Victorian Summer School on Ultracold Atoms

Course: Laser Cooling and Trapping of Atoms

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Lecture 1

Lecture Outline:

1. Background
2. Radiation pressure force
3. Two-level atoms and J=0 → J=1 atoms
4. Cooling limits

<u>Literature:</u>

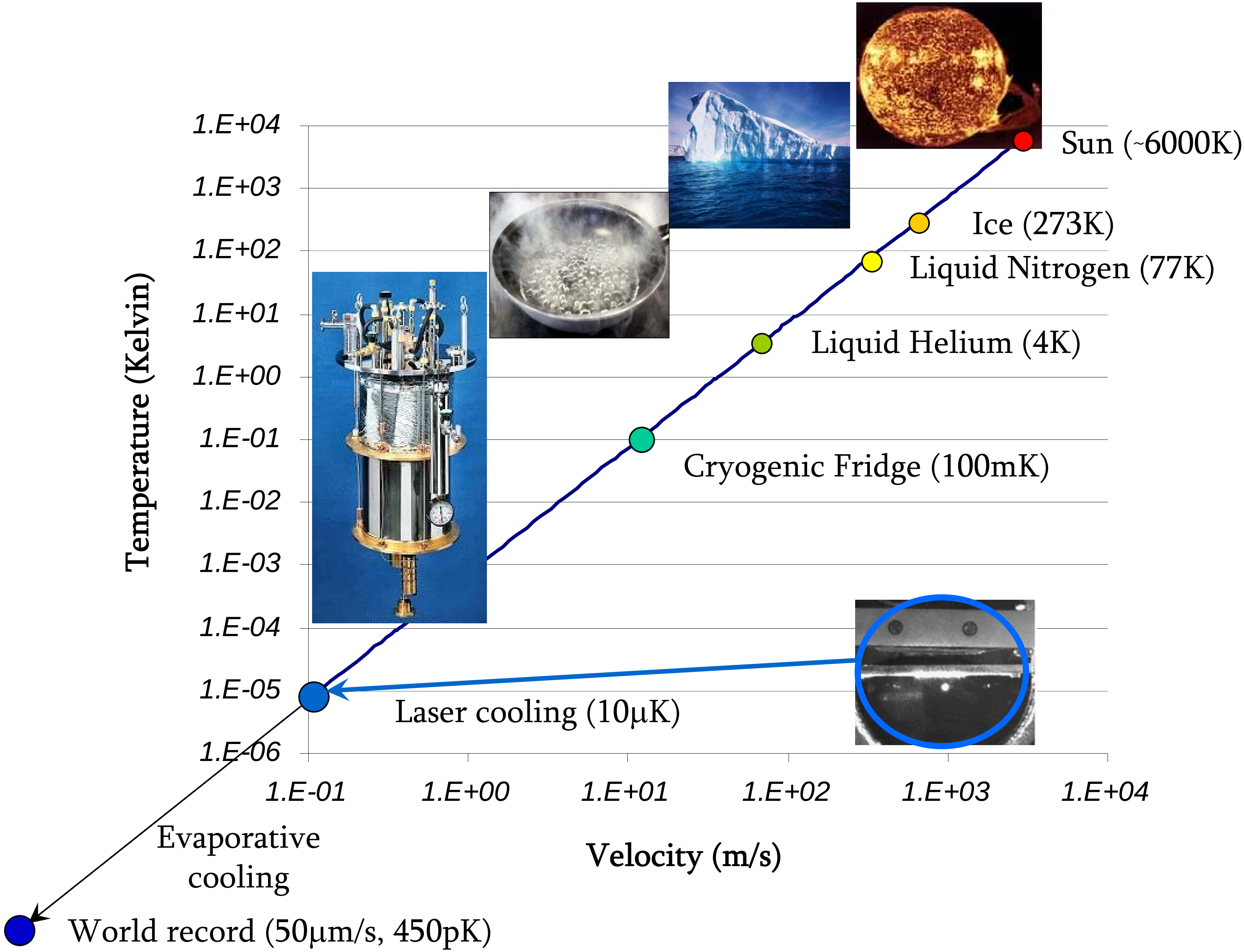
• V.G. Minogin and V.S. Letokhov, "Effects of radiation pressure on atoms" (Nauka, Moscow, 1986)

• J. Dalibard and C. Cohen-Tannoudji, "Dressed-atom approach to atomic motion in laser light: the dipole force revisited", JOSA **B2**, 1707 (1985)

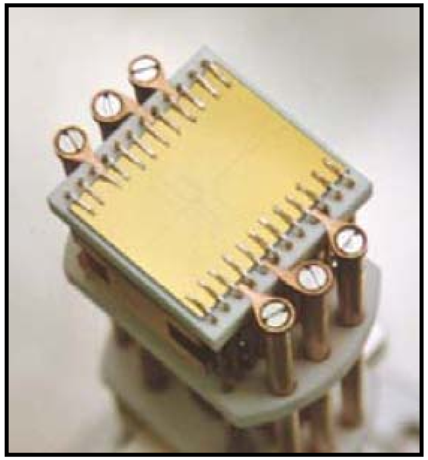
• J. Dalibard and C. Cohen-Tannoudji, "Laser cooling below the Doppler limit by polarization gradients: simple theoretical models", JOSA **B6**, 2023 (1989)

• H.J. Metcalf and P. van der Straten, "Laser cooling and trapping" (Springer, New York, 2002)

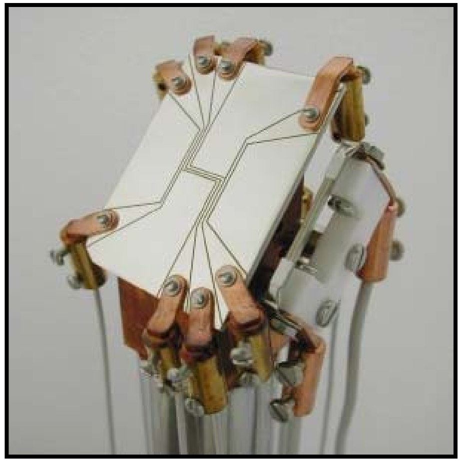
Towards Zero Temperature



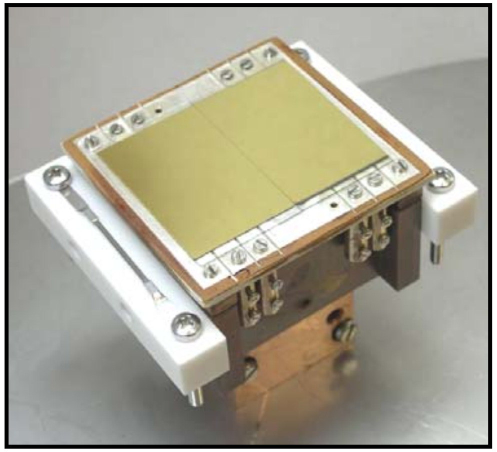
Atom Chips: technology to produce cold atoms



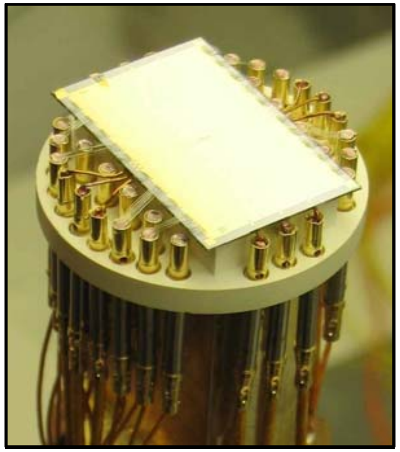
Universität Heidelberg (2002)



University of Queensland (2004)

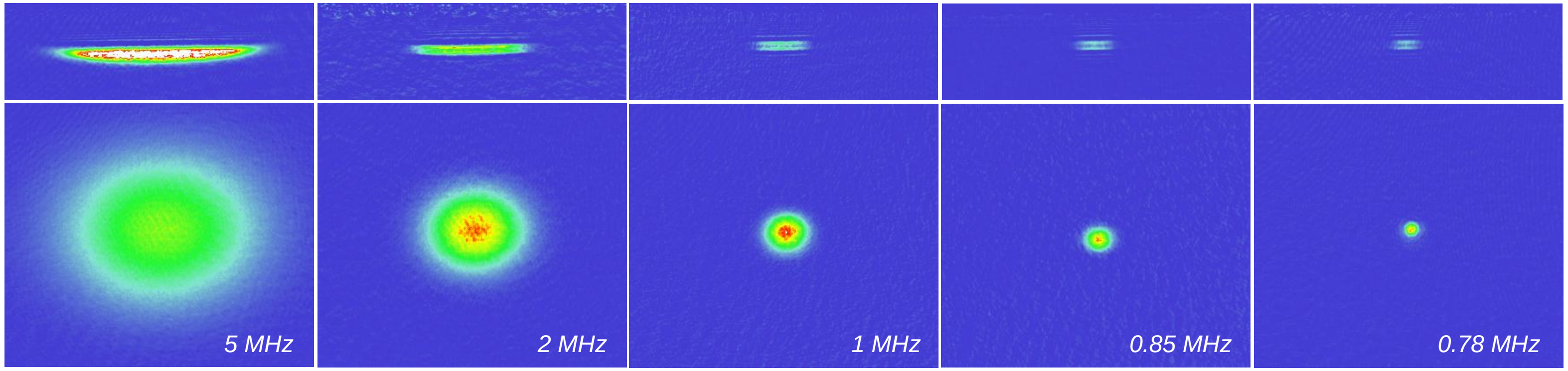


Swinburne University (2005)



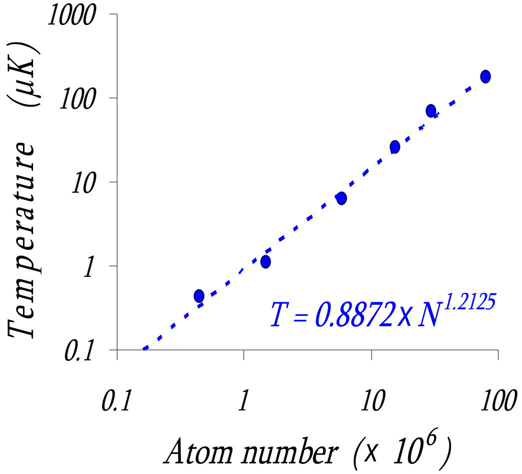
Universiteit van Amsterdam (2005)

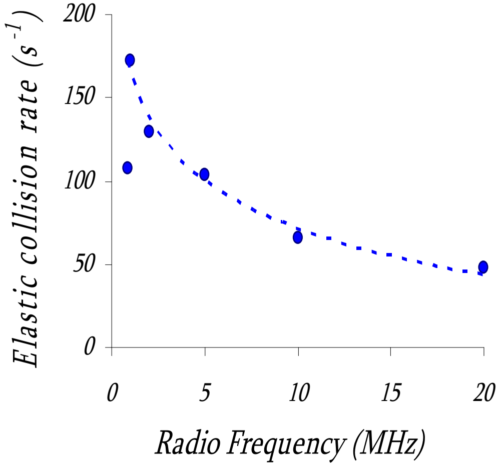
Temperature: measure of motion

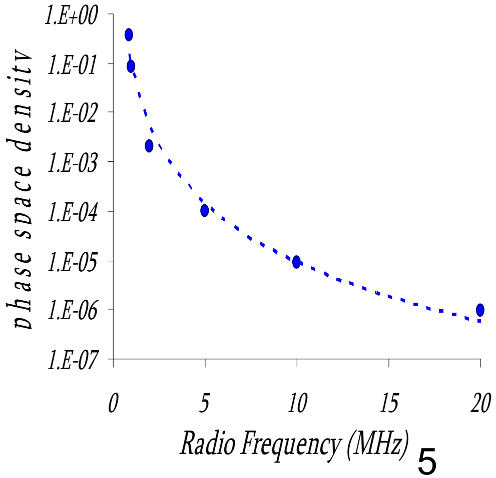


S. Whitlock, 2006

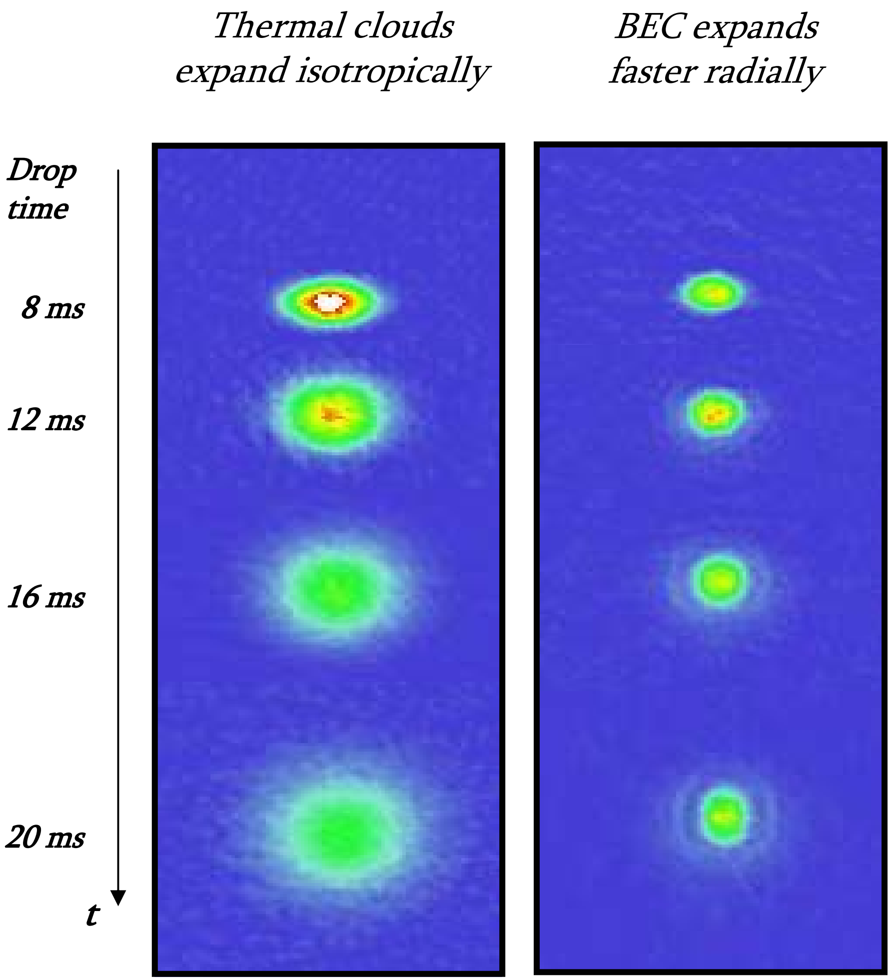
Parameters of evaporation



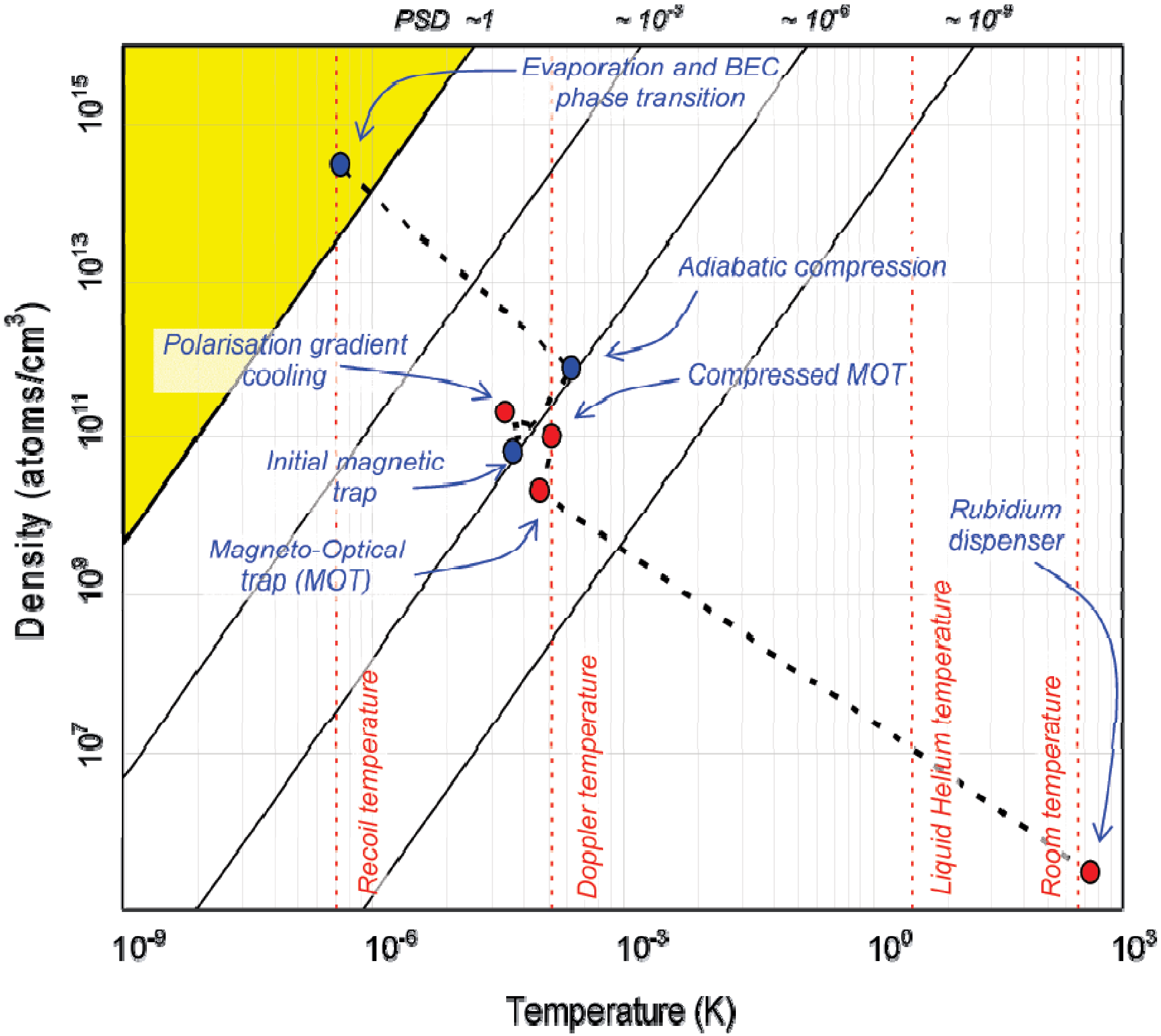




Temperature: measure of motion

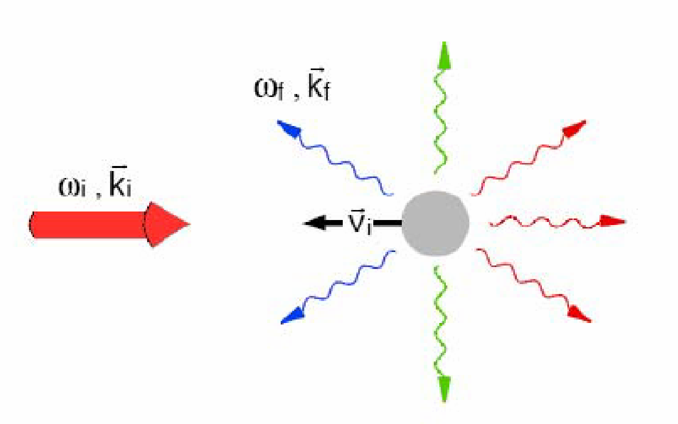


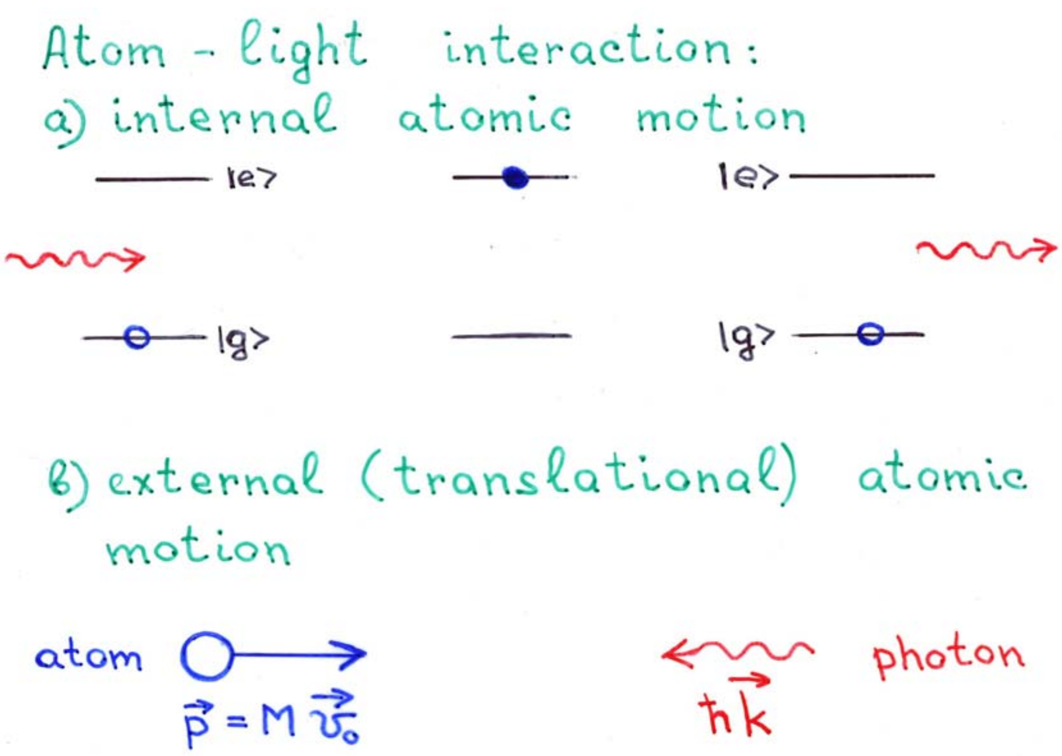
S. Whitlock, 2006



S. Whitlock, PhD thesis 2007

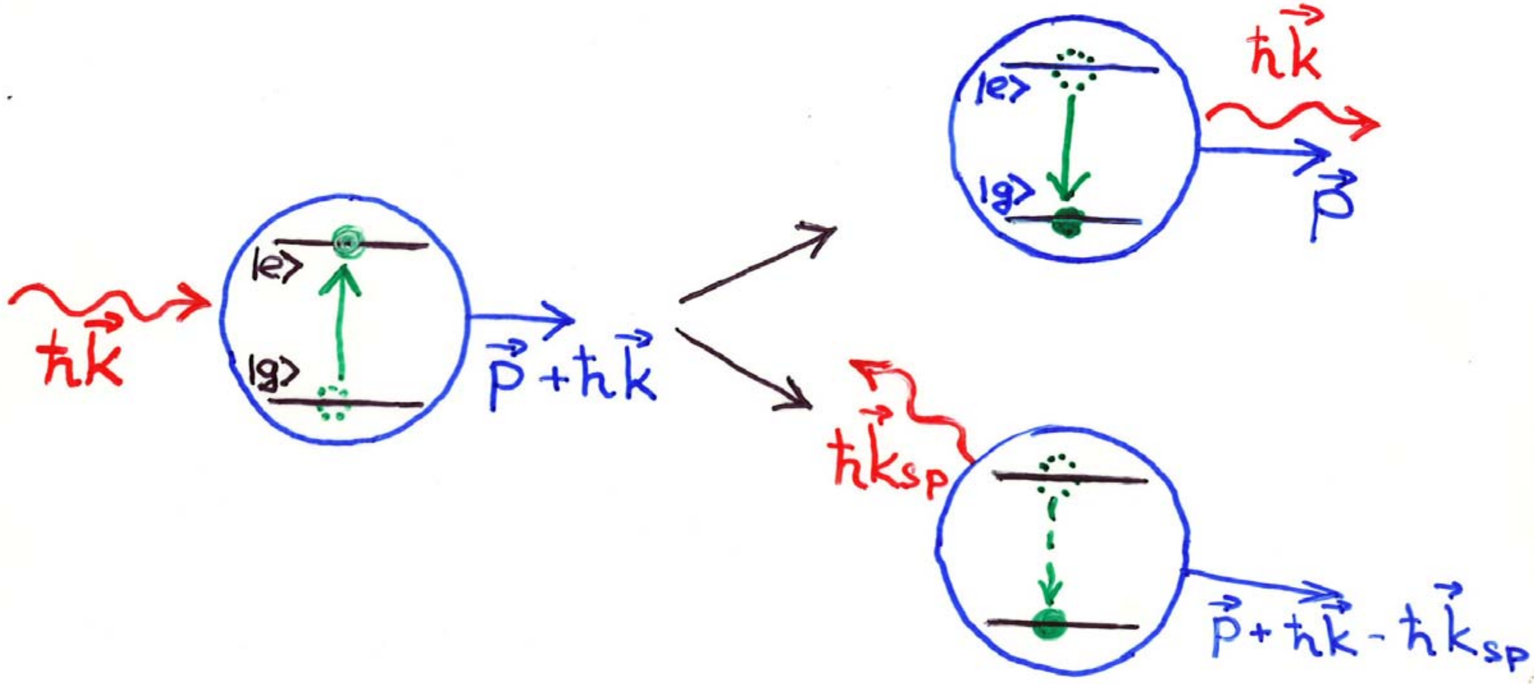
Radiation Pressure Force





v_0 = 600 m/s

$v_{rec} = \hbar k/m$ = 6 mm/s



$$\sum^{N} \hbar \vec{k}_{sp} \simeq 0$$

Spont…

(scattering …orce)

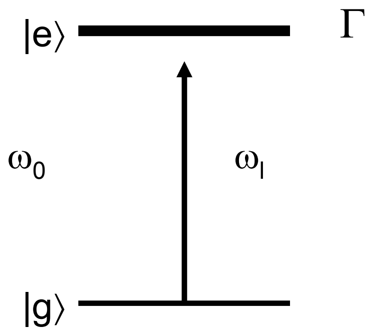
$$F = \frac{\Delta p}{\Delta \tau} = \frac{\hbar k}{2\tau_N} = \frac{\hbar k \Gamma}{2}$$

Rb:

$$F = \frac{\hbar k \Gamma}{2} = \frac{10^{-34} \cdot 8 \cdot 10^{6} \cdot 6.3 \cdot 6 \cdot 10^{6}}{2} = 1.5 \cdot 10^{-20}\ N$$

$$\frac{F}{m_{Rb}} = \frac{1.5 \cdot 10^{-20}}{1.4 \cdot 10^{-25}} \approx 10^{5}\ \frac{m}{s^2} = 10^{4}\ g$$

Cooling with Radiation Pressure



Frequency parameters: $\Gamma, \quad \delta = \omega_l - \omega_0, \quad \Omega = \dfrac{dE_0}{\hbar}$

Characteristic times (^{87}Rb atom):

$$T_0 = \frac{2\pi}{\omega_0} = 2.7\,fs \qquad \tau_N = \Gamma^{-1} = 27ns \qquad \tau_{op} = \Gamma_{op}^{-1} \gg \tau_N$$

$$E_{rec} = \frac{\hbar^2 k^2}{2M} = k_B \times 180nK \qquad \hbar\Gamma \gg E_{rec} \qquad T_{tran} \gg T_{\text{int}}$$

Density matrix equations and mean force

$$i\hbar\frac{d\rho}{dt} = \left[\hat{H},\rho\right] + \quad Losses$$

$$\tilde{\rho}_{ge} \equiv \rho_{ge}e^{-i\delta t}$$

$$\frac{\partial\rho_{gg}}{\partial t} = \frac{i}{2}\left(\Omega^*\tilde{\rho}_{eg} - \Omega\tilde{\rho}_{ge}\right) + \Gamma\rho_{ee}$$

$$\frac{\partial\tilde{\rho}_{ge}}{\partial t} = \frac{i}{2}\Omega^*\left(\tilde{\rho}_{ee} - \tilde{\rho}_{gg}\right) - \left(\frac{\Gamma}{2} + i\delta\right)\tilde{\rho}_{ge}$$

$$\frac{\partial\tilde{\rho}_{eg}}{\partial t} = \frac{i}{2}\Omega\left(\tilde{\rho}_{gg} - \tilde{\rho}_{ee}\right) - \left(\frac{\Gamma}{2} - i\delta\right)\tilde{\rho}_{eg}$$

$$\rho_{gg} + \rho_{ee} = 1$$

$$\mathbf{F} = -\left\langle\nabla\hat{V}\right\rangle \qquad \hat{V} = -\hat{d}\hat{E} \qquad -\nabla\hat{V} = -\frac{\hbar}{2}\left|e\right\rangle\left\langle g\right|e^{-i\omega_l t}\,\nabla\left[\Omega_1(r)e^{-i\Phi(r)}\right] + h.c.$$

$$\nabla\left[\Omega_1(r)e^{-i\Phi(r)}\right] = \Omega_1(r)e^{-i\Phi(r)}\left[\boldsymbol{\alpha}(r) - i\boldsymbol{\beta}(r)\right] \qquad \boldsymbol{\alpha}(r) = \frac{\nabla\Omega_1(r)}{\Omega_1(r)} \qquad \boldsymbol{\beta}(r) = \nabla\Phi(r)$$

$$\mathbf{F}(r,t) = -\hbar\Omega_1(r)\left[u(t)\boldsymbol{\alpha}(r) + v(t)\boldsymbol{\beta}(r)\right]$$

$$u(t) = \mathrm{Re}\left[\rho_{ge}e^{-i(\omega_l t + \Phi)}\right]$$

$$v(t) = \mathrm{Im}\left[\rho_{ge}e^{-i(\omega_l t + \Phi)}\right]$$

Two-level atom at rest

$$u_{st} = \frac{\delta}{\Omega_1}\frac{s}{1+s} \qquad v_{st} = \frac{\Gamma}{2\Omega_1}\frac{s}{1+s} \qquad \omega_{st} = -\frac{s}{2(1+s)} \qquad s = \frac{\Omega_1^2/2}{\delta^2+(\Gamma/2)^2}$$

<u>Travelling wave</u>

$$E(z,t) = \frac{1}{2}E_0\left[e^{i(kz-\omega_l t)} + c.c.\right] \qquad \Omega_1 = \frac{d_{ge}E_0}{\hbar}$$

$$\mathbf{F}_{sc} = \frac{\hbar\mathbf{k}\Gamma}{2}\frac{\Omega_1^2/2}{\Omega_1^2/2+\delta^2+(\Gamma/2)^2}$$

<u>Standing wave</u>

$$E(z,t) = \frac{1}{2}E_0\left[e^{i(kz-\omega_l t)} + e^{-i(kz+\omega_l t)} + c.c.\right]$$

$$\mathbf{F}_{dip} = -\frac{\hbar\delta}{4}\frac{\nabla\Omega_1^2}{\Omega_1^2/2+\delta^2+(\Gamma/2)^2} = -\nabla\left[\frac{\hbar\delta}{2}\ln\left(1+\frac{\Omega_1^2}{2\delta^2}\right)\right]$$

Moving two-level atom

Travelling wave $$E(z,t) = \frac{1}{2}E_0\left[e^{i(kz-\omega_l t)} + c.c.\right] \qquad z = v_0 t$$

$$\Omega_1 = \frac{d_{ge}E_0}{\hbar}$$ - constant

$$\Phi(z) = -kz \qquad \frac{d\Phi}{dt} = \frac{dz}{dt}\nabla\Phi = -kv_0 \qquad \mathbf{F}_{sc} = \frac{\hbar\mathbf{k}\Gamma}{2}\frac{\Omega_1^2/2}{\Omega_1^2/2+(\delta-kv_0)^2+(\Gamma/2)^2}$$

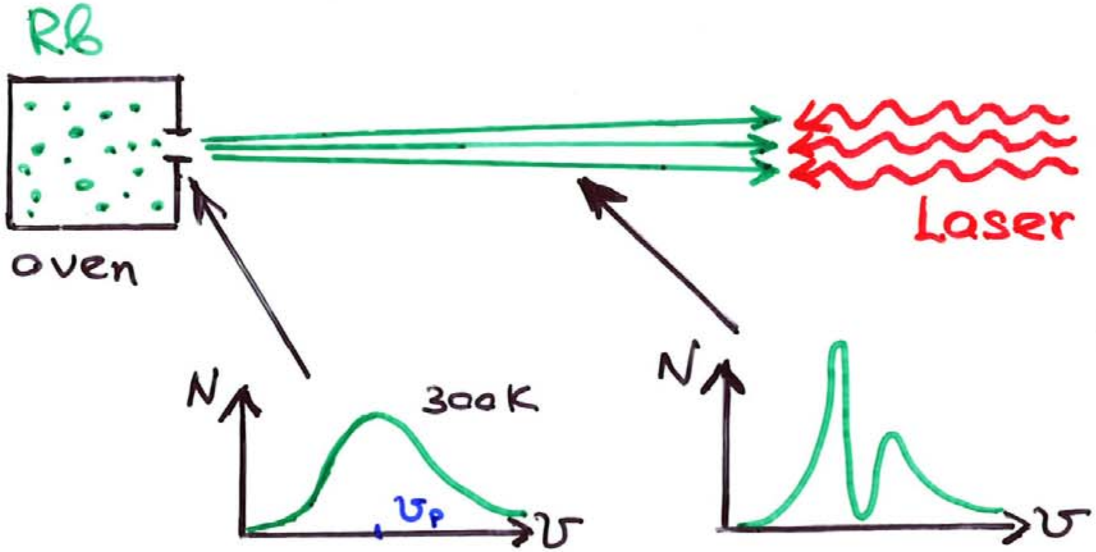
Standing wave $$E(z,t) = \frac{1}{2}E_0\left[e^{i(kz-\omega_l t)} + e^{-i(kz+\omega_l t)} + c.c.\right]$$

$$\Omega_1(z) = 2\Omega_1\cos kz$$

$$\boldsymbol{\alpha} = -\mathbf{k}\tan kz$$

In the limit of small velocities: $kv_0 << \Gamma$ and weak intensity: $s_0 << 1$

$$F_{fr} = -\alpha v_0 \qquad \alpha = -\hbar k^2\Omega_1^2\frac{\delta\Gamma}{\left[\delta^2+(\Gamma/2)^2\right]^2}$$



Scattering force!

For ^{87}Rb: $v_0 \simeq 2.4 \cdot 10^4$ cm/s

$v_{recoil} = \frac{\hbar k}{M} = 0.6 \frac{cm}{s}$

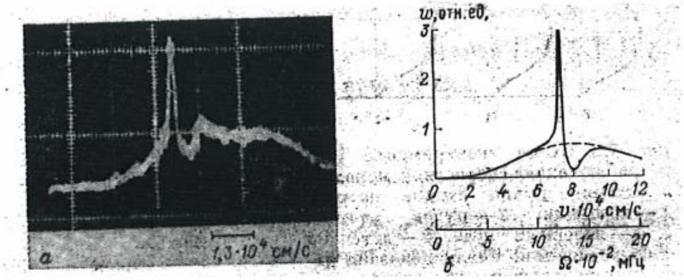
Need about 40000 absorptions to stop a Rb atom.

Resonant acceleration: $a = \frac{\hbar k}{M} \frac{\Gamma}{2} = 1.1 \cdot 10^7 \frac{cm}{s^2}$ (eg $g = 10^3 \frac{cm}{s^2}$)

Stopping distance: $L = \frac{v_0^2}{2a} = 26$ cm (in resonance)

Stopping time: $T = \frac{v_0}{a} = 2.2$ ms (in resonance)

1D laser cooling of atoms (Balykin et al, Moscow, 1981)



Minogin and Letokhov, 1984

if light is in resonance with atom

$$\begin{cases} L_{min} = \dfrac{v_p^2}{2 a_{max}} = \dfrac{v_p^2 \cdot M}{\hbar k \Gamma} \\[2ex] t_{min} = \dfrac{v_p}{a_{max}} = \dfrac{2 v_p M}{\hbar k \Gamma} \end{cases}$$

Atom	T_{oven} (K)	v_p (m/s)	L_{min} (m)	T_{min} (ms)
H	1000	5000	0.012	0.005
He*	4	158	0.03	0.34
Li	1017	2051	1.15	1.12
Na	712	876	0.42	0.96
Rb	568	402	0.75	3.72
Cs	544	319	0.93	5.82

However:

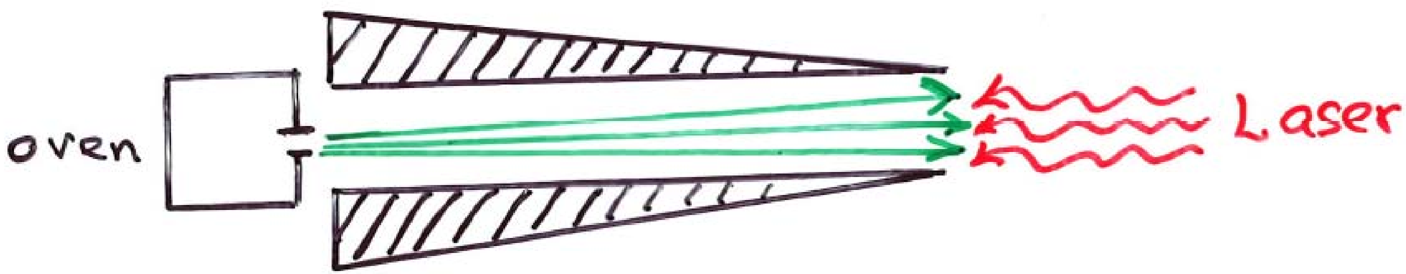
$$F_{scat} = \frac{\hbar k \Gamma}{2} \frac{s(v)}{1+s(v)} = \frac{\hbar k \Gamma}{2} \frac{\Omega^2/2}{\Omega^2/2 + \Gamma^2/4 + (\delta - kv)^2}$$

Efficiency of slowing falls with increasing Doppler shift!

Maintaining resonance condition: $\delta = \omega_\ell - \omega_0$

(a) "Chirp" frequency of laser (ω_ℓ)

(b) Tune frequency ω_0 using inhomogeneous magnetic field (Zeeman effect)



① Laser frequency sweep

$$\omega_e = \omega_{e0} + \dot{\omega}_D \cdot t = \omega_{e0} + k \cdot a \cdot t$$

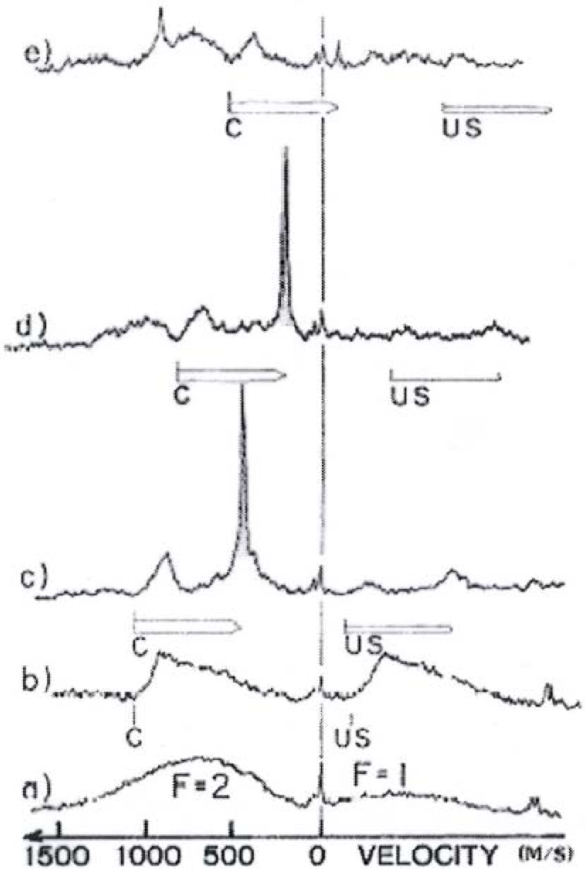
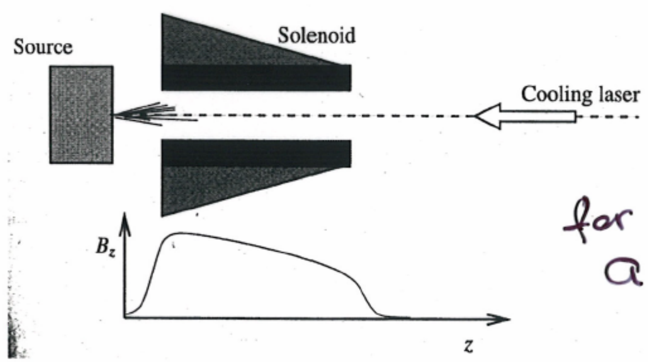


FIG. 3. Sodium-atomic-beam cooling using a frequency-chirped laser. Trace *a*, cooling laser off. The D_2 transition

W. Ertmer et al, PRL, 1984

② Zeeman slower: varying the atomic frequency



for uniform deceleration

$a = \eta a_{max}$

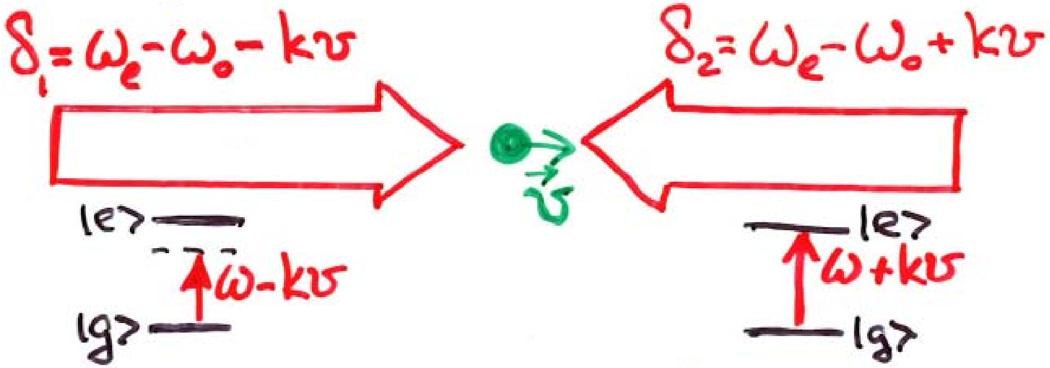
$$B(z) = B_0 \sqrt{1 - z/z_0}, \quad z_0 = \frac{M v_0^2}{\eta \hbar k \Gamma}$$

η - design parameter (< 1)

(8) Doppler cooling of atoms in a standing wave

(Optical molasses)

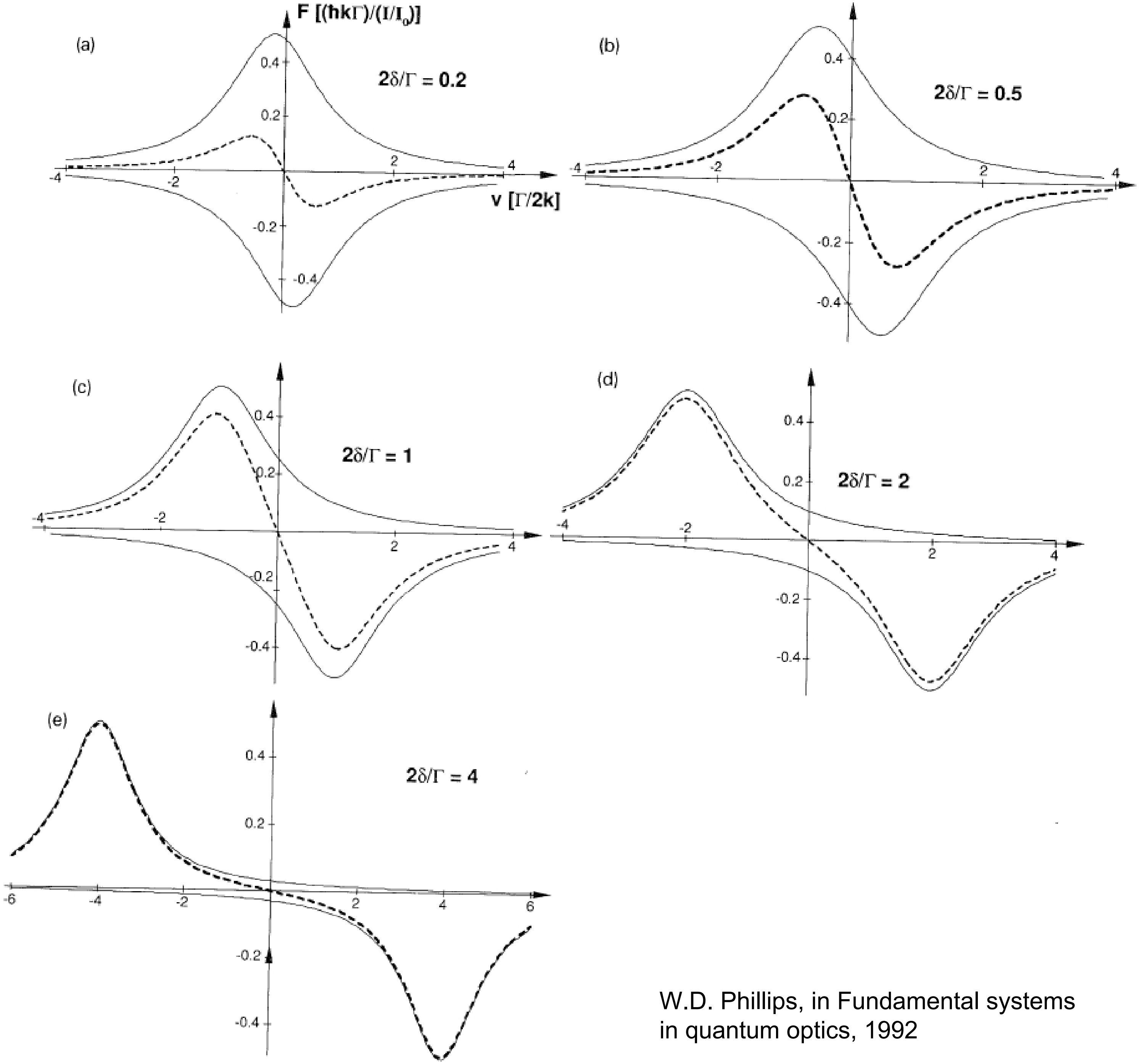
$\omega_e < \omega_0$



For $\Omega^2/2 \ll (\delta_e - kv)^2 + \frac{\Gamma^2}{4}$ add scattering forces

$$F = F_1 - F_2 = \frac{\hbar k \Gamma}{2}\left[\frac{\Omega^2/2}{(\delta_e - kv)^2 + \Gamma^2/4} - \frac{\Omega^2/2}{(\delta_e + kv)^2 + \Gamma^2/4}\right] =$$

$$= -\frac{M}{\tau} v \quad \text{(friction force)}$$

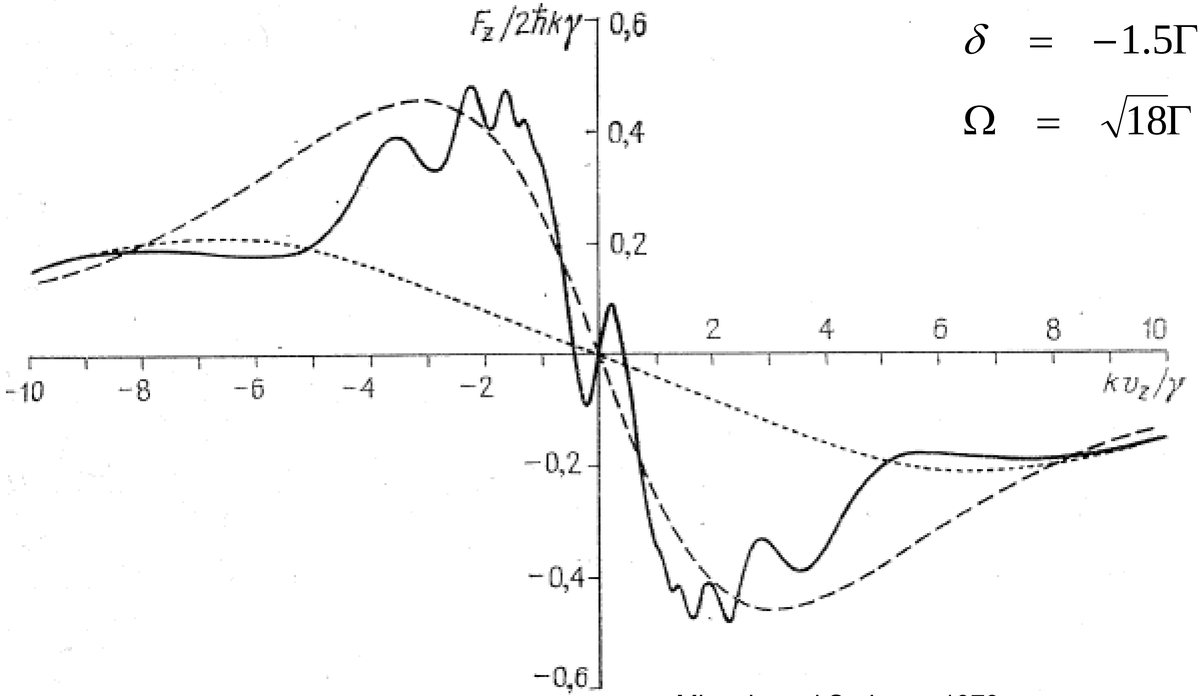


W.D. Phillips, in Fundamental systems in quantum optics, 1992

The case of arbitrary velocities and high intensity

Bloch vector components $u(z) = \sum_n u_n e^{inkz}$ $v(z) = \sum_n v_n e^{inkz}$ $w(z) = \sum_n w_n e^{inkz}$

$$F = -2\hbar k\Gamma \frac{\operatorname{Im} A}{1+2\operatorname{Re} Q} \qquad Q = \cfrac{p_0}{1+\cfrac{p_1}{1+\cfrac{p_2}{1+\dots}}} \qquad A = \frac{\delta}{\Gamma/2+ikv} Q$$

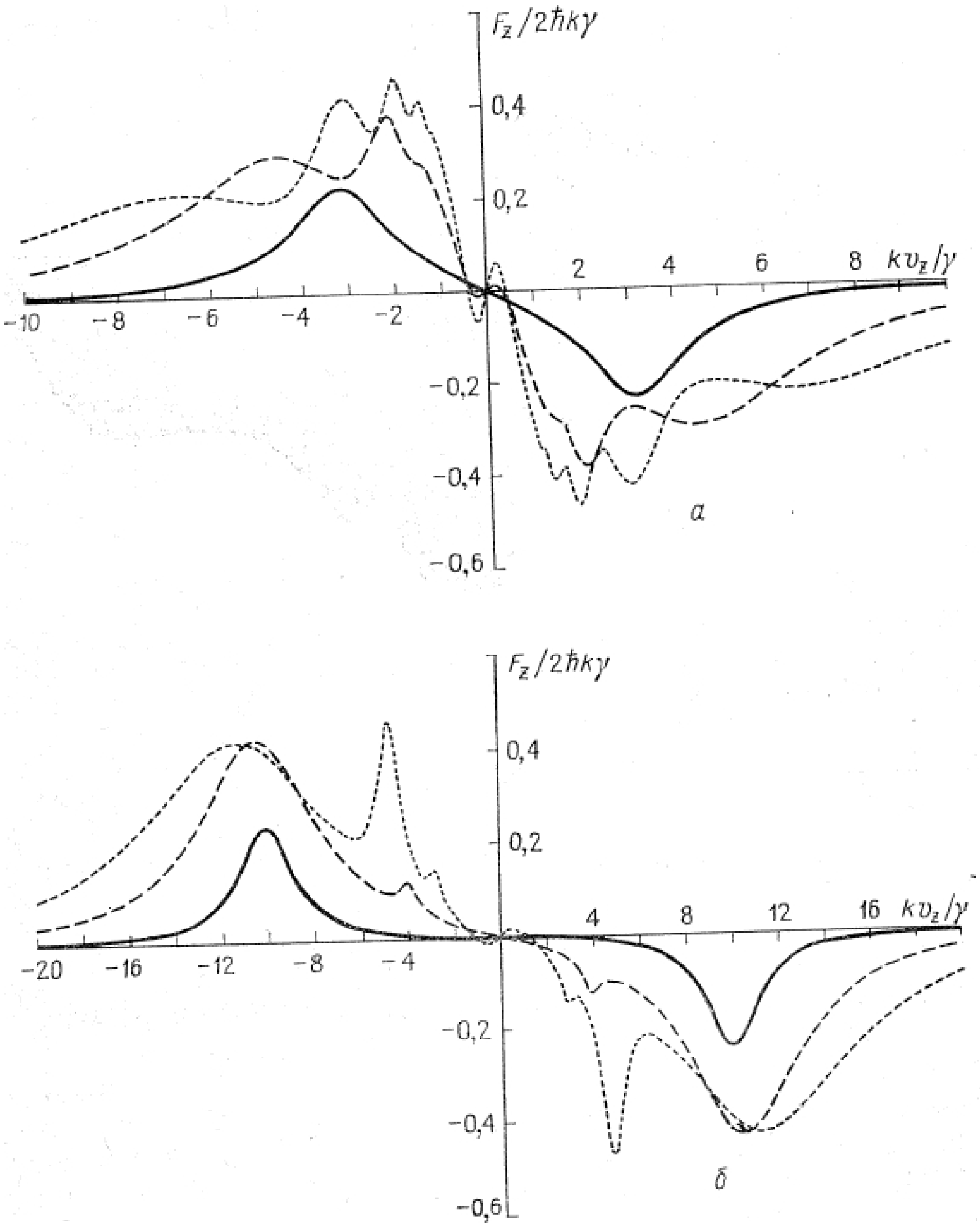


$\delta = -1.5\Gamma$

$\Omega = \sqrt{18}\Gamma$

Minogin and Serimaa, 1979

Minogin and Letokhov, 1984



$$\delta = -1.5\Gamma$$

$$\Omega = \frac{1}{\sqrt{2}}\Gamma$$ solid line

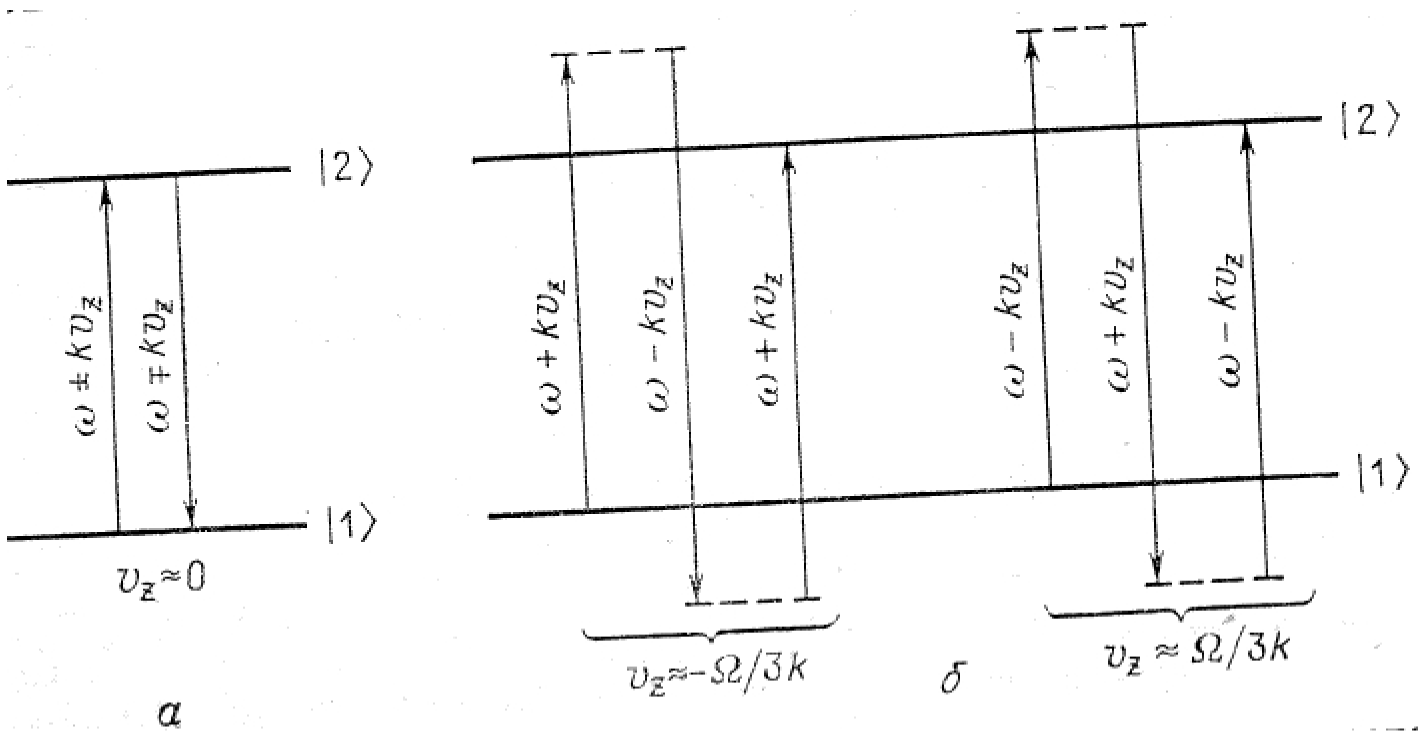
$$\Omega = \sqrt{4.5}\Gamma$$ dashed line

$$\Omega = \sqrt{12.5}\Gamma$$ doted line

$$\delta = -5\Gamma$$

Minogin and Letokhov, 1984

Doppleron (velocity-selective) resonances

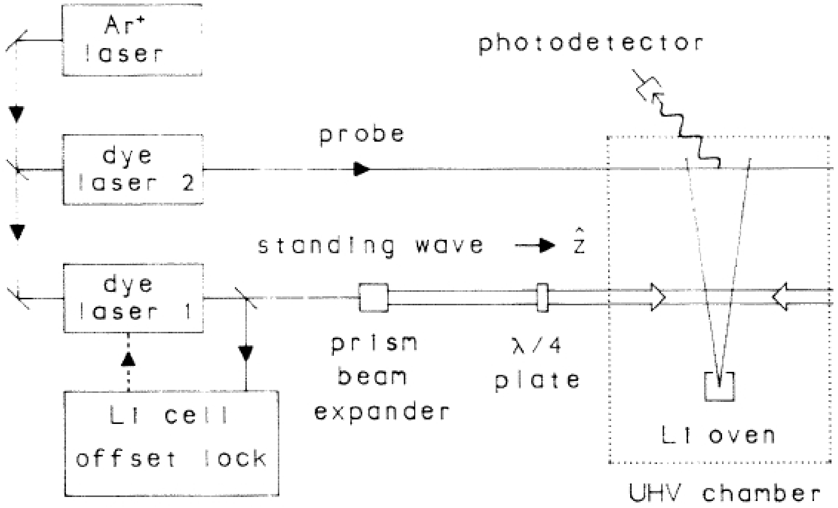


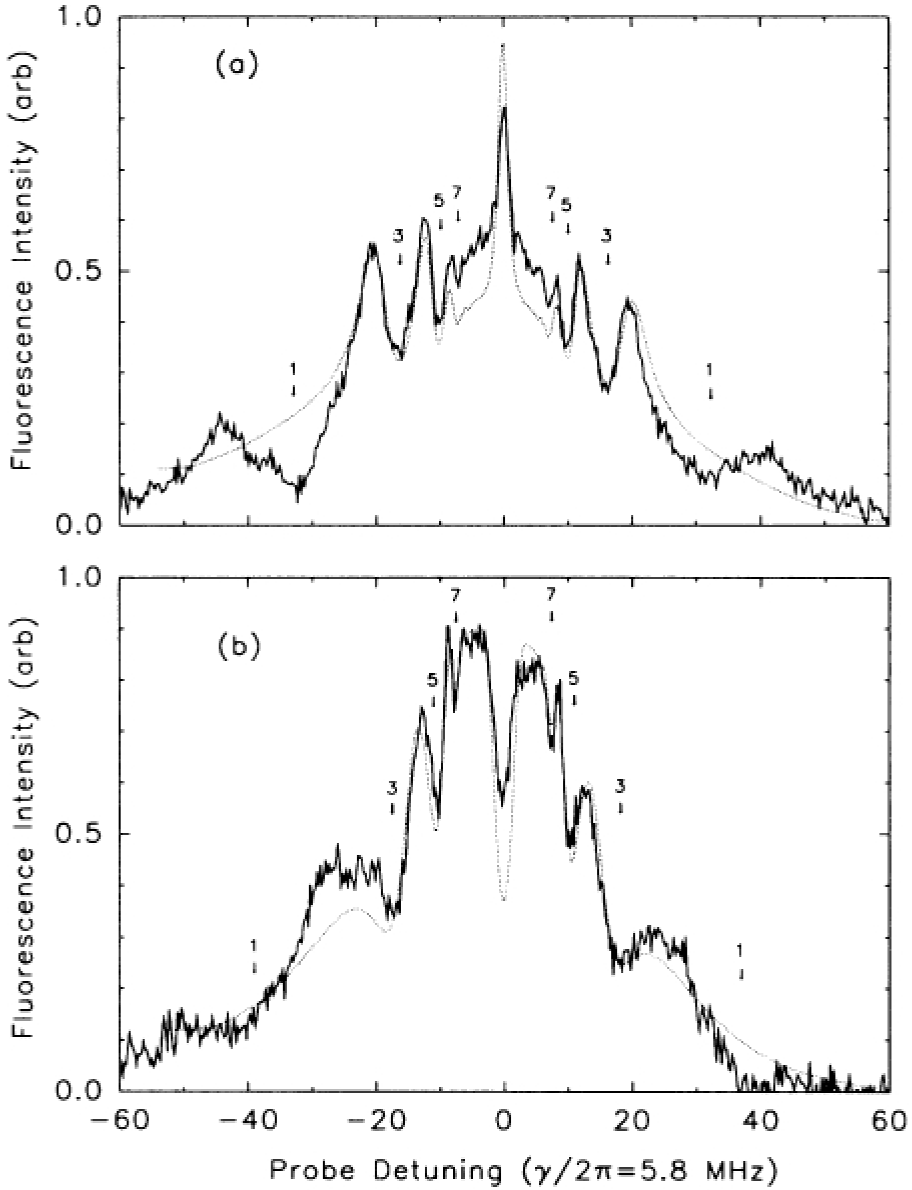
First order $\quad kv = \pm\delta$

Second order $\quad (\omega_l \pm kv) - (\omega_l \mp kv) = 0 \qquad v = 0$

Third order $\quad (\omega_l \pm kv) - (\omega_l \mp kv) + (\omega_l \pm kv) = \omega_0 \qquad kv = \pm\delta/3$

Observation of Doppleron resonances, Hulet et al, PRL 1990



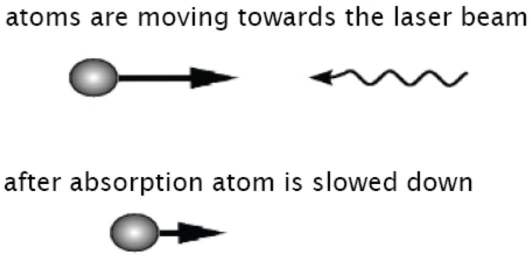


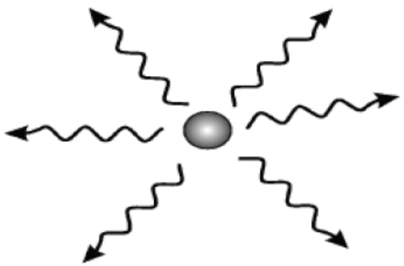
(a) $\delta = +30\Gamma$, $\Omega = 60\Gamma$

(b) $\delta = -30\Gamma$, $\Omega = 60\Gamma$

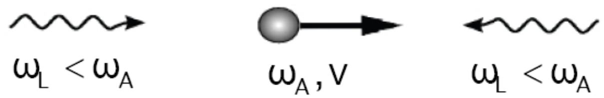
Similar results from atom reflection (Baldwin et al, 1994)

Fokker-Planck equation and cooling limits





Two counterpropagating laser beams



$$F(p,t) \;=\; F_{con}(p) + F_{fluc}(p,t)$$

Moments of the force:

$$M_1 \;=\; \langle F_{con}(p) \rangle$$

$$M_2 \;=\; \langle F_{fluc}(p,t')F_{fluc}(p,t'') \rangle \;=\; 2D(p,t)\delta(t' - t'')$$

D(p,t) is the momentum diffusion coefficient

Fokker-Planck equation: $$\frac{\partial W(p,t)}{\partial t} \;=\; -\frac{[F(p,t)W(p,t)]}{\partial p} + \frac{\partial^2 [D(p,t)W(p,t)]}{\partial p^2}$$

In the case of the friction force $F(v) \;=\; -\alpha v$ and $D(p,t) \;=\; D_0$

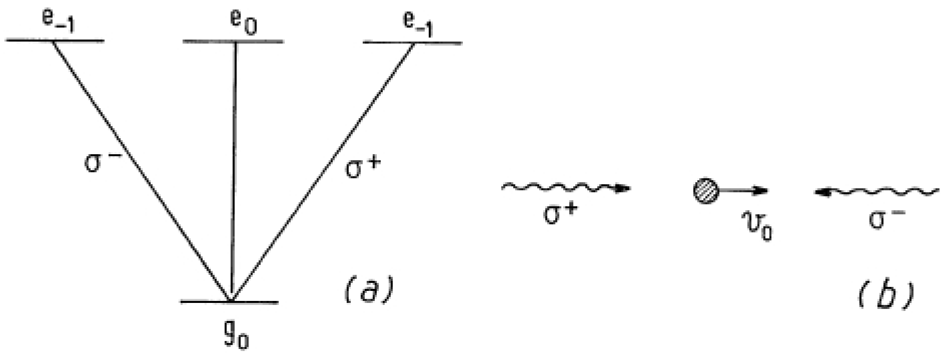
$$W_{st}(p) \;\propto\; e^{-\alpha p^2 / 2MD_0}$$

$$\boxed{k_B T \;=\; D_0/\alpha = \hbar\Gamma/2}$$

Doppler limit

(140 μK for Rb)

σ^+ - σ^- configuration (J. Dalibard et al, 1984)



Exact expression for the force in Dalibard et al, J. Physics B, 1984

$$F = \hbar k \Gamma \left(\rho_{e_{+1}} - \rho_{e_{-1}}\right) = \frac{\hbar k \Gamma}{2} \frac{\delta \Omega^2 k v \left(\Gamma^2 + 4k^2 v^2\right)}{Den}$$

$$Den = \; ... \; long \quad function$$

For small velocities $kv \ll \Gamma$

$$F_{fr} = -\alpha v_0 \qquad \alpha = -\hbar k^2 \Omega_1^2 \frac{\delta \Gamma}{\left[\delta^2 + (\Gamma/2)^2\right]^2} \qquad D = \frac{\hbar^2 k^2 \Gamma^2 \Omega^2}{2\left(\delta^2 + \Gamma^2/4\right)}$$

$$k_B T = \frac{D}{\alpha} = \frac{\hbar \Gamma}{4}\left(\frac{\Gamma}{2\delta} + \frac{2\delta}{\Gamma}\right) \Rightarrow \boxed{k_B T_{Dop} = \frac{\hbar \Gamma}{2}}$$