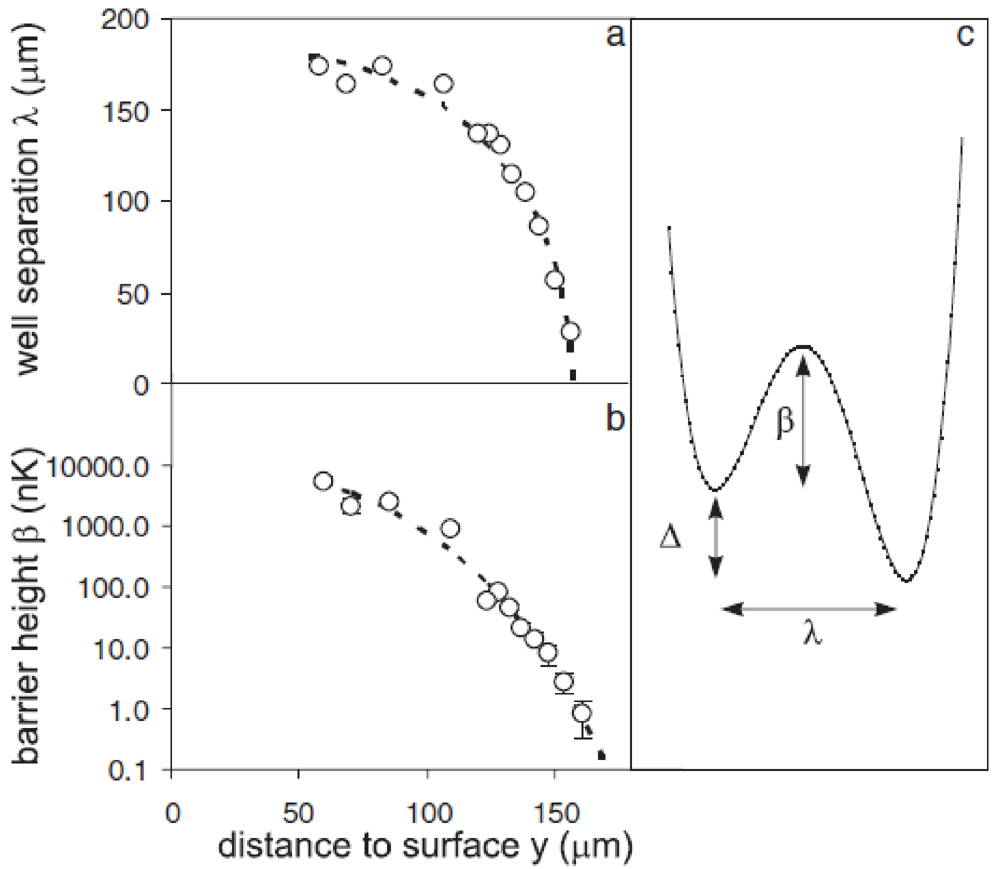


FIG. 2. Characterization of the double well as a function of trap-surface separation is performed using two component clouds. The dashed lines in (a) and (b) are to guide the eye. The well separation λ, barrier height β, and trap asymmetry Δ are shown schematically in (c).

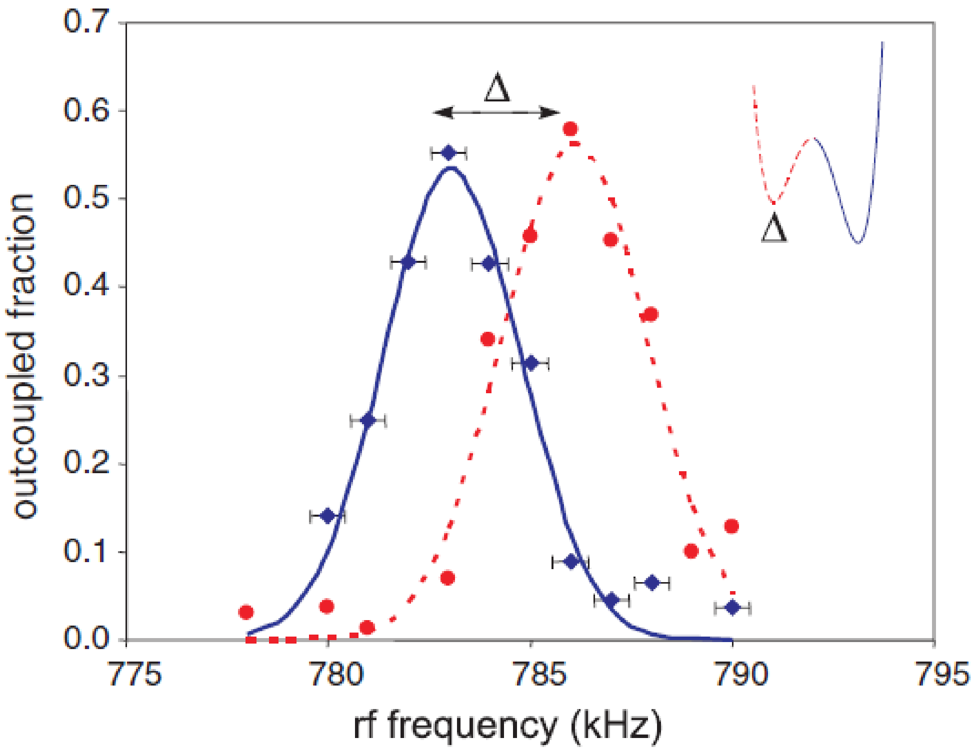
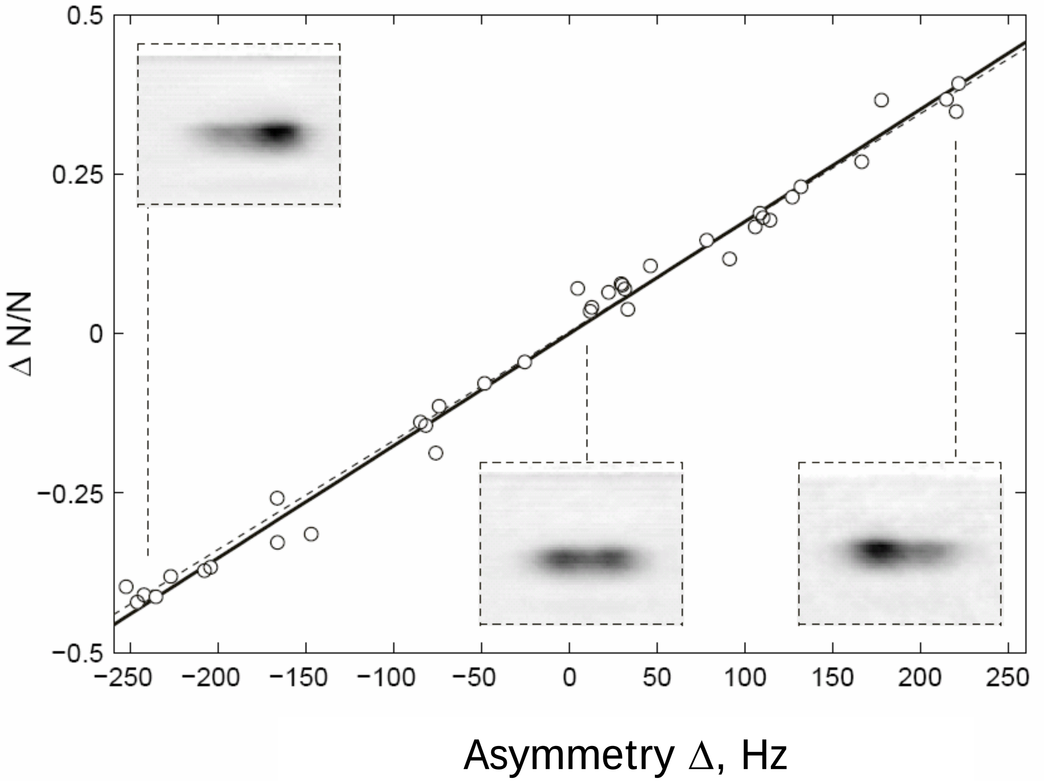


FIG. 3 (color online). Results of pulsed radio-frequency spectroscopy performed on a condensate which has been split symmetrically and then exposed to a large asymmetry. The shift in the output coupled spectra yields a measure of Δ.

Sensitive sensor of asymmetry

$$\frac{\Delta N}{N} = \frac{N_L - N_R}{N_L + N_R}$$



Single shot sensitivity:

16 Hz for 70 μm separation of DW

or 1.5 x 10^{-28} J/m of gradient potential

or $\delta g/g$ = 2 x 10^{-4}

$$\frac{\Delta N}{N} \approx 1.65 \frac{\Delta}{cN^{2/5}}, \qquad c = \frac{\bar{\omega}}{4\pi}\left(\frac{15a}{a_0}\right)^{2/5}$$

B.V. Hall et al, Phys. Rev. Lett. **98**, 030402 (2007)

Similar observations in Gati & Oberthaler, J. Phys B (2007)

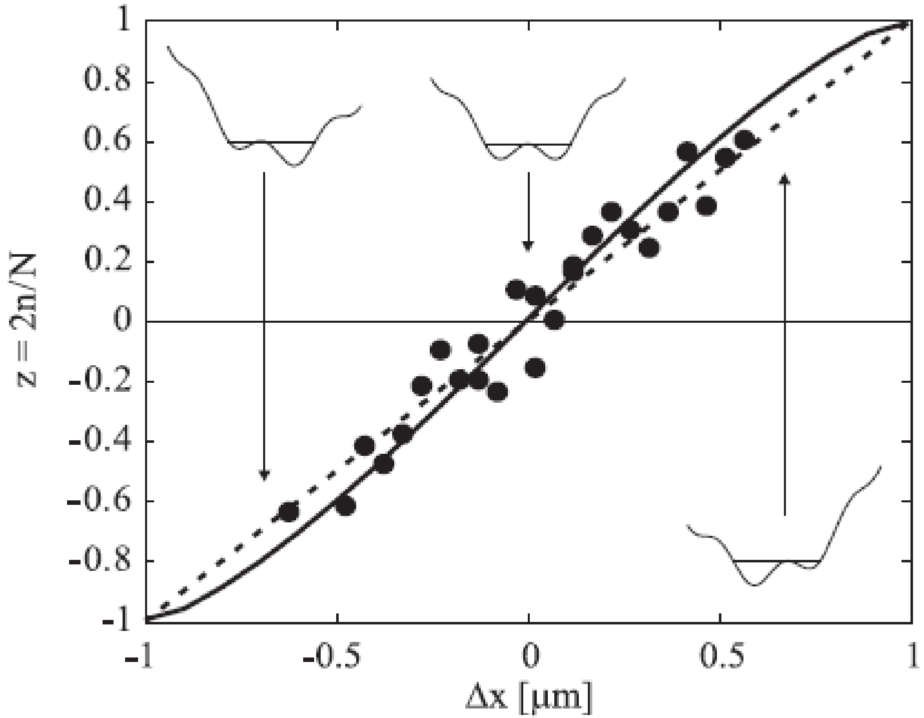


Figure 6. Steady state population imbalance as a function of the shift of the harmonic trapping potential. The solid line is the solution of the 3D Gross–Pitaevskii equation and the dashed line is the prediction of the Bose–Hubbard model.

Summary of Lecture 1

1. Josephson effect can be studied in any two coupled, phase coherent quantum systems
2. Oscillatory exchange of atoms between two non-interacting BECs
3. Double-well interferometer for non-interacting BECs
4. Two-mode approximation and the Bloch vector model
5. Loss of coherence at short and long splitting times
6. Adiabatic splitting of two wells and measure of asymmetric potentials