Victorian Summer School on Ultracold Atoms

Bosonic Josephson Junction

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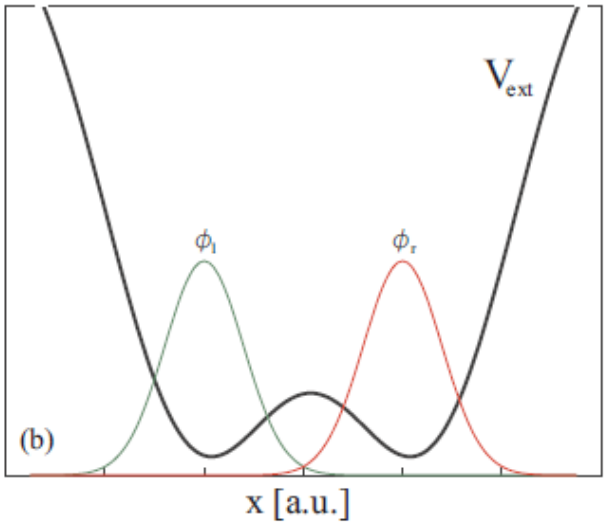
Outline

1. *Two-mode model for interacting BECs*
2. *Josephson oscillations and self-trapping effect*
3. *Oberthaler's experiment*
4. *Two-component BECs*
5. *Spatial evolution of a relative phase*
6. *Self-rephasing and long coherence time*

Interacting BEC in a Double-Well Potential



$$V_{dw}(x) = \frac{1}{2} m\omega^2 x^2 + V_b e^{-x^2/x_0^2}$$

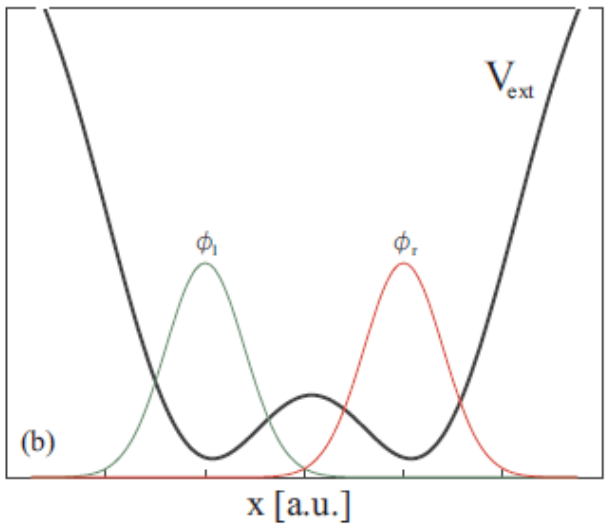
$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

$$\hat{H}_0 = \int dx \left(-\frac{\hbar^2}{2m} \hat{\Psi}^+ \nabla^2 \hat{\Psi} + \hat{\Psi}^+ V_{dw} \hat{\Psi} \right)$$

$$\hat{H}_{\text{int}} = \frac{g}{2} \int dx (\hat{\Psi}^+ \hat{\Psi}^+ \hat{\Psi} \hat{\Psi}) \qquad g = \frac{4\pi\hbar^2 a}{m}$$

Two-mode approximation (following Gati and Oberthaler, J Physics B, 40, R61 (2007)

$$\hat{\Psi} = \hat{a}_L \Phi_L + \hat{a}_R \Phi_R = \frac{1}{\sqrt{2}}\left[\hat{a}_L\left(\Phi_g + \Phi_e\right) + \hat{a}_R\left(\Phi_g - \Phi_e\right)\right]$$



$$\boxed{\hat{H}_{2m} = \frac{E_c}{2}\hat{n}^2 - E_j\hat{\alpha}}$$

where

$$\hat{n} = \frac{\hat{a}_L^+\hat{a}_L - \hat{a}_R^+\hat{a}_R}{2}$$

population imbalance operator

$$\hat{\alpha} = \frac{\hat{a}_R^+\hat{a}_L + \hat{a}_L^+\hat{a}_R}{N}$$

tunnelling operator

$$E_c = 4g\int dx \left|\Phi_g\right|^2 \left|\Phi_e\right|^2$$

local interaction energy

$$E_j = \frac{N}{2}\left(\mu_e - \mu_g\right) - \frac{N(N-1)}{2}\left(\kappa_{e,e} - \kappa_{g,g}\right)$$

tunnelling energy

$$\mu_{g,e} = \int dx\left(-\frac{\hbar^2}{2m}\Phi_{g,e}\nabla^2\Phi_{g,e} + \Phi_{g,e}\left(V_{dw} + gN\left|\Phi_{g,e}\right|^2\right)\Phi_{g,e}\right)$$

$$\kappa_{i,j} = \frac{g}{2}\int dx \left|\Phi_i\right|^2 \left|\Phi_j\right|^2$$

Two-mode mean field model

The wavefunction $$\Psi(x,t) = \sqrt{N_L(t)}e^{i\phi_L}\Phi_L(x) + \sqrt{N_R(t)}e^{i\phi_R}\Phi_R(x)$$

$N_{L,R}$ populations of L (R) wells

$\phi_{L,R}$ phases of L (R) wavefunctions

We introduce: $$n = \frac{N_R - N_L}{2}$$ population imbalance

$$\phi = \phi_R - \phi_L$$ relative phase

Can evaluate the Hamilton function $$H_{2m} = \frac{E_c}{2}n^2 - E_j\sqrt{1-\frac{4n^2}{N^2}}\cos\phi$$

The equations of motion $$\frac{dn}{dt} = -\frac{1}{\hbar}\frac{\partial H_{2m}}{\partial\phi} = -\frac{E_j}{\hbar}\sqrt{1-\frac{4n^2}{N^2}}\sin\phi$$

$$\frac{d\phi}{dt} = \frac{1}{\hbar}\frac{\partial H_{2m}}{\partial n} = \frac{E_c}{\hbar}n + \frac{E_j}{\hbar}\frac{4n}{N^2\sqrt{1-4n^2/N^2}}\cos\phi$$

For $n = 0$ & $\phi = 2\pi$ the system is in the ground state and does not change

If $n \neq 0$ or $\phi \neq 2\pi$ current will start to flow through the barrier

1) For small initial amplitudes $n_0^2 << N^2$ and $\sin\phi_0 \approx \phi_0$ the equations of motion

$$\frac{dn}{dt} = -\frac{E_j}{\hbar}\phi \qquad \frac{d\phi}{dt} = \left(\frac{E_c}{\hbar} + \frac{4E_j}{\hbar N^2}\right)n$$

The solution is harmonic (Josephson) oscillations of the population imbalance

$$n(t) = n_* \sin\left(\omega_p t + \theta_{in}\right) \qquad \omega_p = \frac{1}{\hbar}\sqrt{E_j\left(E_c + \frac{4E_j}{N^2}\right)}$$

2) Self-trapping effect: if the initial population imbalance is high to make

$$H_{2m}\left(n_c, \phi = 0\right) = H_{2m}\left(0, \phi = \pi\right) \qquad H_{2m} = \frac{E_c}{2}n^2 - E_j\sqrt{1 - \frac{4n^2}{N^2}}\cos\phi$$

$$\left|n_c\right| = 2\sqrt{\frac{E_j}{E_c}\left(1 - \frac{4E_j}{N^2 E_c}\right)}$$

Experiment on Josephson oscillations (Rudolf Gati, PhD thesis 2007)

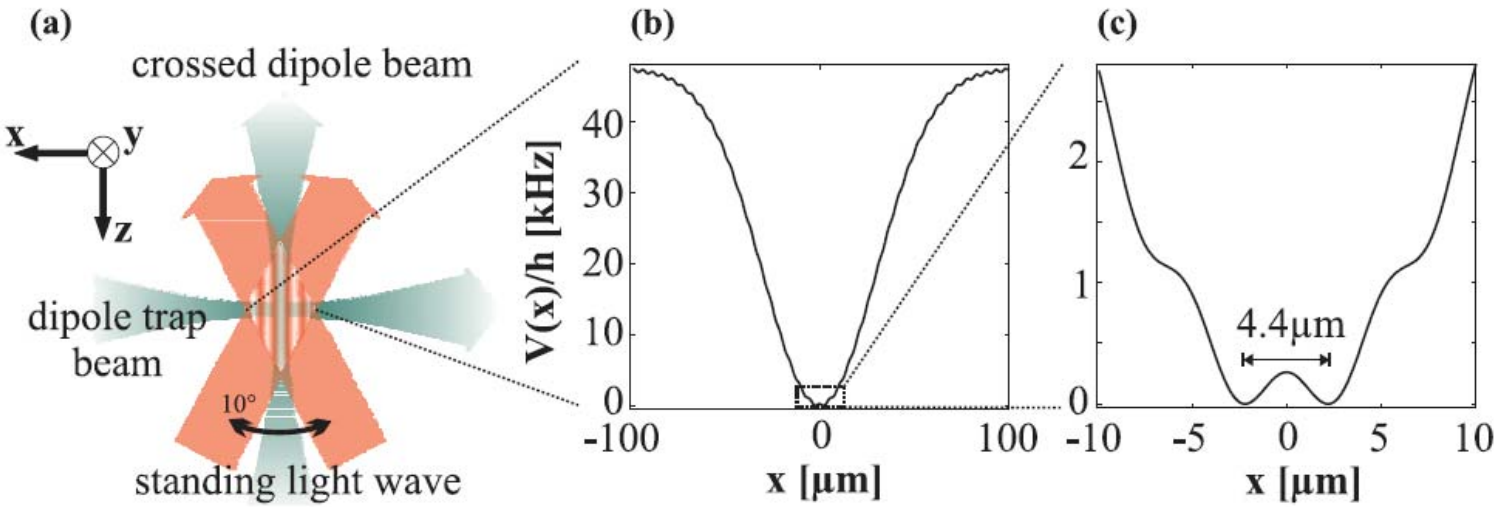


Figure 3.1: Experimental setup and realization of the double-well potential by the superposition of a harmonic trap and an optical lattice with large periodicity. (a) is a cut through the center of the laser beams generating the optical potentials. Two orthogonal dipole trap beams at 1064nm (gray) create a 3-D harmonic confinement and two laser beams at 830nm crossing under an angle of about 10° generate the optical lattice (red) with a periodicity of $\lambda \approx 5\ \mu$m. (b) shows the effective potential resulting from the superposition of the dipole trap and the optical lattice on the scale of the Gaussian dipole trap beam. (c) is a zoom onto the potential in the center. It reveals that the potential can effectively be described as a double-well potential with a separation of the two wells of about 4.4 μm,

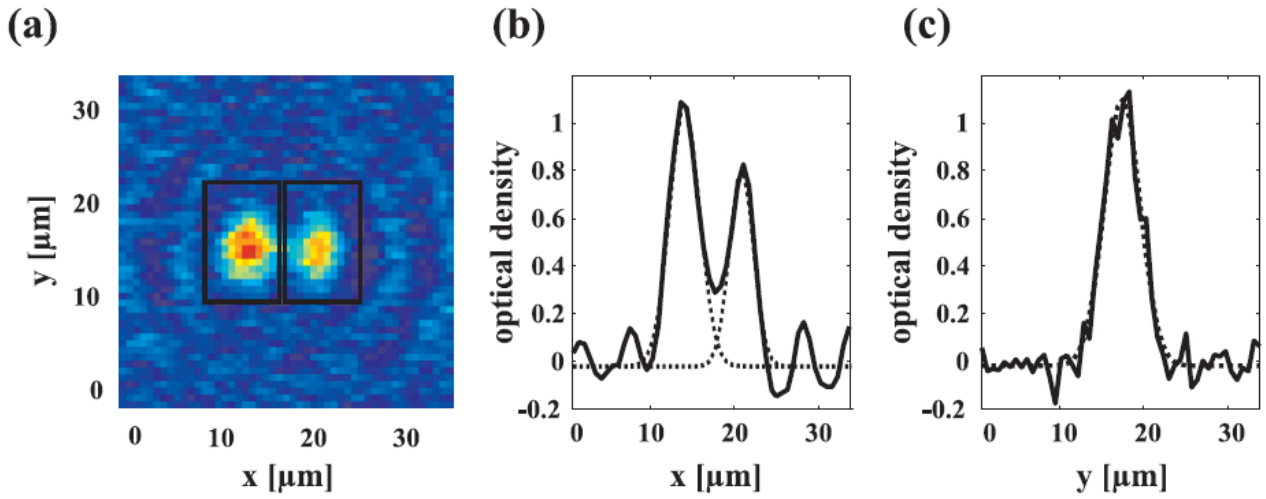


Figure 3.15: Measurement of the population imbalance. In (a) absorption image of the BEC in the double-well potential are shown. Here, the distance of the two matter wave packets is increased by

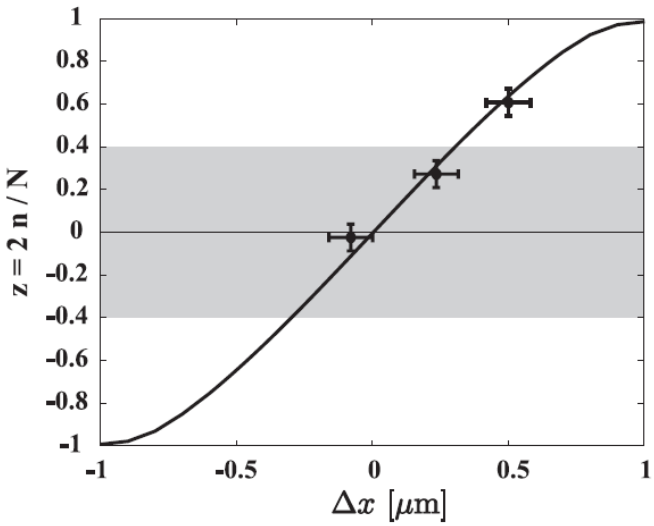


Figure 5.4: Initial population imbalance as a function of the shift of the harmonic trapping potential. The three data points correspond to the initial population imbalances in the symmetric double-well, in the plasma-oscillation regime and in the self-trapping regime. The solid line is a prediction resulting from the numerical solution of the 3-D Gross-Pitaevskii equation. The gray shaded area shows the plasma-oscillation regime, below and above there is the self-trapping regime.

R. Gati, 2007

R. Gati, 2007

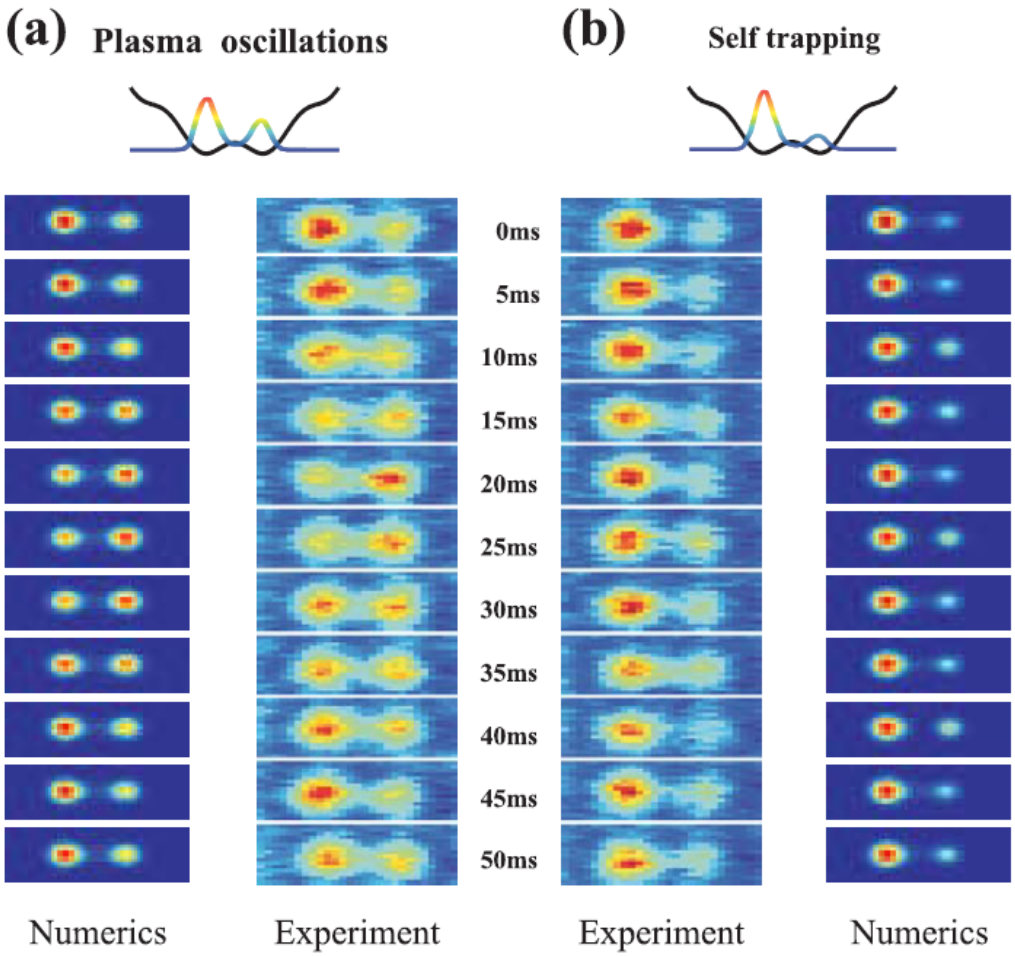


Figure 5.5: Tunneling dynamics in the double-well trap. The temporal evolution of the density distribution in the double-well trap is shown for a small initial imbalance (a) and a large initial imbalance (b). For an imbalance below the critical value for self trapping the BJJ is in the plasma-oscillation regime and shows a tunneling dynamics of atoms from the left well to the right and back. If

Number fluctuations and coherence

$$\hat{H}_{2m} = \frac{E_c}{2}\hat{n}^2 - E_j\hat{\alpha}$$

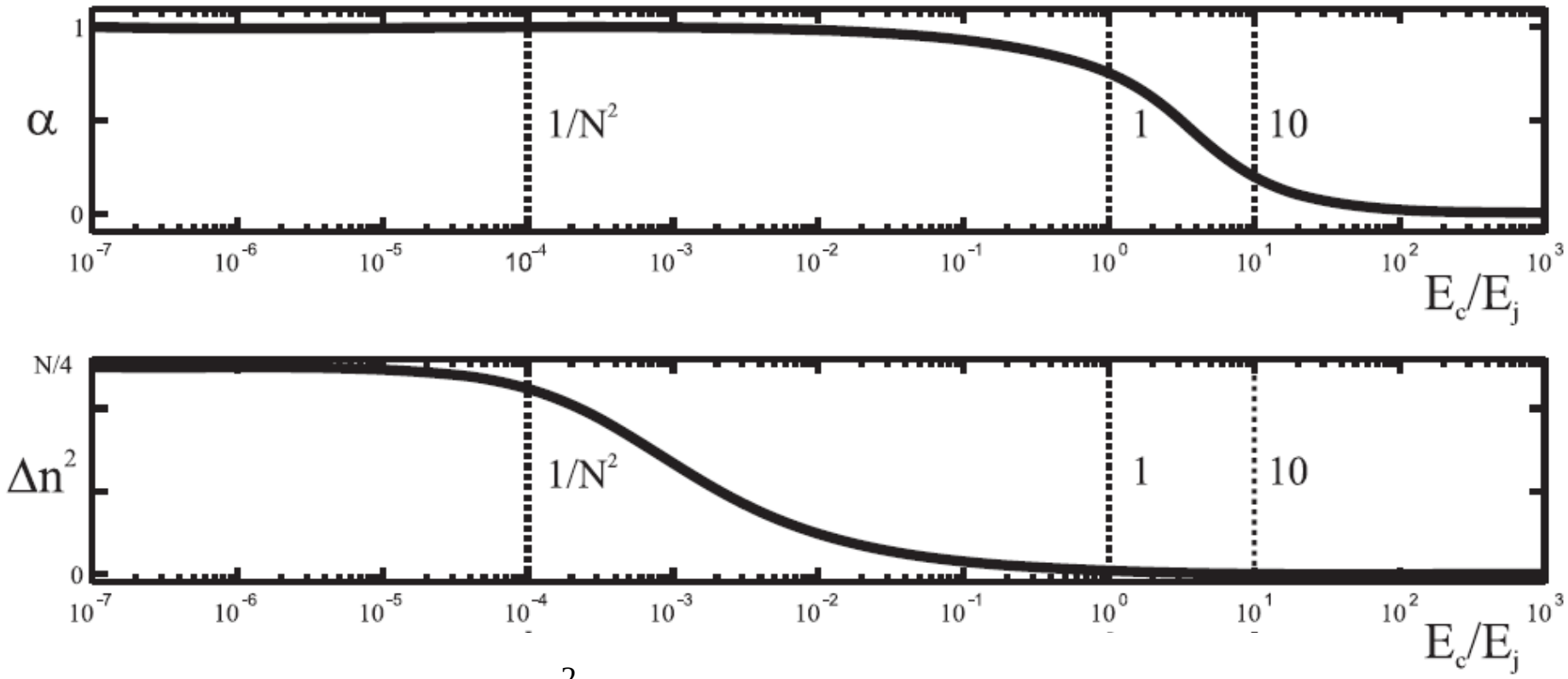
Fluctuations of the atom number difference

$$\Delta n^2 = \left\langle \hat{n}^2 \right\rangle - \left\langle \hat{n} \right\rangle^2 = \left\langle \frac{(\hat{a}_R^+\hat{a}_R - \hat{a}_L^+\hat{a}_L)2}{4} \right\rangle - \left\langle \frac{\hat{a}_R^+\hat{a}_R - \hat{a}_L^+\hat{a}_L}{2} \right\rangle^2$$

Coherence

$$\alpha = g^{(1)}(x,x') = \frac{\left\langle \hat{a}_L^+\hat{a}_R + \hat{a}_R^+\hat{a}_L \right\rangle}{N} = \left\langle \hat{\alpha} \right\rangle$$

Rabi, Josephson and Fock regimes



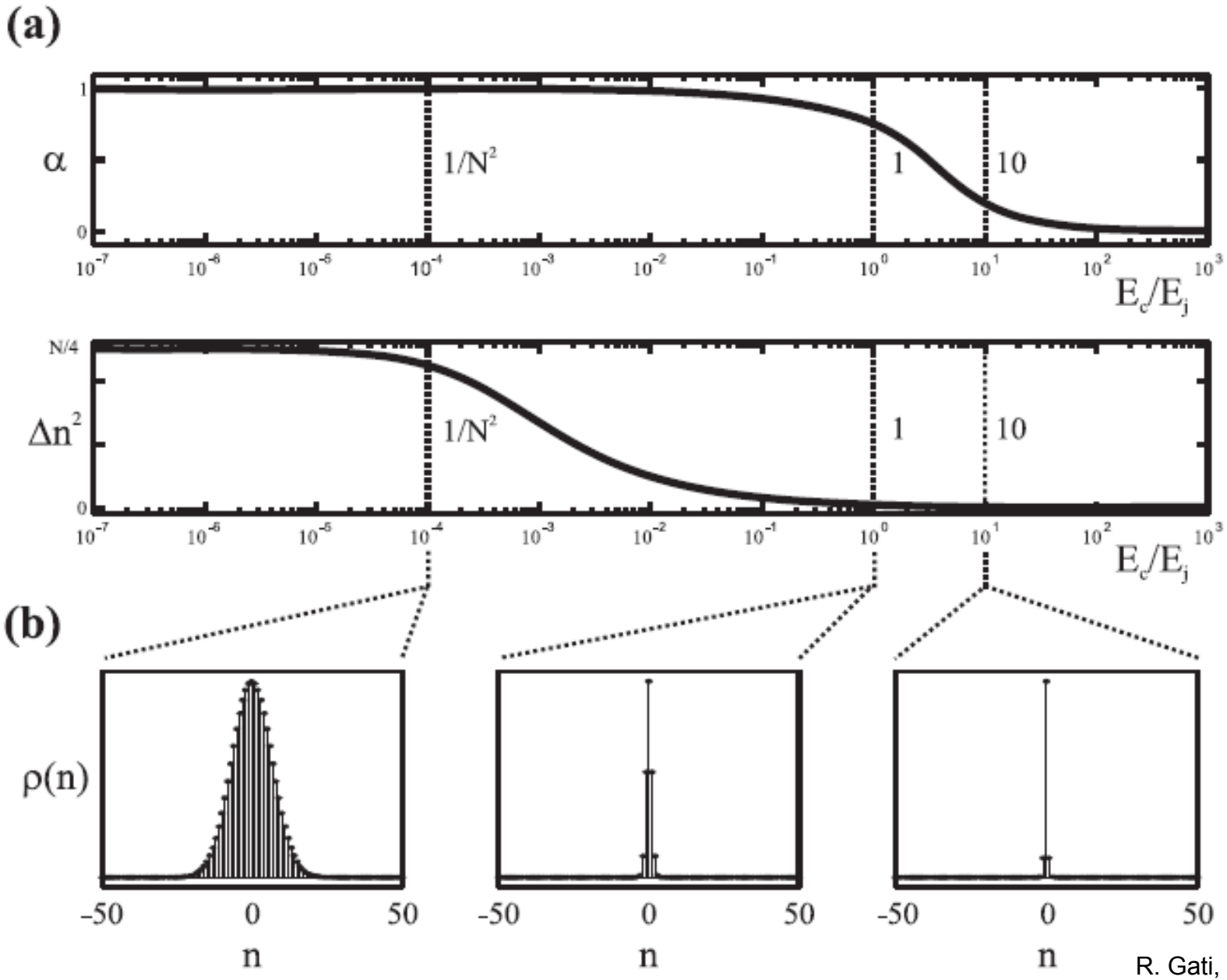
Rabi: $E_c / E_j \quad << \quad N^{-2}$

R. Gati, 2007

Josephson: $N^{-2} \quad << \quad E_c / E_j \quad << \quad 1$

Fock: $E_c / E_j \quad >> \quad 1$

Rabi, Josephson and Fock regimes



R. Gati, 2007

Rabi, Josephson and Fock regimes

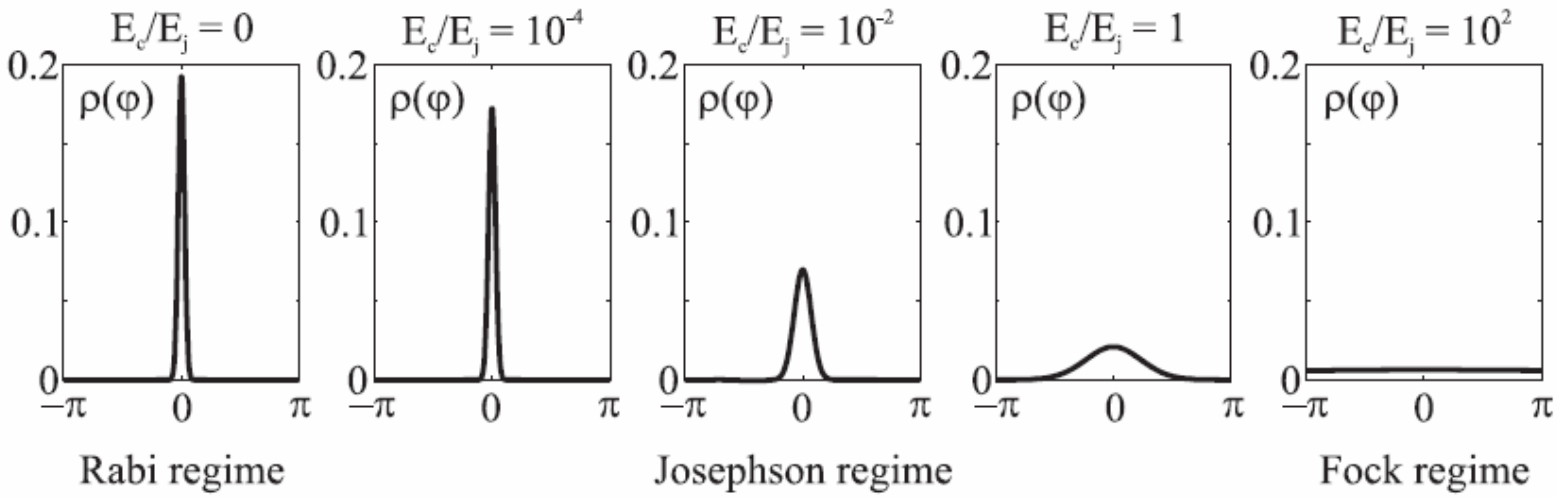


Figure 2.4: Distribution of the relative phases for different ratio of E_c/E_j for $N = 100$. The five graphs show typical distribution of the relative phase for the three regimes, the Rabi regime on the left, the Josephson regime in the center and the Fock regime on the right. In the Rabi regime the width of the distribution is only determined by the number of particles. In the Josephson regime also the ratio of the interaction energy and the coupling strength plays a role. In the Fock regime the relative phase becomes random.

R. Gati, 2007

Measurements of relative phase

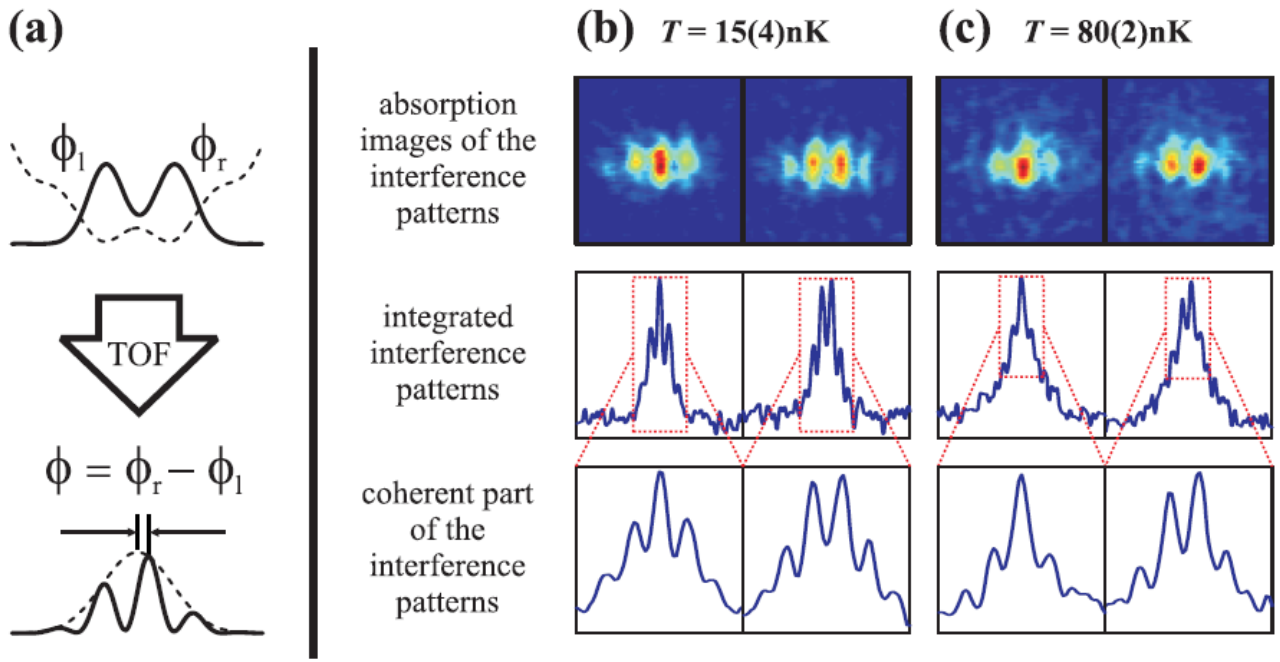


Figure 3.16: matter-wave interference patterns. (a) is a sketch of the interference experiments. Once the double-well trap is turned off, the matter-wave packets expand, overlap and interfere revealing the relative phase as a shift of the interference peaks with respect to their envelope. (b) corresponds

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Phase fluctuations

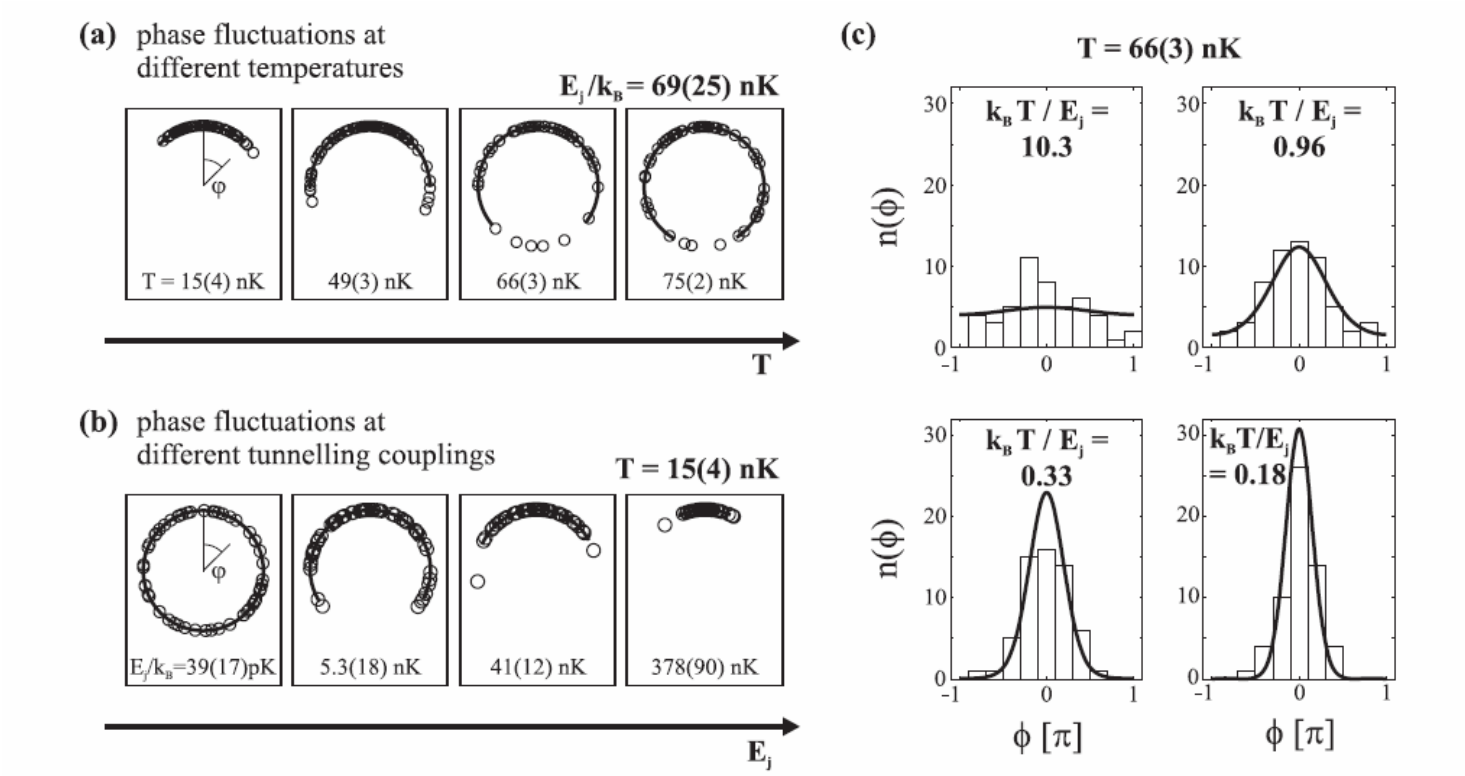

Figure 4.2: Thermally induced fluctuations. (a) shows polar plots of relative phase measurements for a fixed barrier height leading to a constant tunneling coupling and for different temperatures. Every open circle corresponds to a single realization of the interference measurement. The solid lines indicate twice the standard deviation of the phase in both directions. The amount of fluctuations increases with temperature. (b) corresponds to similar experiments, but here the temperature is fixed and the barrier height varied, in order to realize different tunneling couplings. The fluctuations decrease with the tunneling coupling. (c) shows four typical distribution functions in form of histograms for different

R. Gati, 2007

PHYSICAL REVIEW A VOLUME 59, NUMBER 1 JANUARY 1999

Nonlinear Josephson-type oscillations of a driven, two-component Bose-Einstein condensate

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(Received 10 September 1998)

We propose an experiment that would demonstrate nonlinear Josephson-type oscillations in the relative population of a driven, two-component Bose-Einstein condensate. An initial state is prepared in which two condensates exist in a magnetic trap, each in a different hyperfine state, where the initial populations and relative phase between condensates can be controlled within experimental uncertainty. A weak driving field is then applied, which couples the two internal states of the atom and consequently transfers atoms back and forth between condensates. We present a model of this system and investigate the effect of the mean field on the dynamical evolution. [S1050-2947(99)50801-9]

2CBEC:

$$\frac{dn}{dt} = -\kappa\sqrt{1-\frac{4n^2}{N^2}}\sin\phi$$

$$\frac{d\phi}{dt} = -\left[(\mu_2-\mu_1)-\delta\right]+\kappa\frac{4n/N^2}{\sqrt{1-\frac{4n^2}{N^2}}}\cos\phi$$

$$\kappa = 2\Omega\int dx\Phi_2(x)\Phi_1(x)$$

DW system:

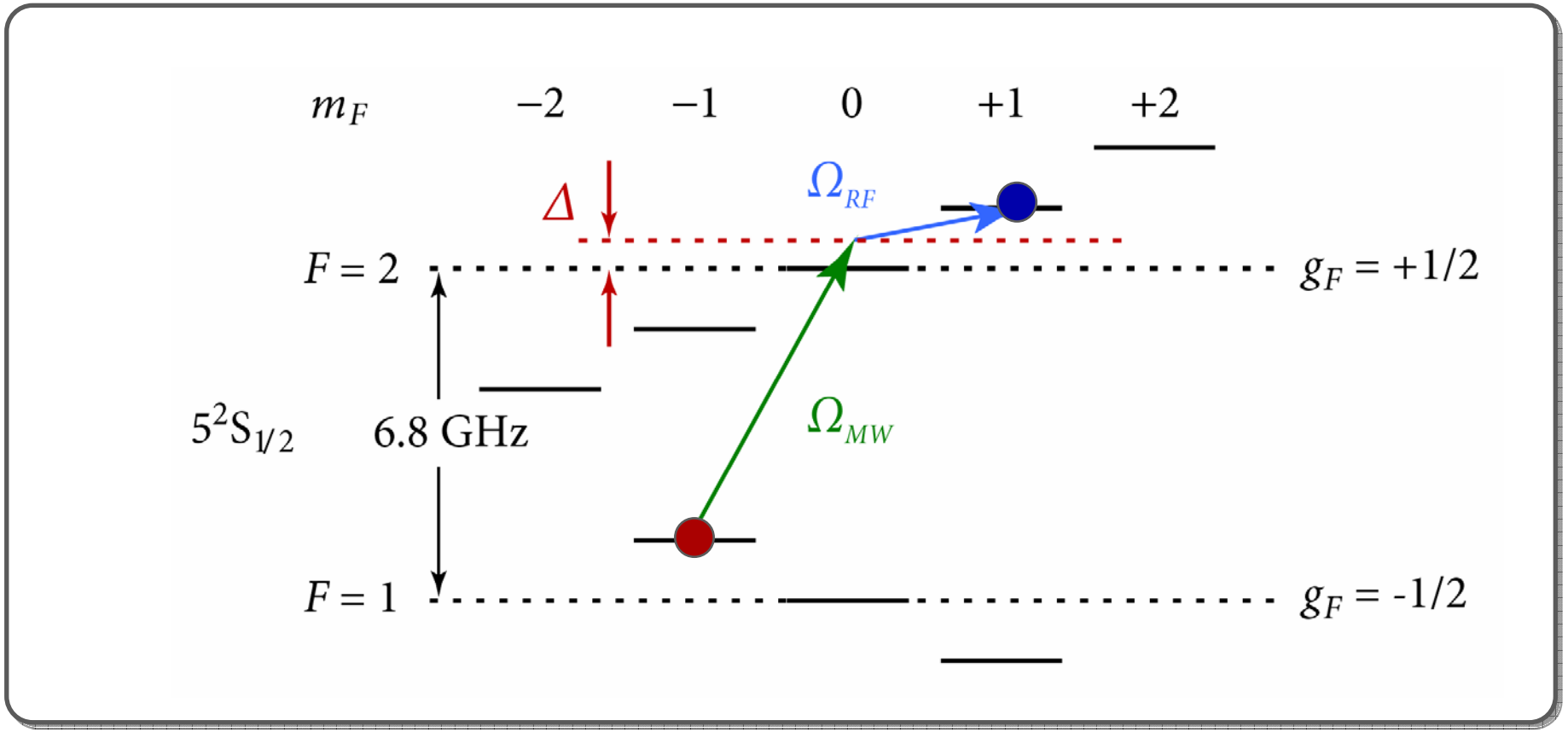
$$\frac{dn}{dt} = -\frac{E_j}{\hbar}\sqrt{1-\frac{4n^2}{N^2}}\sin\phi$$

$$\frac{d\phi}{dt} = \frac{E_c}{\hbar}n+\frac{E_j}{\hbar}\frac{4n/N^2}{\sqrt{1-\frac{4n^2}{N^2}}}\cos\phi$$

of the condensates and so also varies slowly in time. These are nonlinear versions of the usual Josephson-junction equations [9] and are nearly identical in form to those obtained in Refs. [16] and [17] describing the double-well tunneling

tion region of the barrier. In contrast, the interaction between condensates due to their significant overlap plays an important role in the evolution of the system described in this paper. In particular, it is this mutual interaction that causes the system to move out of resonance.

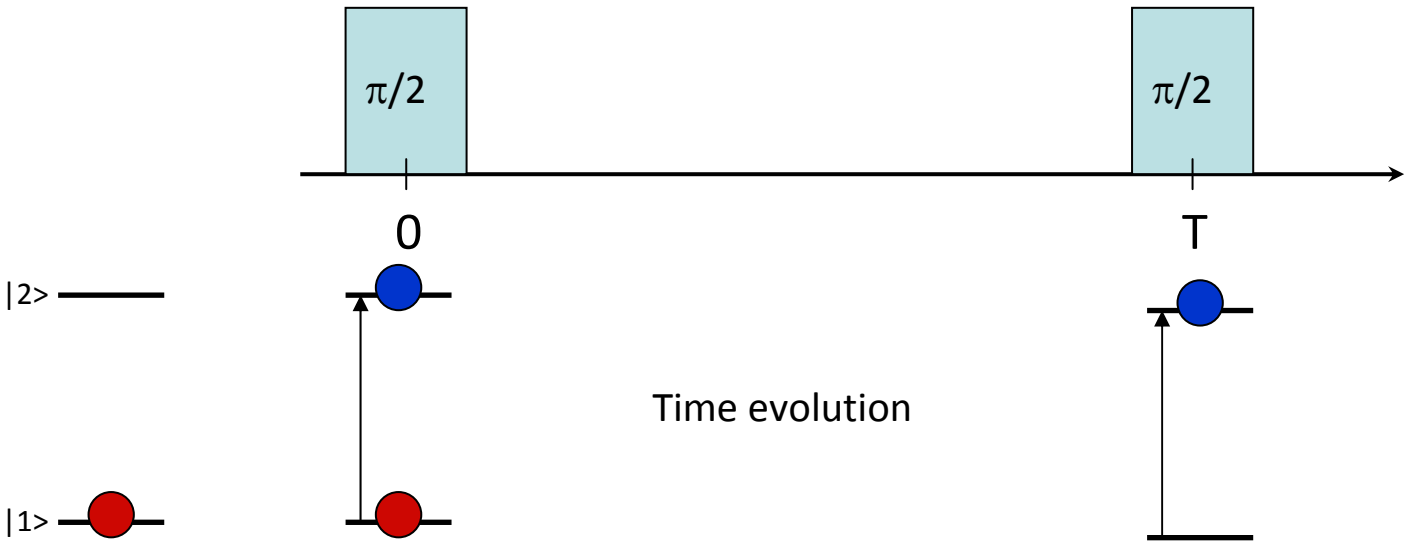
Two-Component BEC in Rubidium-87



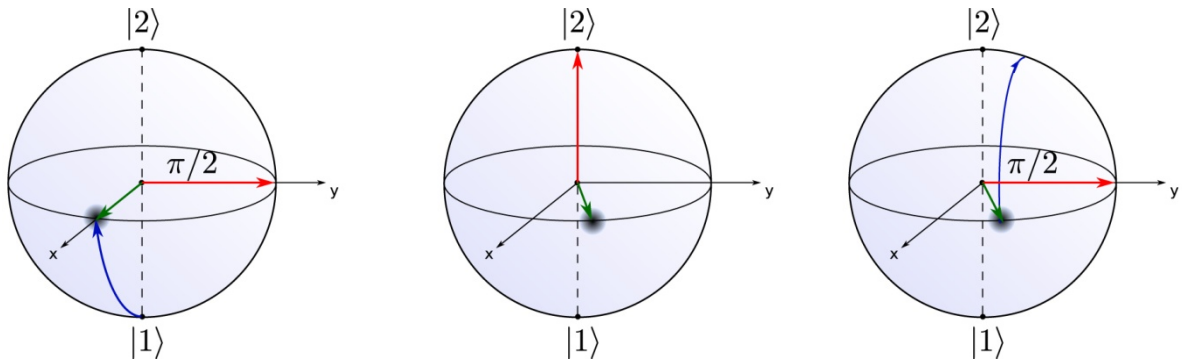
Interactions include collisions between atoms in:

states |1> and |1> (scattering length a_{11}),
states |2> and |2> (scattering length a_{22}) and
states |1> and |2> (scattering length a_{12}).

Ramsey method of separated oscillatory fields



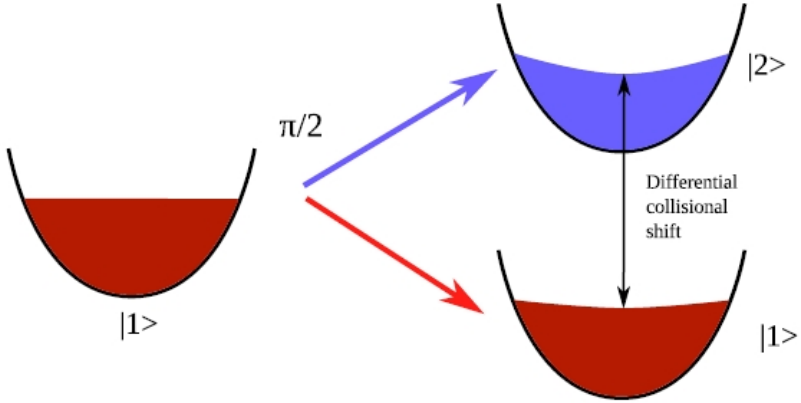
Bloch vector evolution



measure $P_z = \frac{N_1 - N_2}{N_1 + N_2} = V(T)\cos(\Delta T + \phi(T))$

Nonlinear Evolution of Amplitude and Phase

- Spatially dependent chemical potentials:
  - change in spatial dependence of amplitude
  - phases of both states will develop spatial dependence
  - three s-wave scattering lengths determine dynamics of the system



$$\delta\mu_2 = \mu_2 - \mu_0 = \frac{4\pi\hbar^2}{2m}(a_{22} + a_{12} - 2a_{11})n_0(x)$$

$$\delta\mu_1 = \mu_1 - \mu_0 = \frac{4\pi\hbar^2}{2m}(a_{12} - a_{11})n_0(x)$$

$$|\Psi(\boldsymbol{r},0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\sqrt{n_0(\boldsymbol{r})} \qquad |\Psi(\boldsymbol{r},t_{\pi/2})\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}\sqrt{n_0(\boldsymbol{r})}$$

$$\mu_1 - \mu_2 \quad = \quad 0.0247 \times g_{11} n_{01}(z) \quad \approx h \times 17\,Hz$$

in the centre for 50,000 atoms

Simulations of Nonlinear Dynamics

Set of two coupled Gross-Pitaevskii equations

$$i\hbar\frac{\partial\Psi_1}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + V_1 + g_{11}|\Psi_1|^2 + g_{12}|\Psi_2|^2 - i\hbar\,\Gamma_1 + \frac{\hbar\,\Delta}{2}\right]\Psi_1 + \frac{\hbar\,\Omega}{2}\Psi_2\,,$$

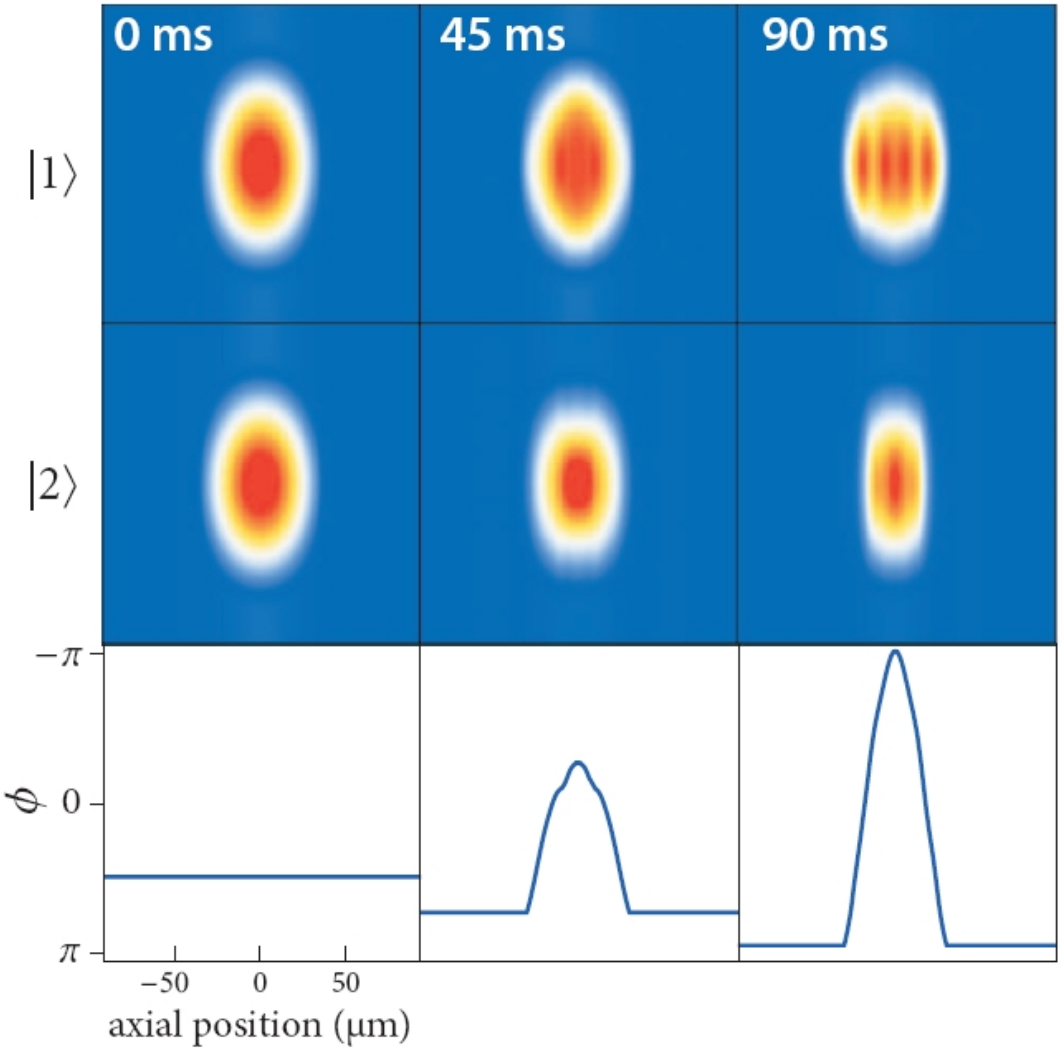
$$i\hbar\frac{\partial\Psi_2}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + V_2 + g_{22}|\Psi_2|^2 + g_{12}|\Psi_1|^2 - i\hbar\,\Gamma_2 - \frac{\hbar\,\Delta}{2}\right]\Psi_2 + \frac{\hbar\,\Omega}{2}\Psi_1$$

with decay terms

$$\Gamma_1 = (\gamma_{111}n_1^2 + \gamma_{12}n_2)/2\,,\ \text{and}$$

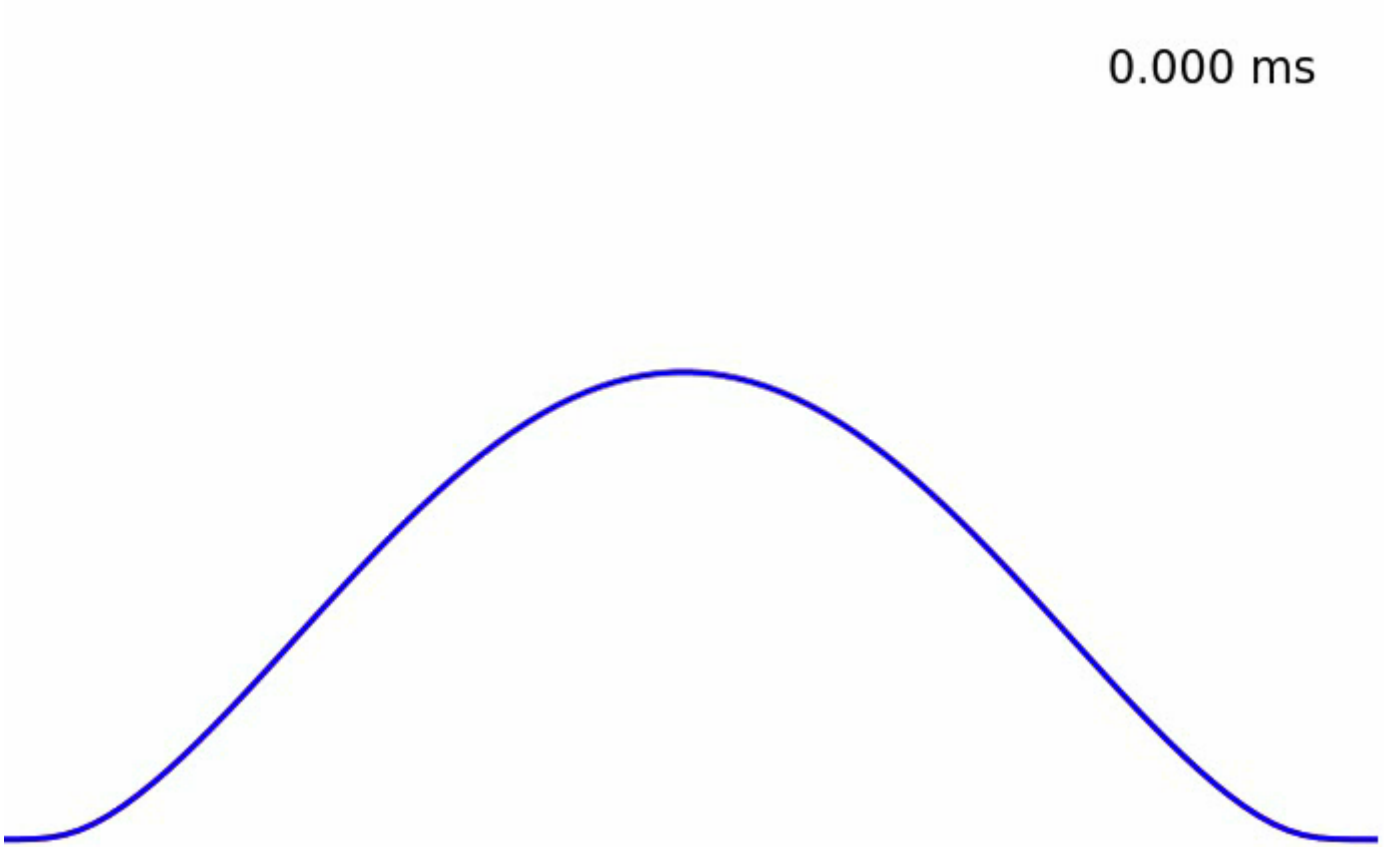
$$\Gamma_2 = (\gamma_{22}n_2 + \gamma_{12}n_1)/2\,,$$

Spatially Non-Uniform Phase



R. P. Anderson et al, *Physical Review A* **80**, 023603 (2009)

Collective Oscillations of 2CBEC

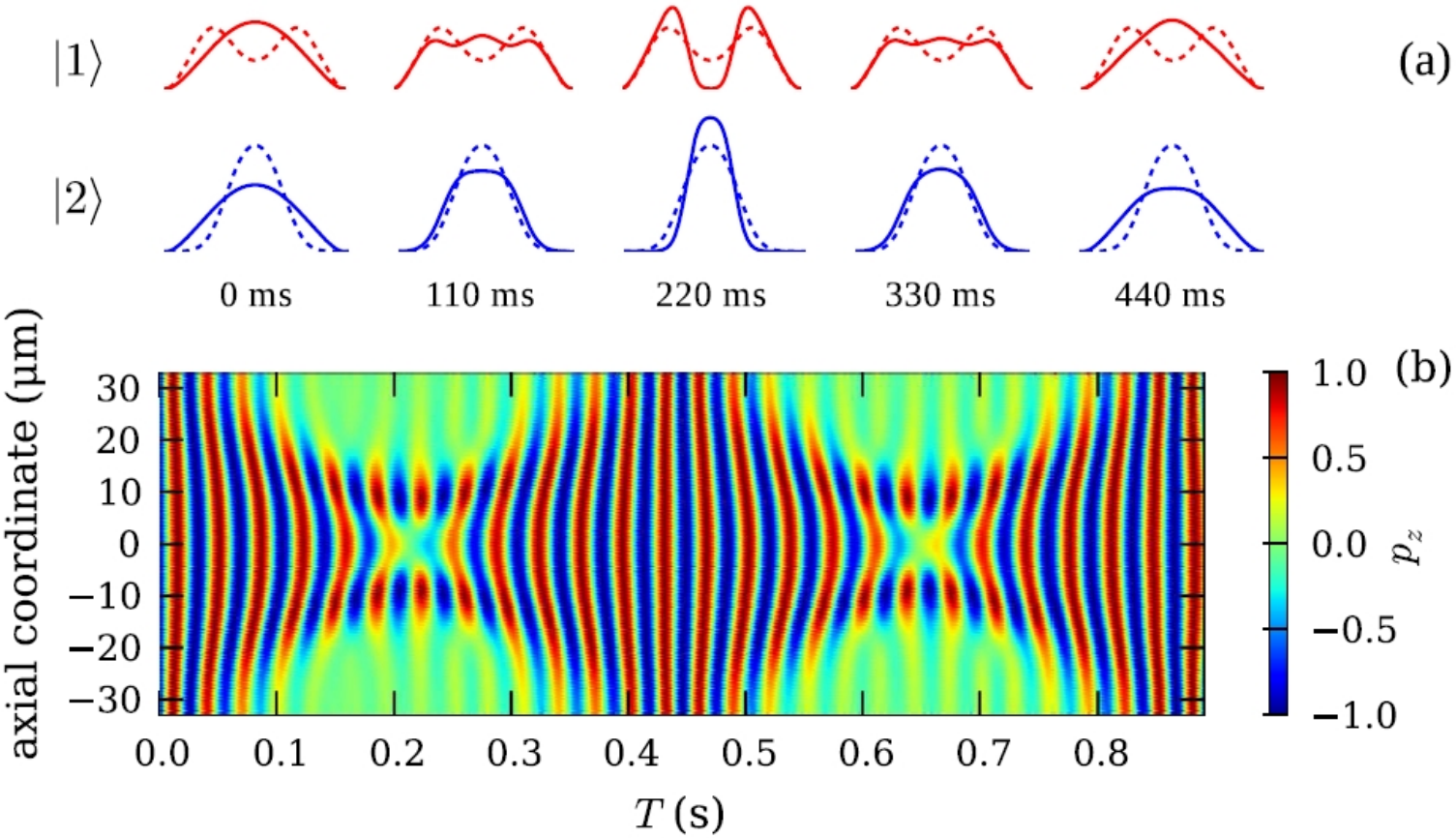


State |2>

State |1>

M. Egorov, 2011

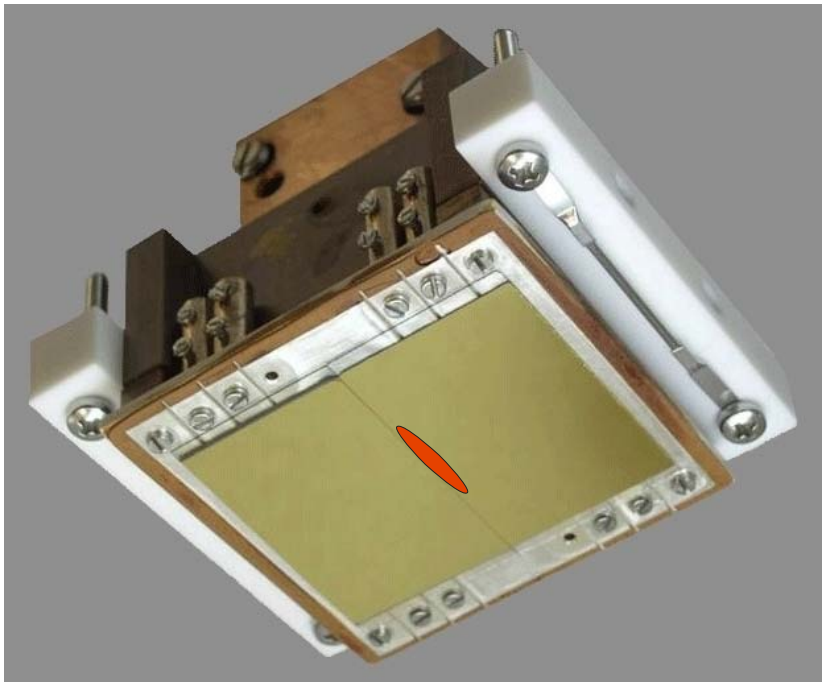
Collective Oscillations and Self-Rephasing of BEC



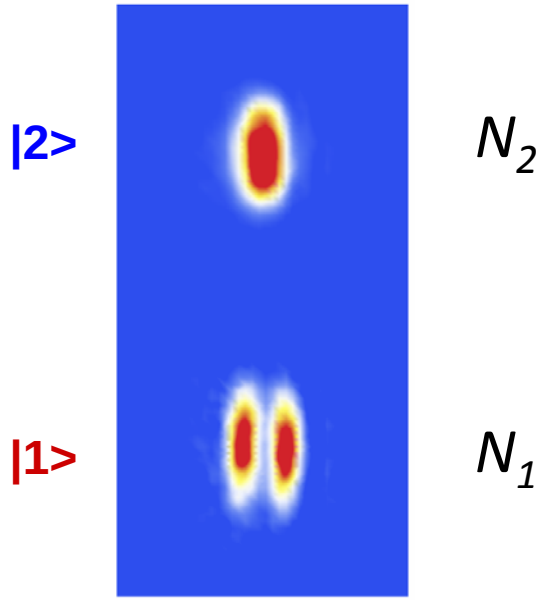
$$p_z = \frac{n_2(z) - n_1(z)}{n_2(z) + n_1(z)}$$

M. Egorov et al, *Physical Review A* **84**, 021605(R) (2011)

Two-component BEC on Atom Chip



Absorption imaging



N = 55,000 atoms

radial frequency 97.0 Hz

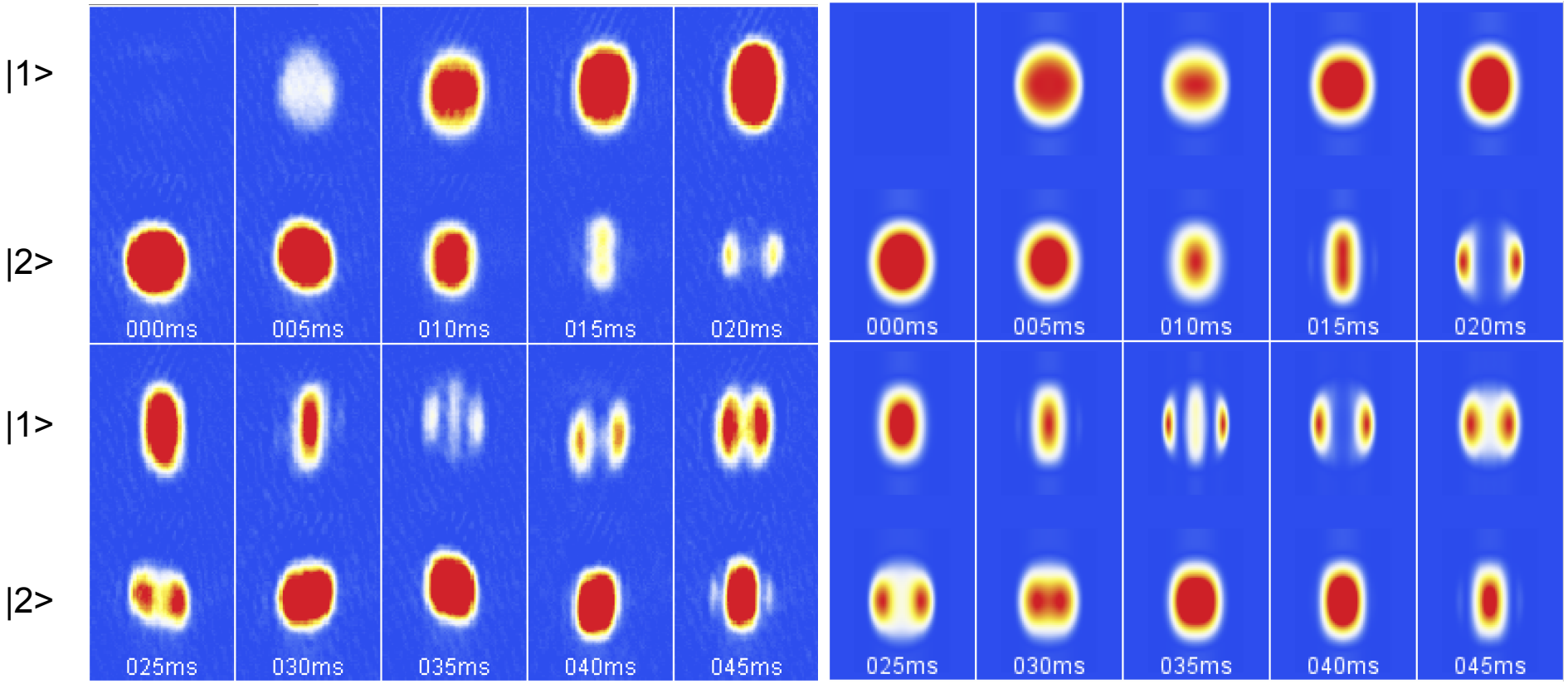
axial frequency 11.69 Hz

$$P_z = \frac{N_1 - N_2}{N_1 + N_2}$$

Ramsey Interference in TOF

Experiment

Theory (GPE)



R. Anderson, 2009

Ramsey Interference in TOF

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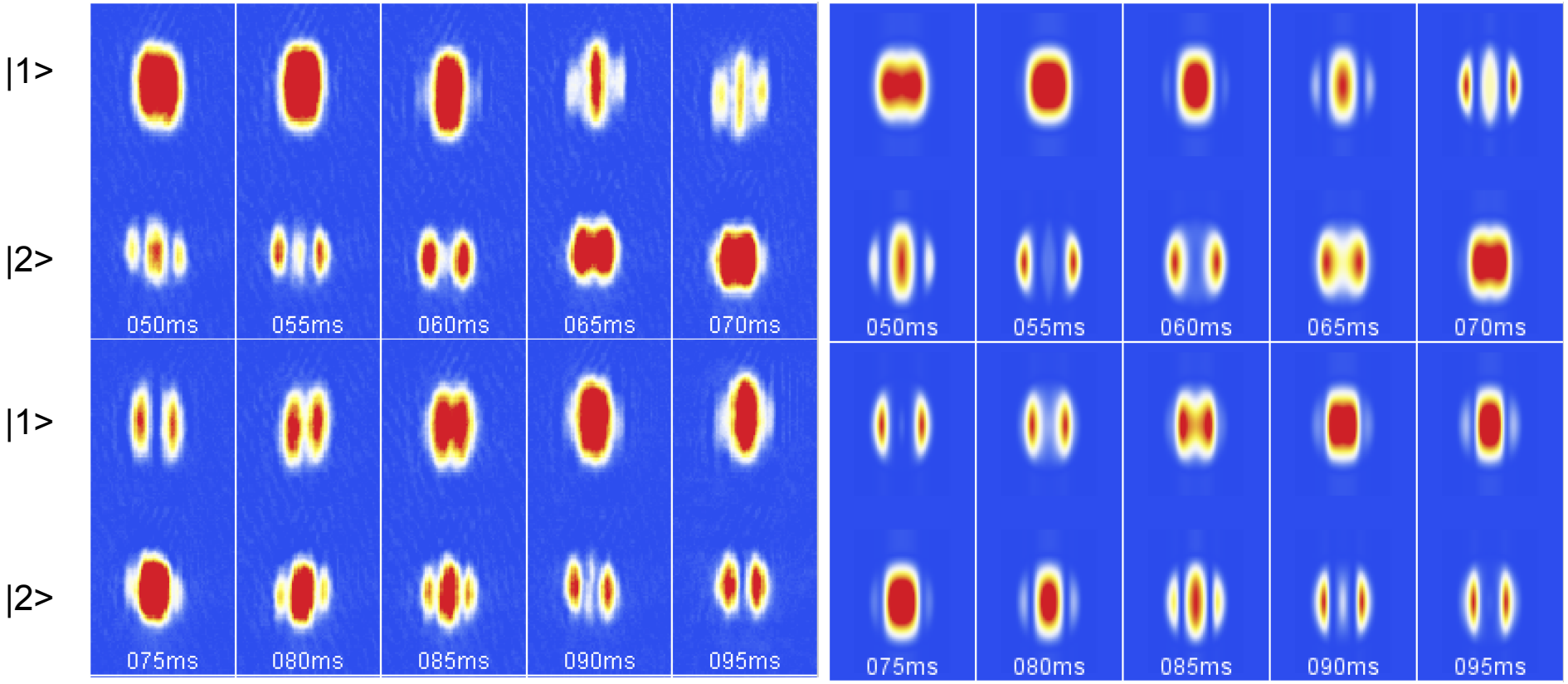
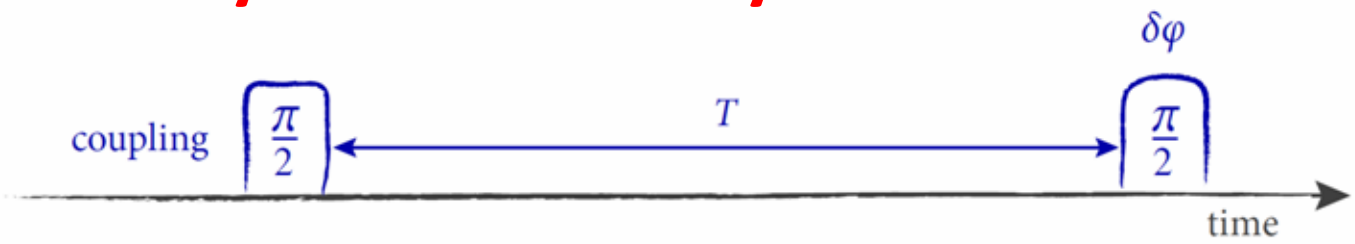
Theory (GPE)

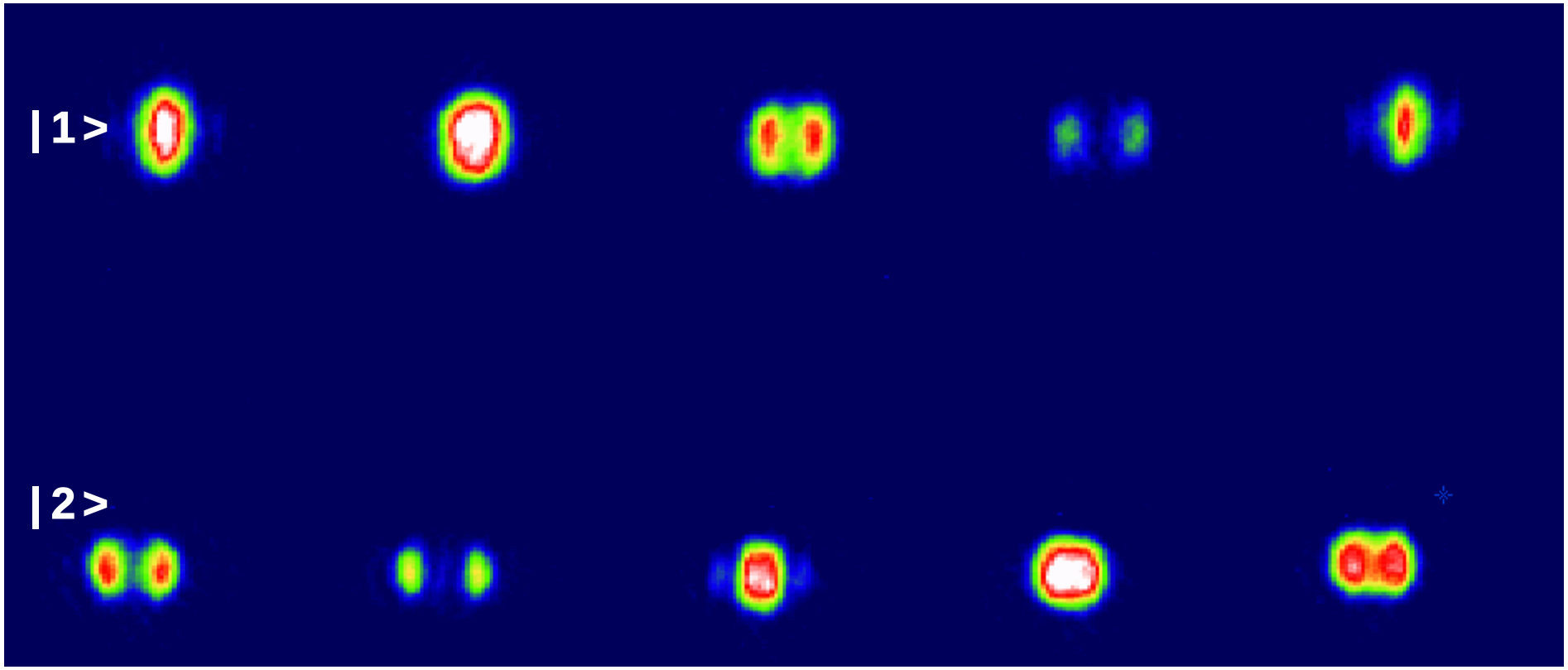


Long evolution time T between two $\pi/2$ pulses

R. Anderson, 2009

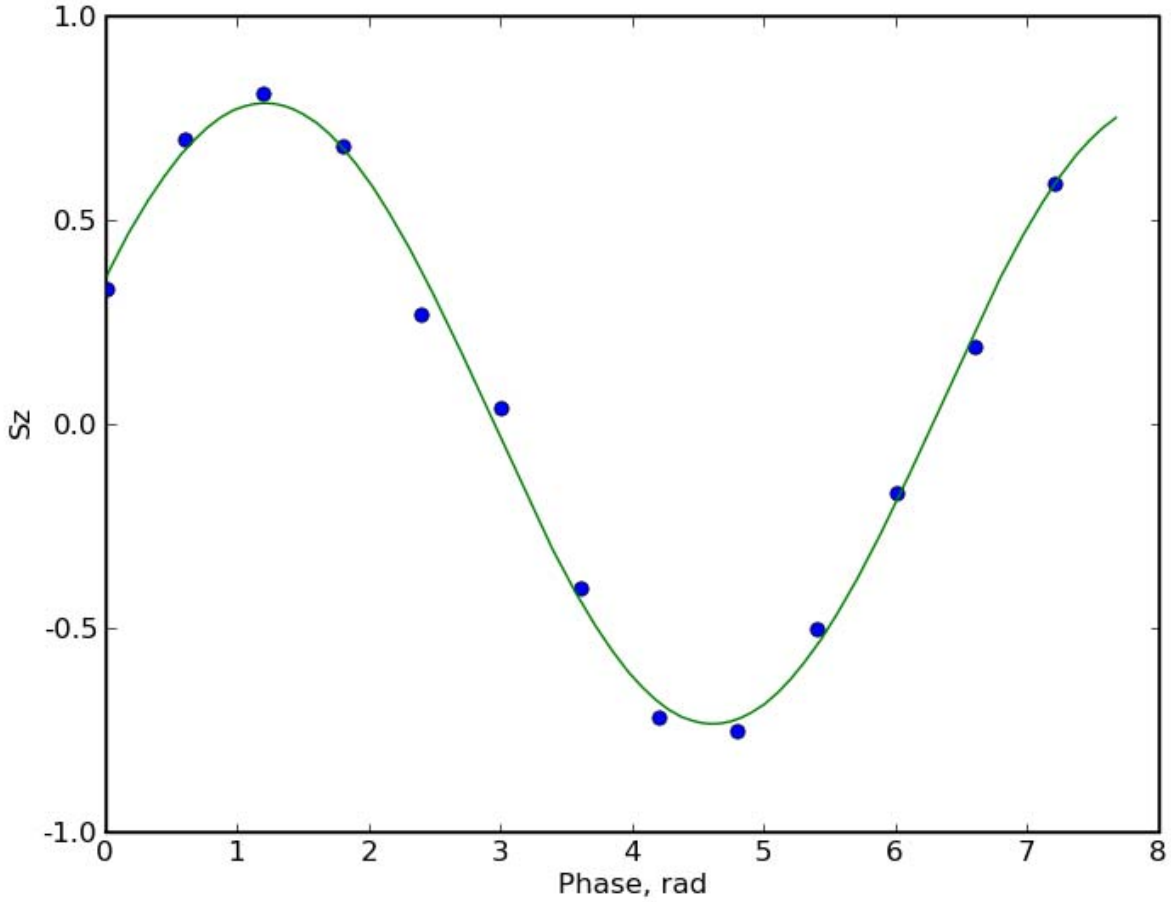
Ramsey Interferometry in Phase Domain





$\phi = \pi/3$ $2\pi/3$ $7\pi/6$ $5\pi/3$ 2π

Ramsey Interferometry in Phase Domain



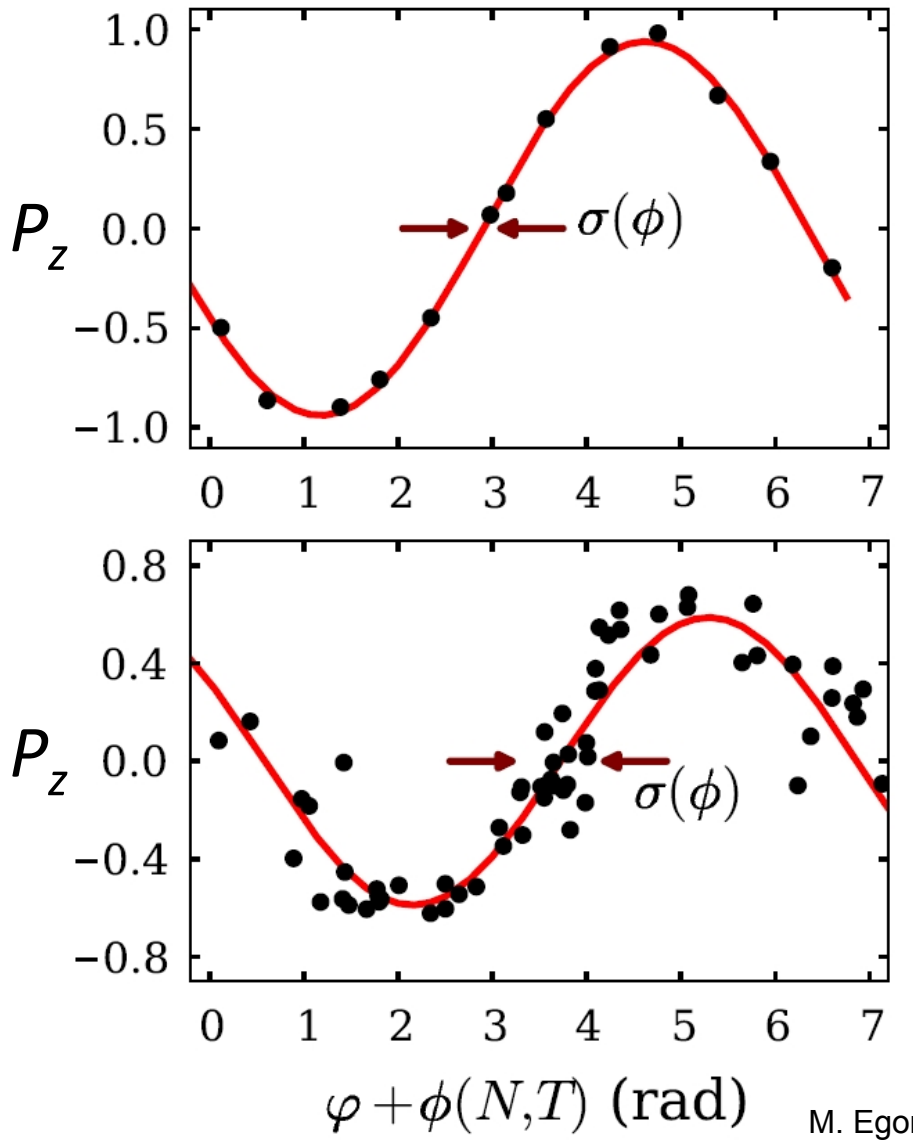
T = 20 ms, Visibility 0.8

Ramsey Interference Fringes

- exp points
- fit

Fitting with

$$P_z = V(T)\cos[\Delta T + \phi(T)]$$

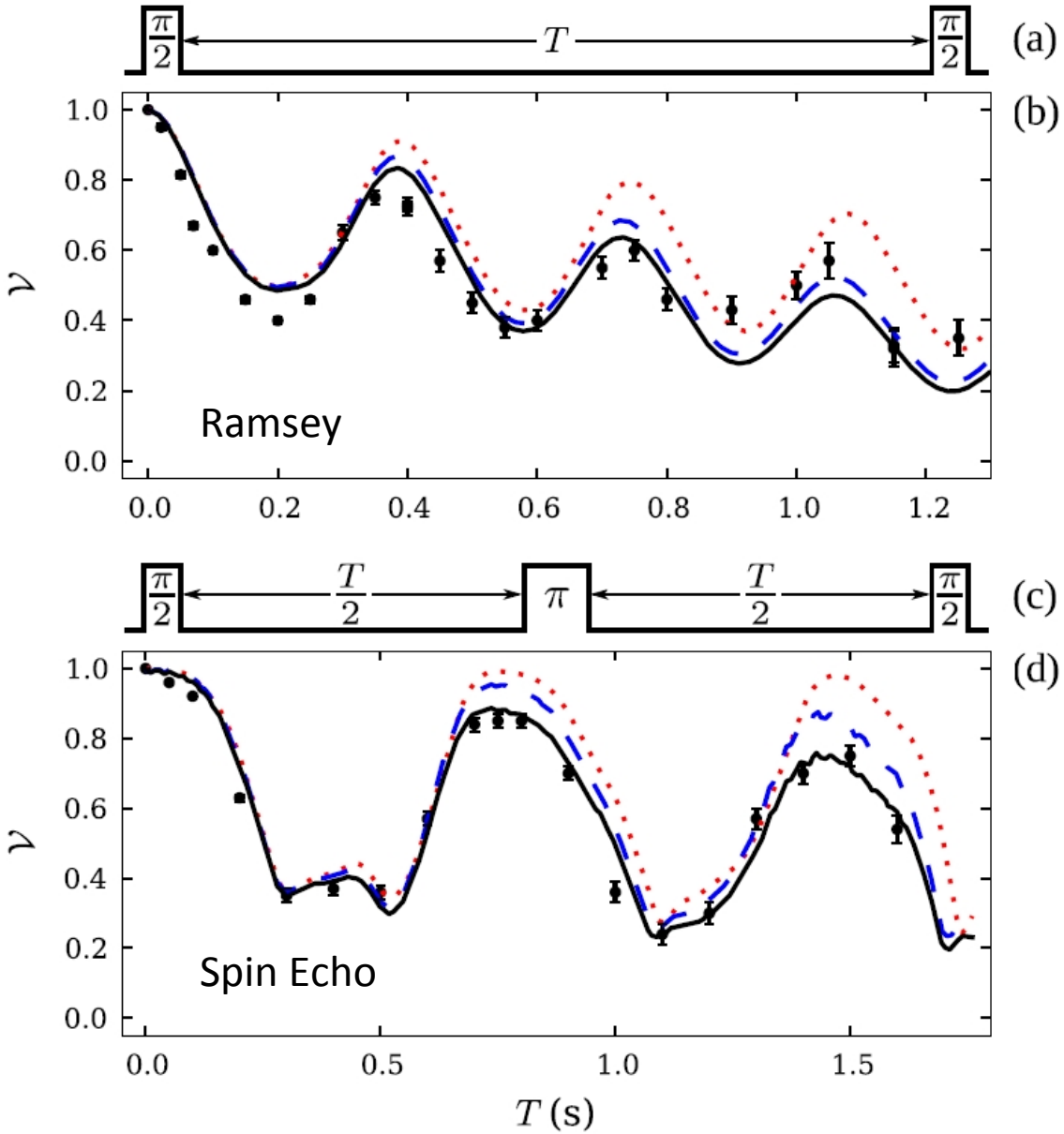


T = 20 ms

T = 450 ms

M. Egorov et al, 2011

Visibility Decay



- exp points
- GPE model
- Wigner model
- Wigner + exp(-T/τ)

τ = 10 s

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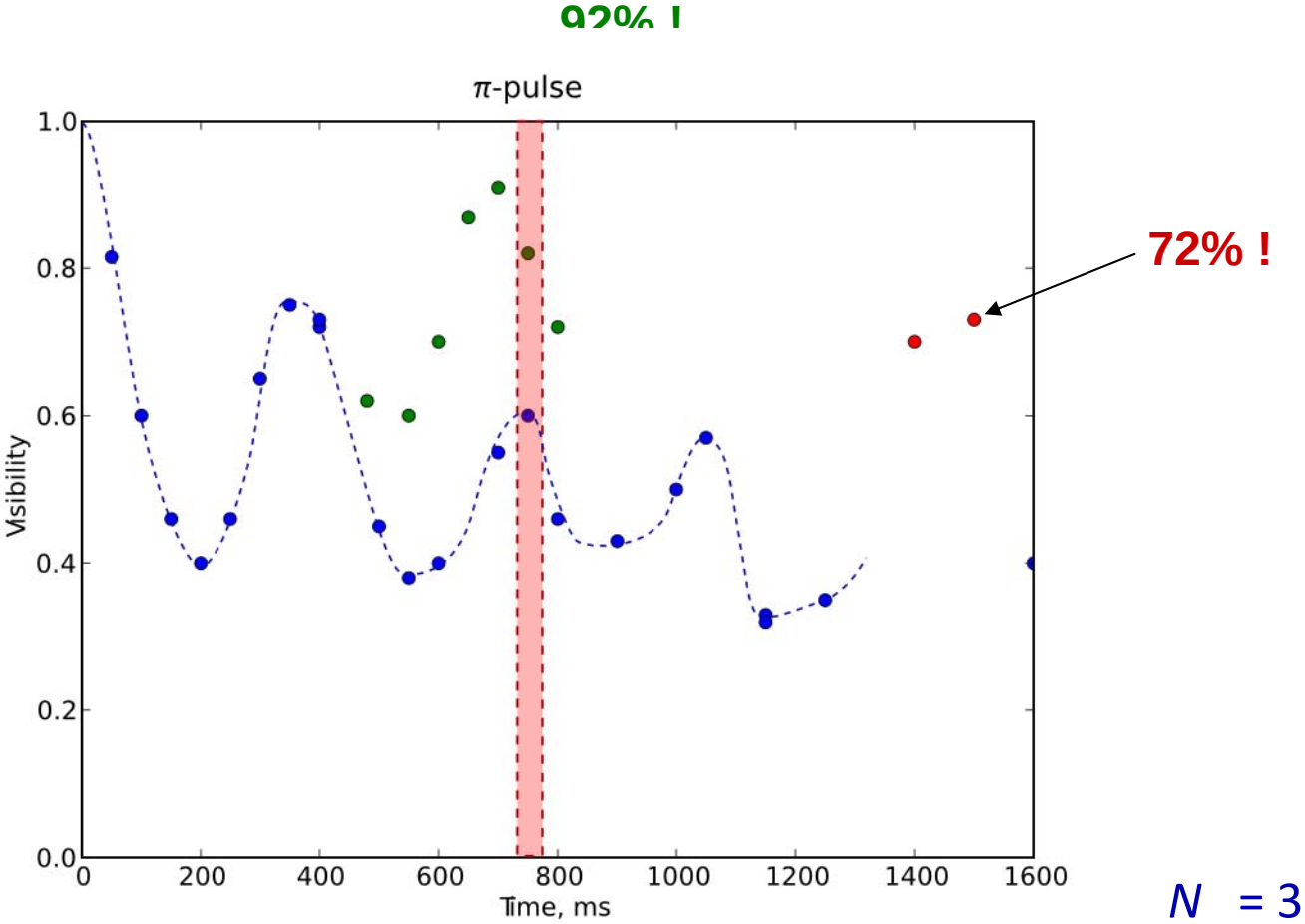
Self-Revival + Spin Echo

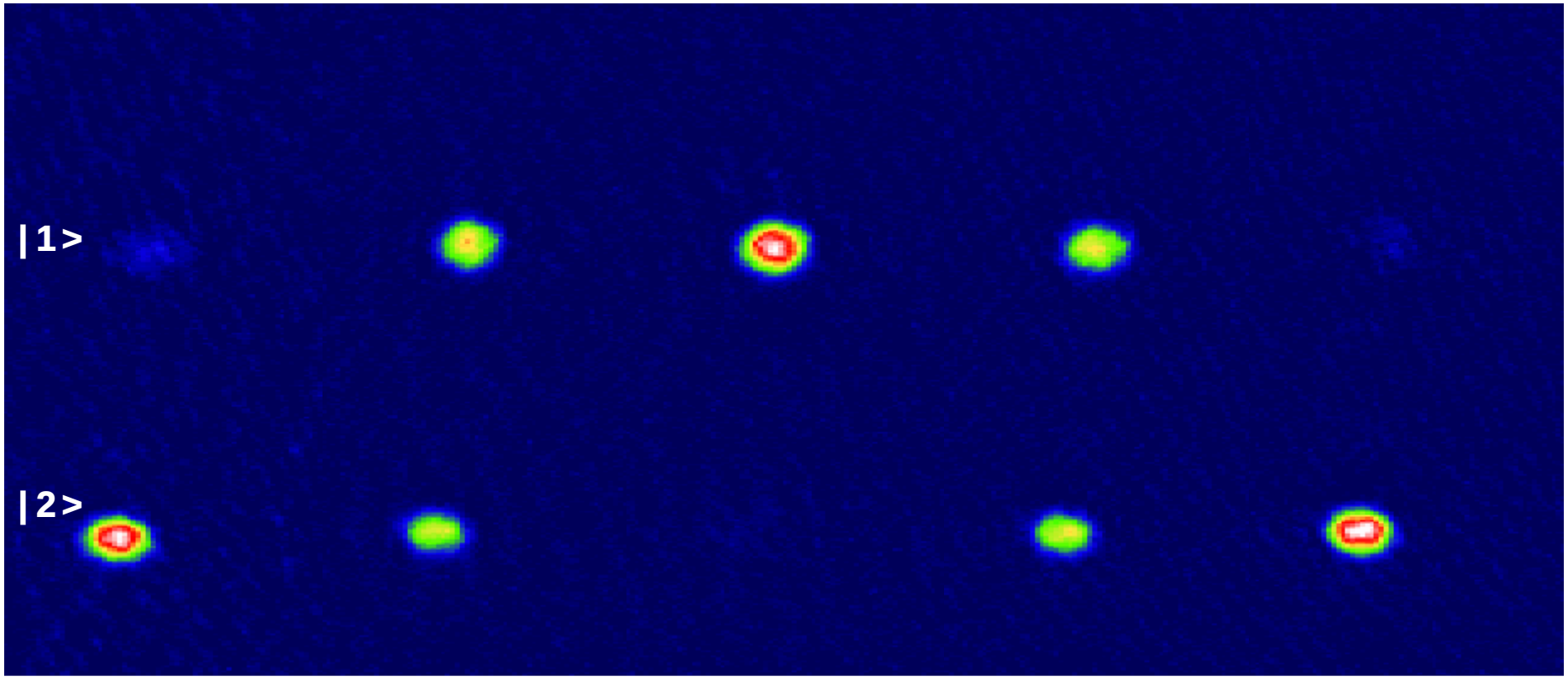


N = 30,000 atoms

T_{dp} = 200 ms (dephasing time)

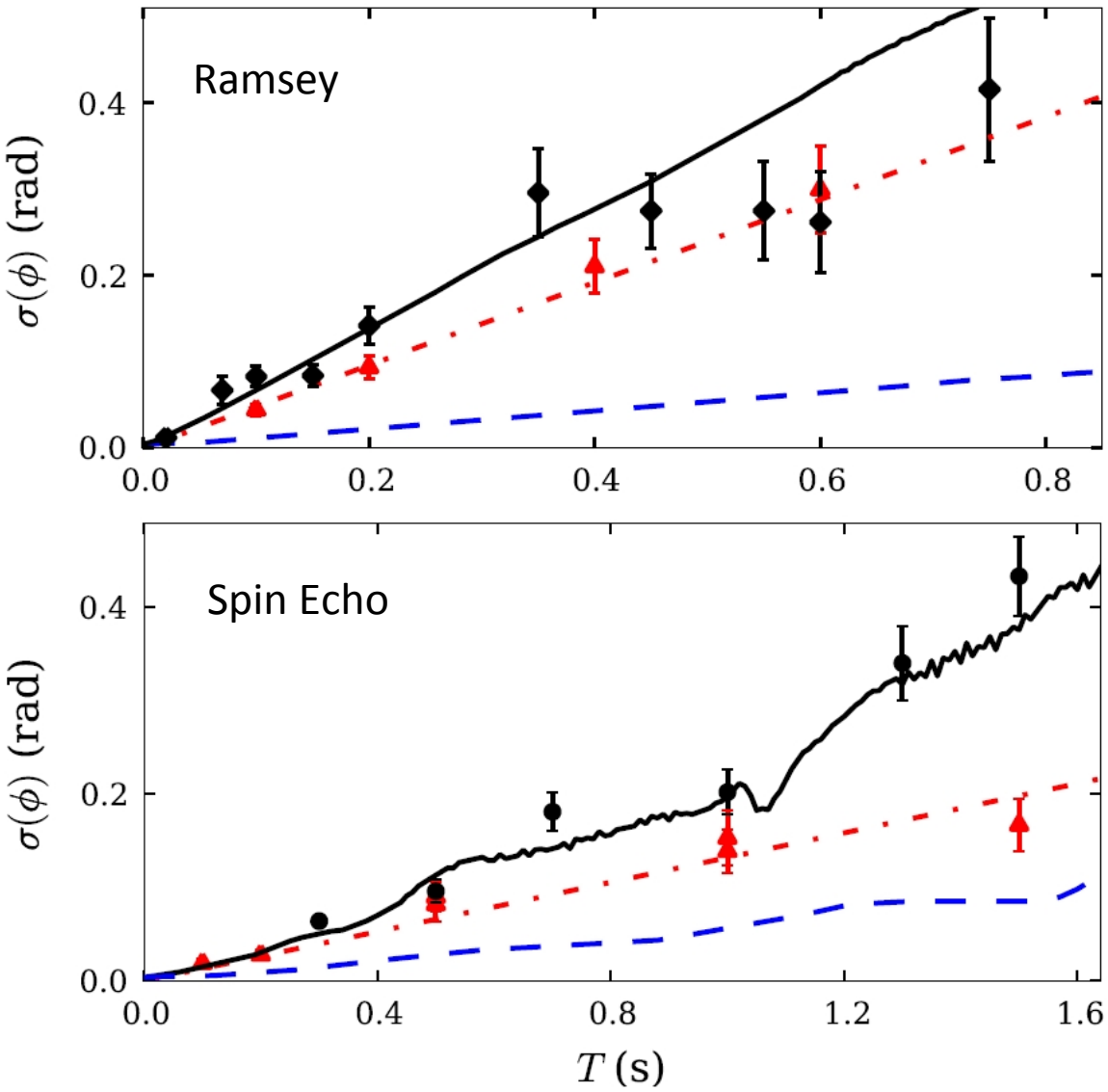
T_1 > 5 s (coherence time)

Spin Echo Profiles at 700 ms



$\phi = 0$ $2\pi/3$ π $3\pi/2$ 2π

Evolution of Phase Noise



- exp points
- local oscillator noise
- Quantum Noise
- Classical + Quantum Noise

M. Egorov et al, 2011

We are looking for new PhD students