

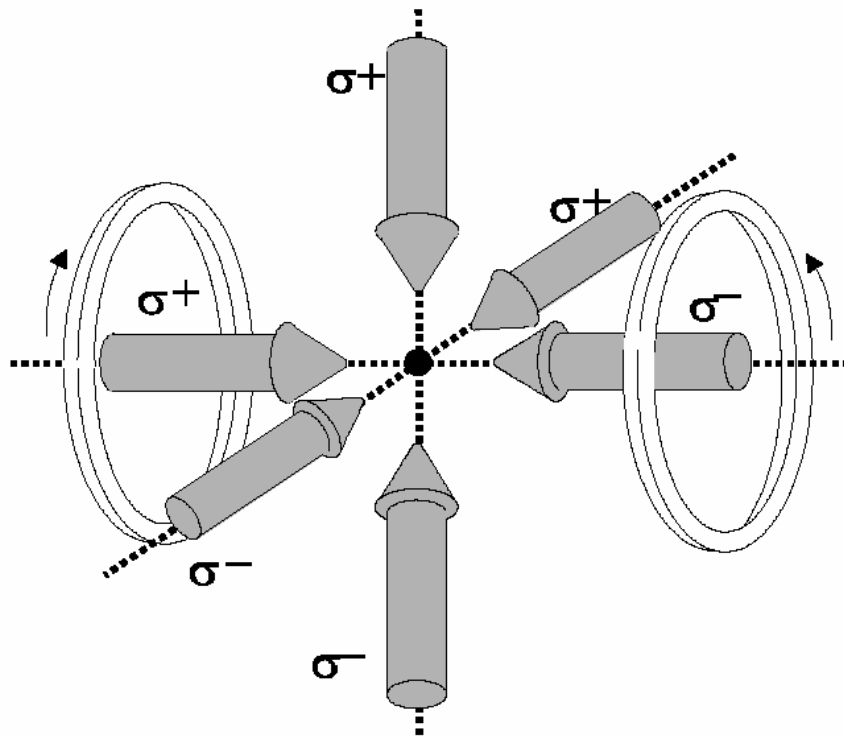
Victorian Summer School on Ultracold Atoms
Course: Laser Cooling and Trapping of Atoms

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1. Magneto-optical trap
2. Sub-Doppler temperatures
3. Polarisation-gradient cooling
4. Laser cooling and BEC on atom chip

Magneto-optical trap



- Three pairs of counterpropagating σ^+ - σ^- laser beams (power 5 mW – 50 mW per beam),
- Quadrupole magnetic field provided by a pair coils in anti-Helmholtz configuration (optimal gradient ~ 10 G/cm)
- Idea proposed by Jean Dalibard in 1987
- Realised by Raab, Prentiss, Cable, Chu and Pritchard in 1987 (Bell labs/ MIT collaboration)

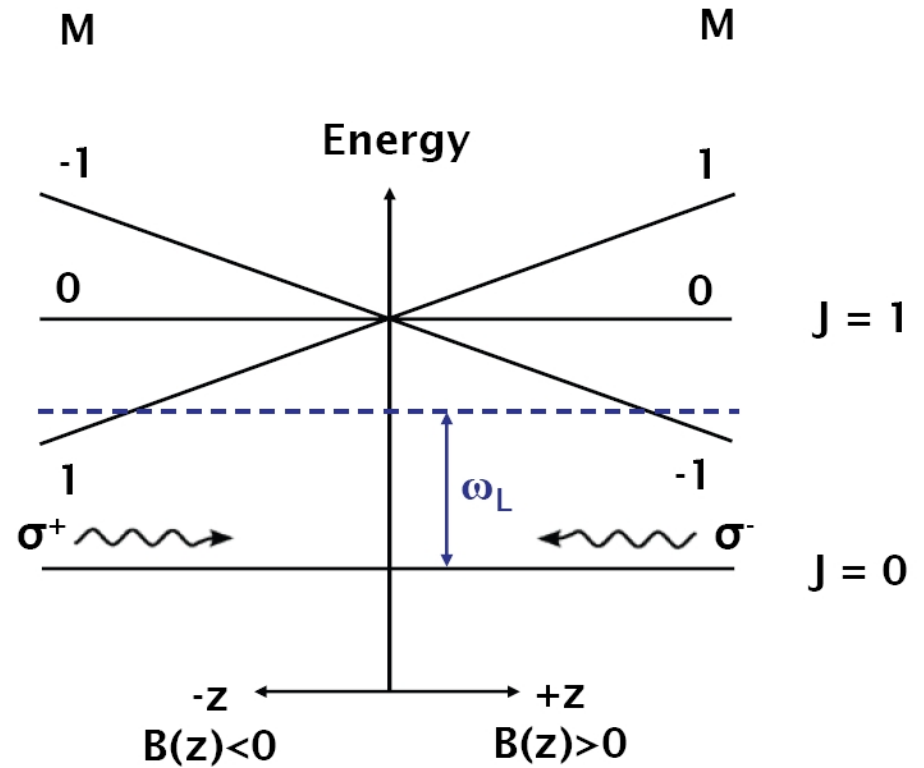
Consider low intensity limit: $\Omega \ll \Gamma$

$$b = \nabla B$$

$$F = F_{\sigma^+} - F_{\sigma^-}$$

$$= \frac{\hbar k \Gamma}{2} \left[\frac{\Omega^2 / 2}{(\delta - kv - bz)^2 + (\Gamma/2)^2} - \frac{\Omega^2 / 2}{(\delta + kv + bz)^2 + (\Gamma/2)^2} \right]$$

$$F(v, z) = -\hbar k \Omega^2 \frac{\delta \Gamma}{(\delta^2 + \Gamma^2 / 4)^2} (kv + bz)$$



Damped harmonic oscillator

$$\ddot{z} + \beta\dot{z} + \omega^2 z = 0$$

$$\beta = \frac{\hbar k^2 \Omega^2 \delta \Gamma}{M [\delta^2 + \Gamma^2 / 4]^2} \quad \omega^2 = \frac{\hbar k \Omega^2 \delta \Gamma}{M [\delta^2 + \Gamma^2 / 4]^2} b$$

$$\frac{\beta^2}{\omega^2} = \frac{\hbar k^3 \Omega^2 \delta \Gamma}{b M [\delta^2 + \Gamma^2 / 4]^2}$$

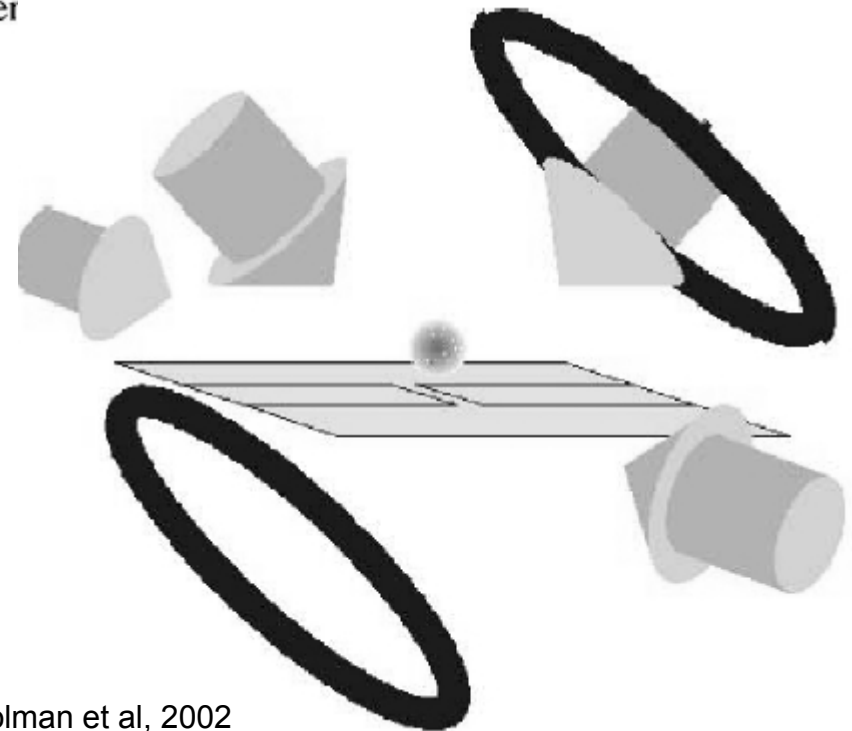
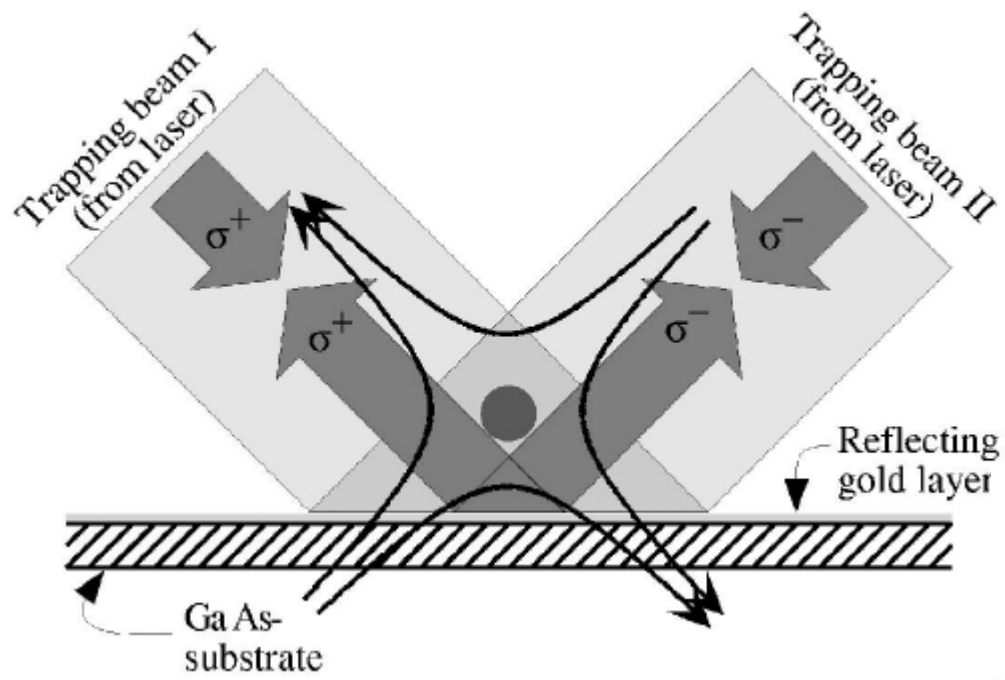
In optimal conditions for cooling (damping) $2\delta/\Gamma = -1$ and $4\Omega^2/\Gamma^2 = 1$

$$\frac{\gamma^2}{4\omega^2} = \frac{\pi E_{rec}}{4\hbar\beta\lambda} \quad \begin{array}{l} = 25 \text{ for Na} \\ = 2.5 \text{ for Cs} \end{array}$$

$$U_{tr} \approx k_B \times 2K$$

$$\text{Doppler limit for Rb} \quad k_B T_{Dop} = \hbar\Gamma/2 = k_B \times 140 \mu K$$

Mirror MOT (Reichel, Munich, 2001)



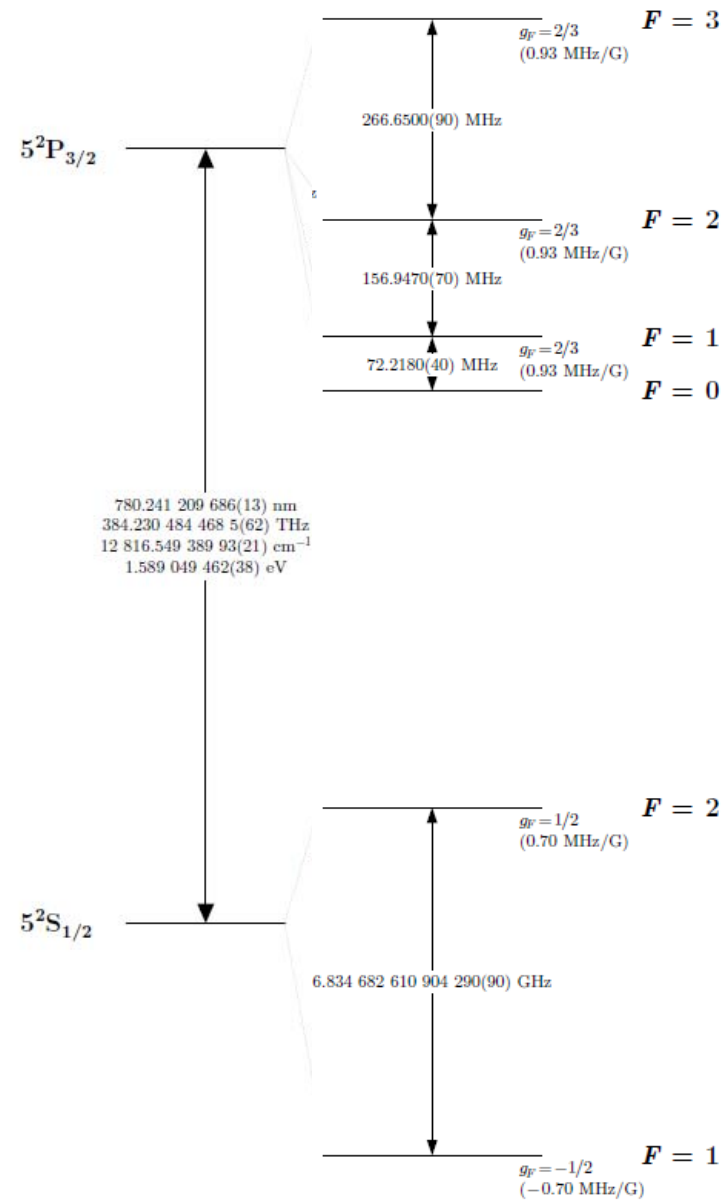
R. Folman et al, 2002

Cooling parameters

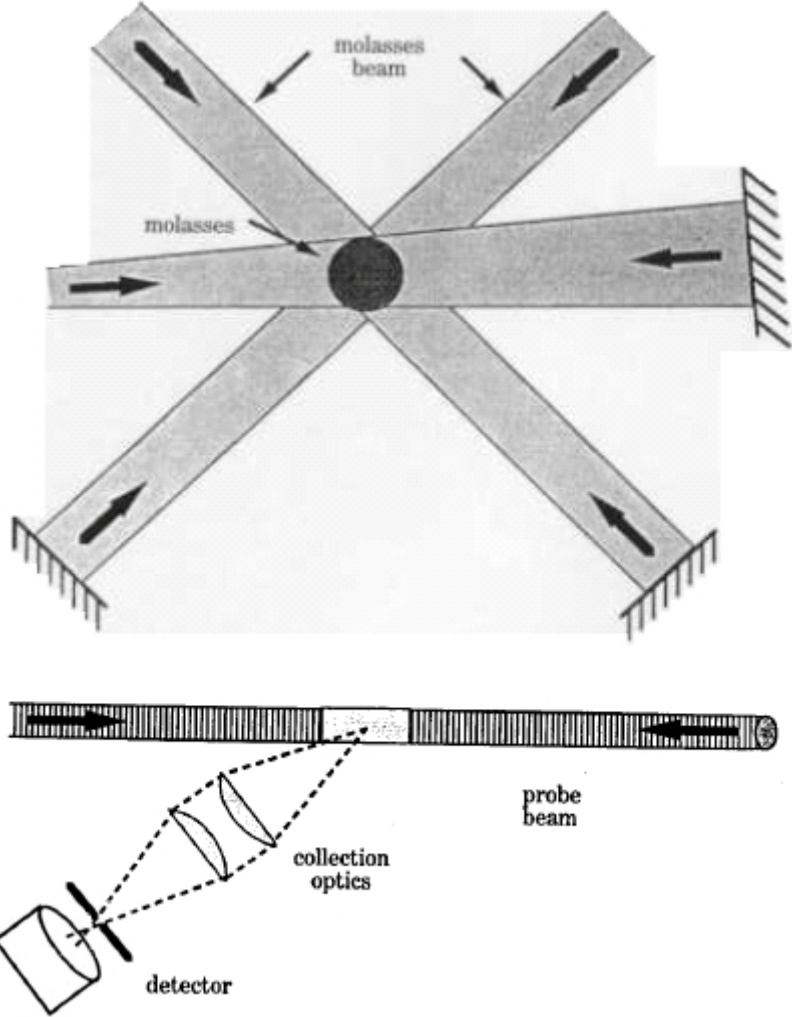
	⁷ Li	²³ Na	⁸⁷ Rb	¹³³ Cs
$\lambda_0 (nm)$	671	589	780	852
$\Gamma / 2\pi (MHz)$	5.9	9.9	5.9	5.3
$I_0 = \frac{\hbar\omega\Gamma k^2}{12\pi} (mW/cm^2)$	2.5	6.3	1.6	1.1
$T_{Dop} = \frac{\hbar\Gamma}{2k_B} (\mu K)$	140	240	140	120
$\frac{E_{rec}}{k_B} = \frac{\hbar^2 k^2}{2Mk_B} (\mu K)$	3	1.2	0.18	0.1

For Na: $v_\Gamma = \frac{\Gamma}{k} = 6m/s$ $v_{rec} = 3cm/s$

Quantum level diagram in ^{87}Rb

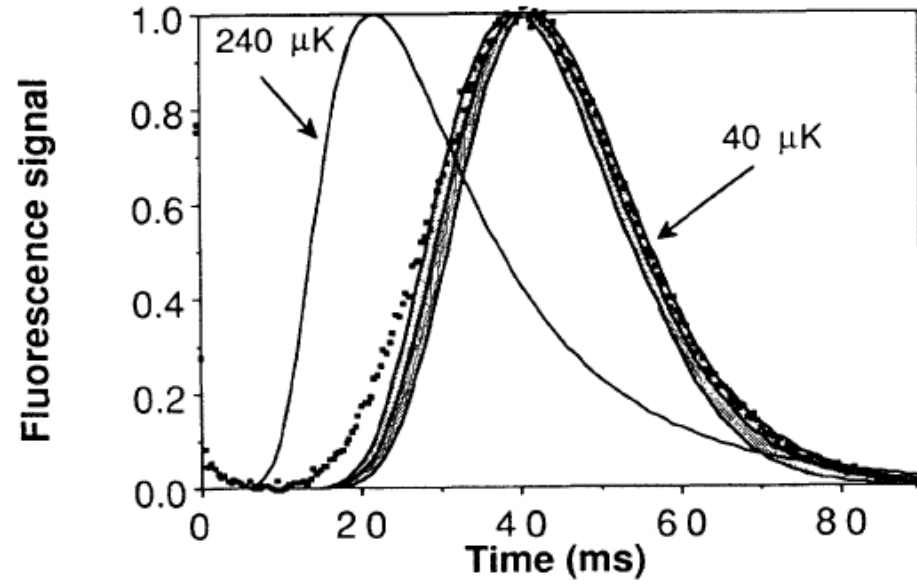


Sub-Doppler temperature in optical molasses (W. Phillips et al, 1988)



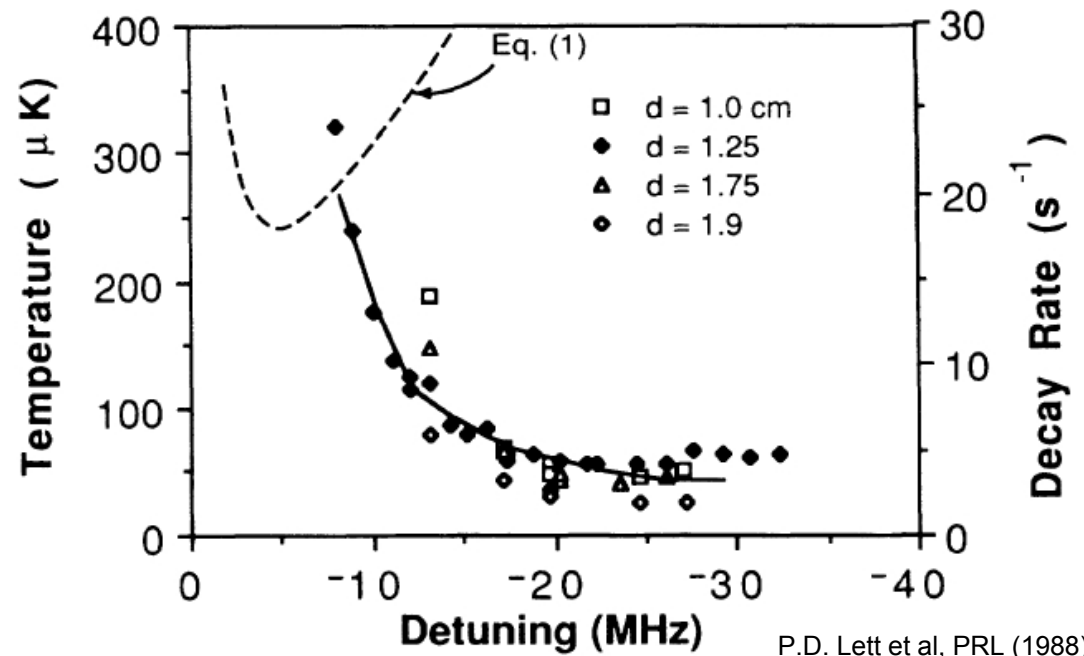
P.D. Lett et al, PRL (1988)

1) Time-of-flight

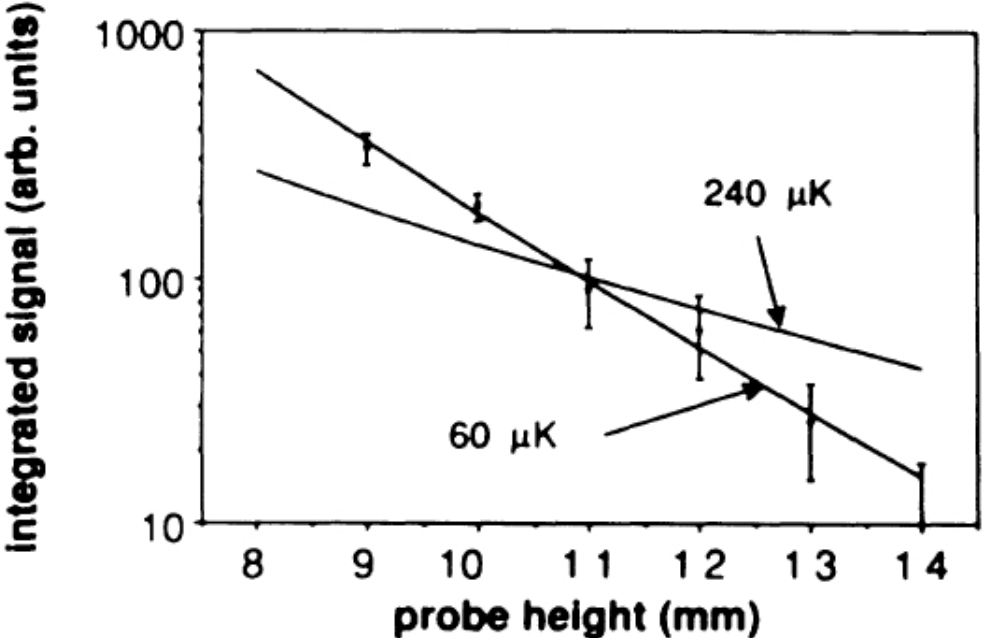


2) Dependence on detuning

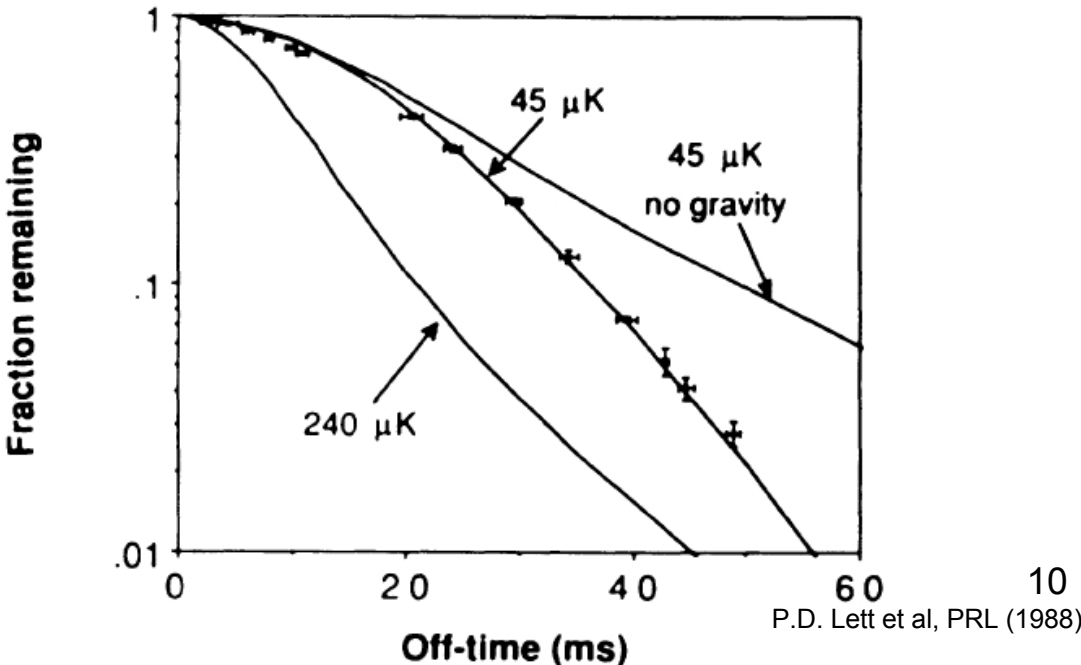
$$k_B T = \frac{D}{\alpha} = \frac{\hbar \Gamma}{4} \left(\frac{\Gamma}{2\delta} + \frac{2\delta}{\Gamma} \right)$$



3) Fountain



4) Release and recapture



Laser cooling below the Doppler limit by polarization gradients: simple theoretical models

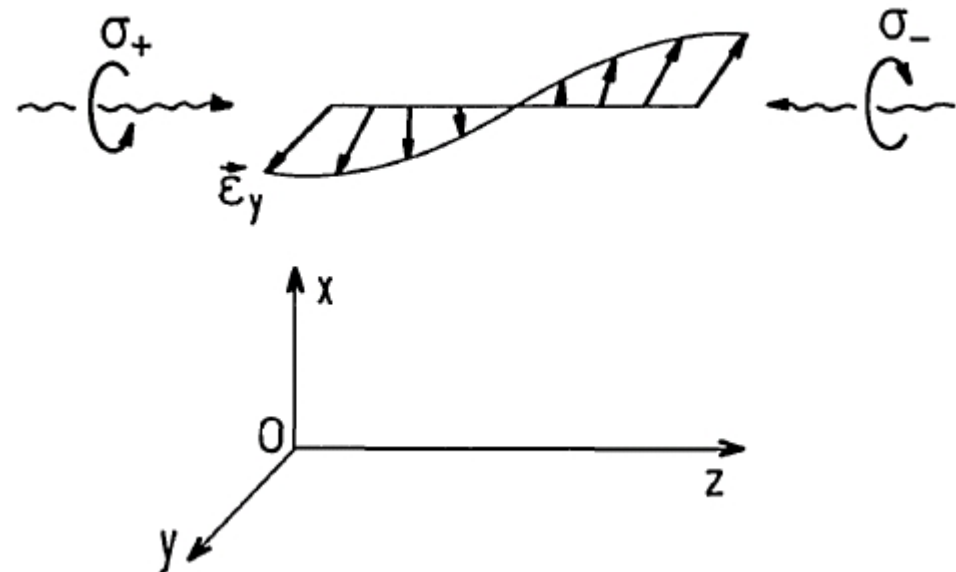
J. Dalibard and C. Cohen-Tannoudji

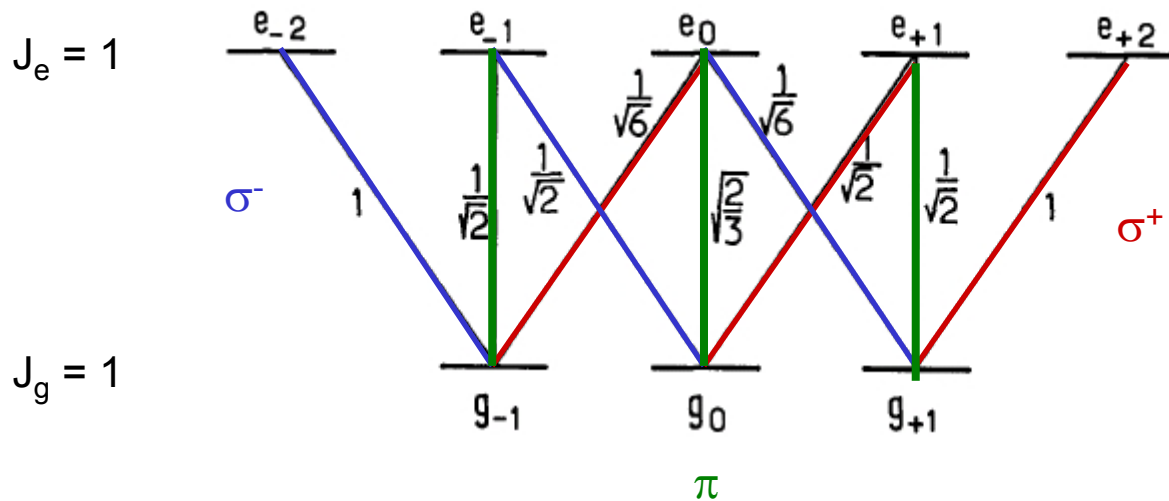
$\sigma^+ - \sigma^-$ configuration

$$\mathbf{E}(z,t) = \mathbf{E}^+(z)e^{-i\omega t} + c.c. \quad \mathbf{E}^+(z) = \mathbf{e}_+ E_0 e^{ikz} + \mathbf{e}_- E_0 e^{-ikz}$$

$$\mathbf{e}_+ = -\frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y) \quad \mathbf{e}_- = \frac{1}{\sqrt{2}}(\mathbf{e}_x - i\mathbf{e}_y)$$

$$\mathbf{E}^+(z) = -i\sqrt{2}E_0(\mathbf{e}_x \sin kz + \mathbf{e}_y \cos kz)$$





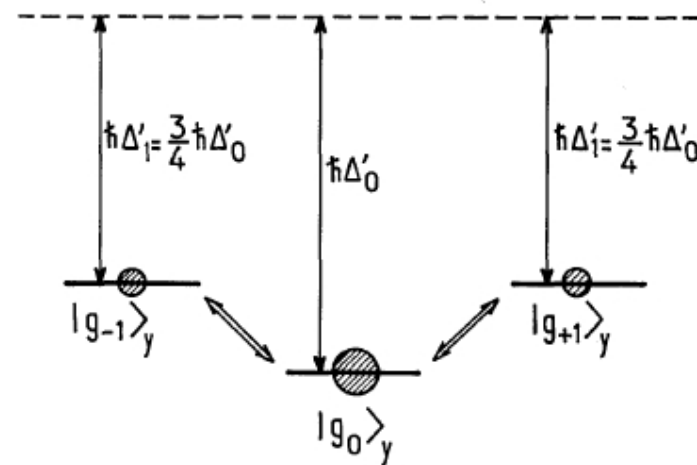
Atom at rest ($z = 0$)

π polarization leads to accumulation of atoms in g_0 (pumping rates $1/4$ and $1/9$).

Steady state populations $4/17$, $9/17$, $4/17$.

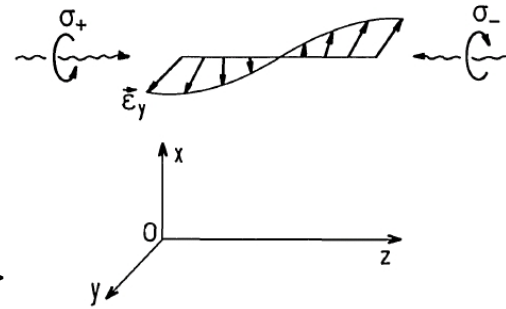
Stronger coupling leads to the light shift

$$\Delta'_0 = \frac{4}{3}\Delta'_1$$



J. Dalibard et al, JOSA (1989)

Moving atom and motion-induced orientation



$$z = vt \longrightarrow \varphi = -kz = kv t$$

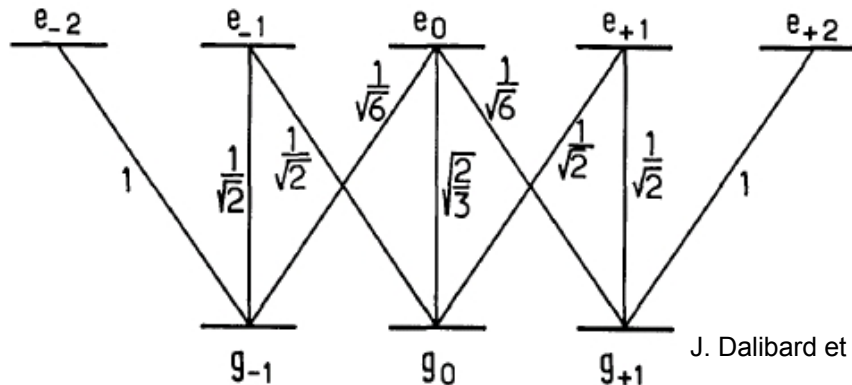
In the rotating frame the new $|g_0\rangle$ is contaminated by $|g_1\rangle$ and $|g_{-1}\rangle$

$$|g_0\rangle_{rot} = |g_0\rangle + \frac{kv}{\sqrt{2}(\Delta'_0 - \Delta'_1)} |g_{+1}\rangle + \frac{kv}{\sqrt{2}(\Delta'_0 - \Delta'_1)} |g_{-1}\rangle$$

$$\langle J_z \rangle_{st} = \frac{40 \hbar kv}{17 \Delta'_0} \longrightarrow \Pi_{+1} - \Pi_{-1} = \frac{40 kv}{17 \Delta'_0}$$

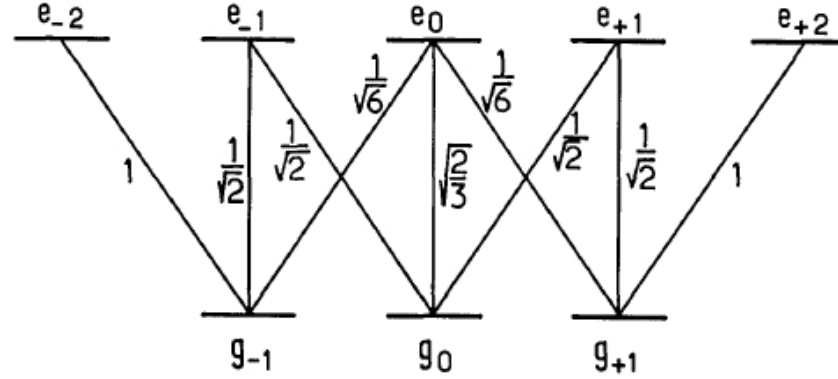
$$\text{For } v > 0 \text{ and } \delta < 0 \longrightarrow \Pi_{-1} > \Pi_{+1}$$

Different populations lead to unbalanced radiation pressure, eg for $v > 0$ an atom will scatter more counter-propagating σ^- photons than co-propagating σ^+ photons.



Does not work for $J_g = 1/2$

J. Dalibard et al, JOSA (1989)



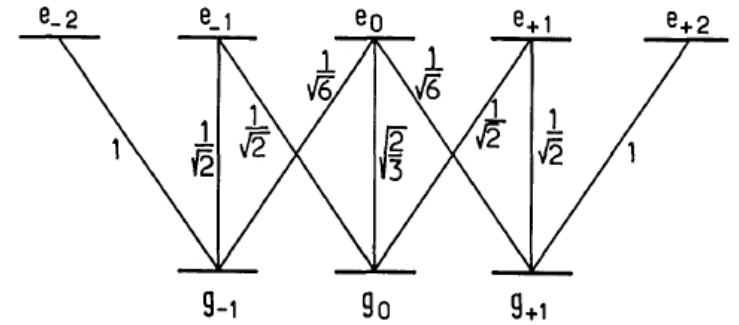
$$\hat{V} = -\frac{\hbar\Omega}{2} \left(|g_1\rangle\langle e_2| + \frac{1}{\sqrt{2}} |g_0\rangle\langle e_1| + \frac{1}{\sqrt{6}} |g_{-1}\rangle\langle e_0| \right) e^{i(\omega_- t - kz)} \quad \Omega = \frac{2dE_0}{\hbar}$$

$$-\frac{\hbar\Omega}{2} \left(|g_{-1}\rangle\langle e_{-2}| + \frac{1}{\sqrt{2}} |g_0\rangle\langle e_{-1}| + \frac{1}{\sqrt{6}} |g_1\rangle\langle e_0| \right) e^{i(\omega_+ t - kz)} + h.c. \quad \omega_{\pm} = \omega_l \pm kv$$

$$\mathbf{F}_{\sigma} = \left\langle -\frac{dV}{dz} \right\rangle = -i\hbar\mathbf{k}\Omega \left[\rho(e_2, g_1) + \frac{1}{\sqrt{2}} \rho(e_1, g_0) + \frac{1}{\sqrt{6}} \rho(e_0, g_{-1}) \right]$$

$$+ i\hbar\mathbf{k}\Omega \left[\rho(e_{-2}, g_{-1}) + \frac{1}{\sqrt{2}} \rho(e_{-1}, g_0) + \frac{1}{\sqrt{6}} \rho(e_0, g_1) \right] + c.c.$$

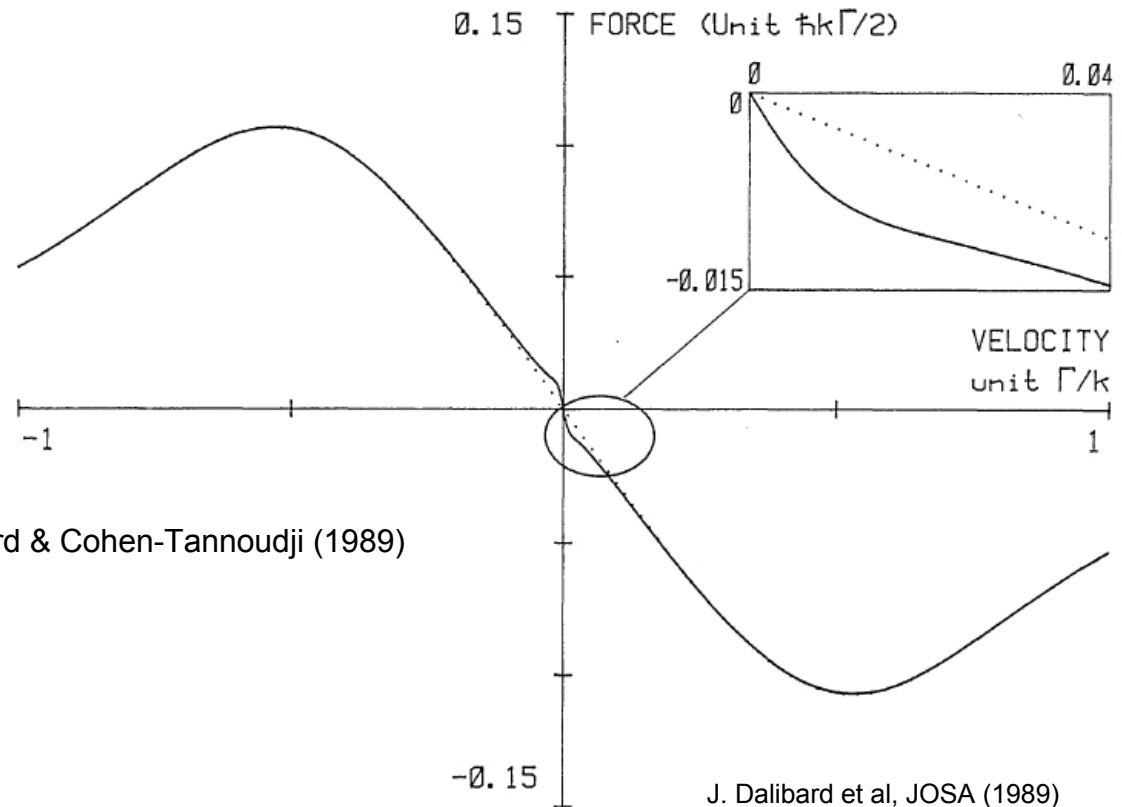
$$\mathbf{F}_\sigma = \frac{\hbar \mathbf{k} \Gamma}{2} \left[\Pi_1 \left(s_+ - \frac{s_-}{6} \right) + \Pi_0 \left(\frac{s_+ - s_-}{2} \right) + \Pi_{-1} \left(\frac{s_+}{6} - s_- \right) \right] + C_r \left(\frac{s_+ - s_-}{6} \right) - \frac{1}{3} C_i \left(s_+ \frac{\delta - kv}{\Gamma} + s_- \frac{\delta + kv}{\Gamma} \right)$$



$$C_r = \text{Re} \left[\langle g_1 | \rho | g_{-1} \rangle e^{-2ikvt} \right]$$

$$C_i = \text{Im} \left[\langle g_1 | \rho | g_{-1} \rangle e^{-2ikvt} \right]$$

$$s_\pm = \frac{\Omega^2 / 2}{(\delta \mp kv)^2 + \Gamma^2 / 4}$$



Evaluation of Π_i and C_i is given in Dalibard & Cohen-Tannoudji (1989)

In the low velocity domain

$$F_{\sigma} = \frac{\hbar k \Gamma}{2} s_0 \left[\frac{5}{6} (\Pi_1 - \Pi_{-1}) - \frac{2\delta}{3\Gamma} C_i \right]$$

$$s_0 = \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4}$$

$$\mathbf{F}_{\sigma} \cong -\alpha v \quad \alpha = \frac{120}{17} \frac{-\delta\Gamma}{5\Gamma^2 + 4\delta^2} \hbar k^2$$

← Independent on $\Omega!!!$

$$D = \left[\frac{18}{170} + \frac{4}{17} + \frac{36}{17} \frac{1}{1 + 4\delta^2/5\Gamma^2} \right] \hbar^2 k^2 \Gamma s_0$$

$$k_B T = \frac{\hbar \Omega^2}{|\delta|} \left[\frac{29}{300} + \frac{254}{75} \frac{\Gamma^2/4}{\delta^2 + \Gamma^2/4} \right]$$

$J = 1 \rightarrow J = 2$ atom

$$k_B T = \frac{D}{\alpha} = \frac{\hbar \Gamma}{4} \left(\frac{\Gamma}{2\delta} + \frac{2\delta}{\Gamma} \right)$$

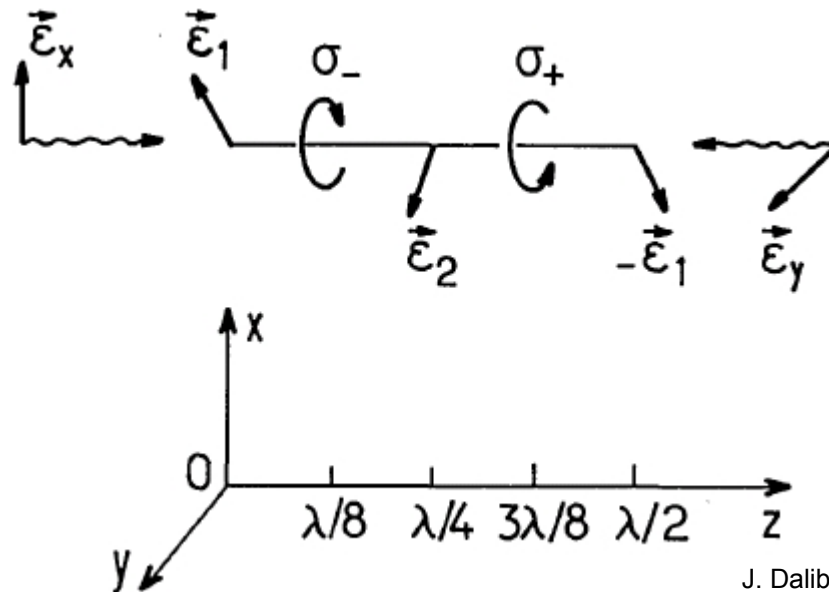
compare to $J = 0 \rightarrow J = 1$ atom

Lin \perp Lin laser configuration

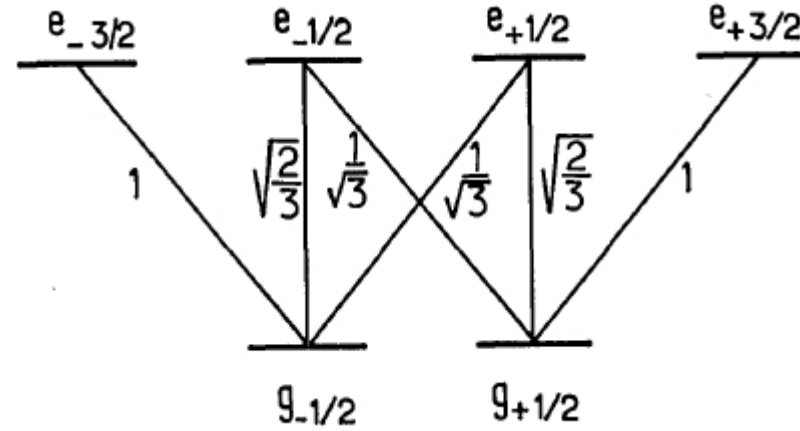
Again follow J. Dalibard and C. Cohen-Tannoudji, JOSA B **6**, 2025 (1989)

$$\mathbf{E}(z,t) = \mathbf{E}^+(z)e^{-i\omega t} + c.c.$$

$$\begin{aligned} \mathbf{E}^+(z) &= \mathbf{e}_x E_0 e^{ikz} + \mathbf{e}_y E_0 e^{-ikz} \\ &= E_0 \sqrt{2} \left(\cos kz \frac{\mathbf{e}_x + \mathbf{e}_y}{\sqrt{2}} - i \sin kz \frac{\mathbf{e}_x - \mathbf{e}_y}{\sqrt{2}} \right) \end{aligned}$$



J. Dalibard et al, JOSA (1989)



$$\hat{V} = -\frac{\hbar\Omega}{\sqrt{2}}\sin kz\left(|e_{3/2}\rangle\langle g_{1/2}| + \frac{1}{\sqrt{3}}|e_{1/2}\rangle\langle g_{-1/2}|\right)e^{-i\omega t}$$

$$- \frac{\hbar\Omega}{\sqrt{2}}\cos kz\left(|e_{-3/2}\rangle\langle g_{-1/2}| + \frac{1}{\sqrt{3}}|e_{-1/2}\rangle\langle g_{1/2}|\right)e^{-i\omega t} + h.c.$$

$$\Omega = \frac{2dE_0}{\hbar}$$

$$F_{\perp} = \left\langle -\frac{d\hat{V}}{dz} \right\rangle$$

$$= \frac{\hbar k\Omega}{\sqrt{2}}\cos kz\left[\rho(g_{1/2}, e_{3/2}) + \frac{1}{\sqrt{3}}\rho(g_{-1/2}, e_{1/2}) + c.c.\right]$$

$$- \frac{\hbar k\Omega}{\sqrt{2}}\sin kz\left[\rho(g_{-1/2}, e_{-3/2}) + \frac{1}{\sqrt{3}}\rho(g_{1/2}, e_{-1/2}) + c.c.\right]$$

In the limits of low intensity ($\Omega \ll \Gamma$) and low velocities ($kv \ll \Gamma$)

The optical coherence adiabatically follows the ground-state population

$$\tilde{\rho}(g_{1/2}, e_{3/2}) = \frac{\Omega/\sqrt{2}}{\delta - i\Gamma/2} \Pi_{1/2} \sin kz$$

$$F_{\perp} = -\frac{2}{3} \hbar k \delta s_0 (\Pi_{1/2} - \Pi_{-1/2}) \sin 2kz$$

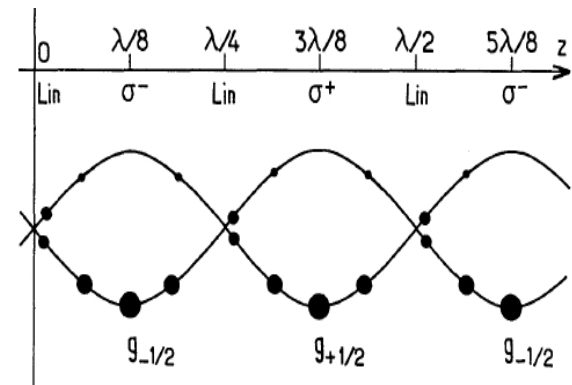
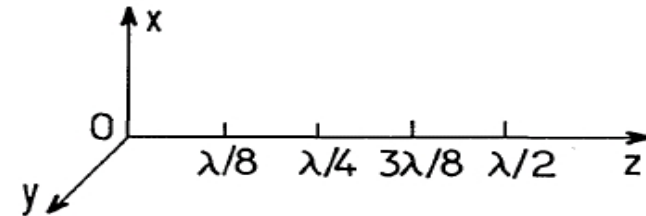
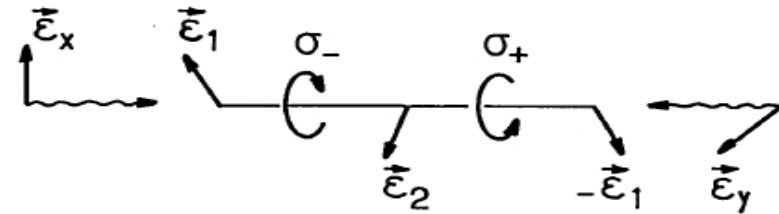
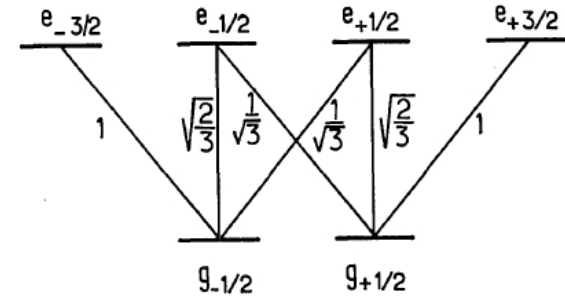
$$\Delta E_{1/2} = \hbar \Delta'_+ = E_0 - \frac{\hbar \delta s_0}{3} \cos 2kz$$

$$\Delta E_{-1/2} = \hbar \Delta'_- = E_0 + \frac{\hbar \delta s_0}{3} \cos 2kz$$

$$E_0 = \frac{2}{3} \hbar \delta s_0$$

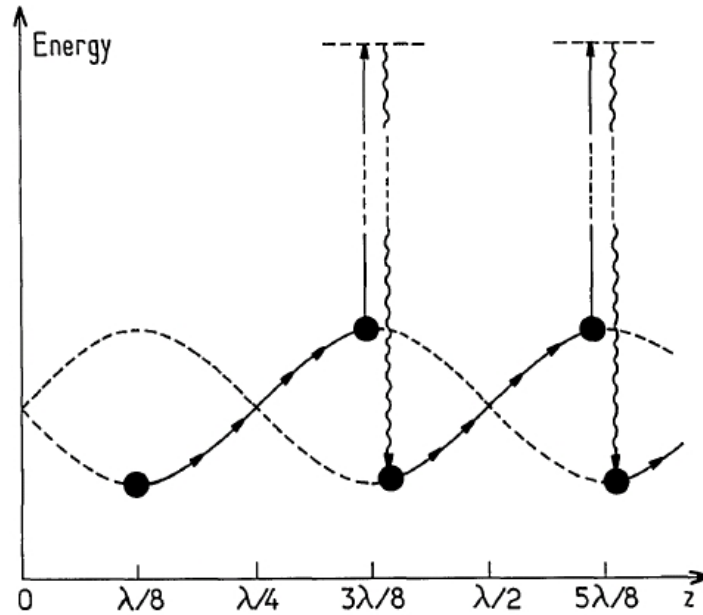
$$f_{\pm 1/2} = -\frac{d}{dz} \Delta E_{\pm 1/2}$$

$$F_{\perp} = f_{1/2} \Pi_{1/2} + f_{-1/2} \Pi_{-1/2}$$



light shift

Sisyphus cooling

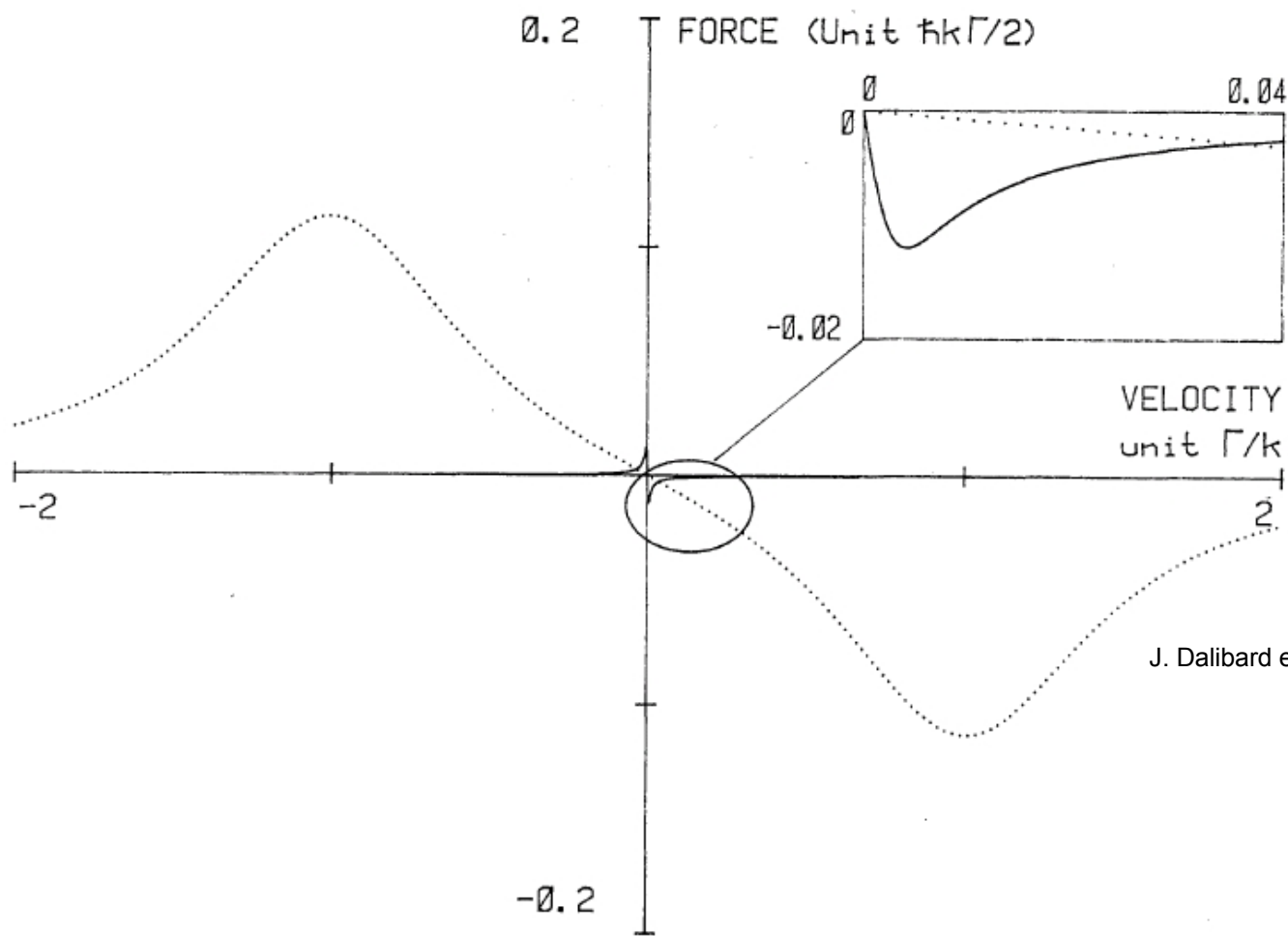


J. Dalibard et al, JOSA (1989)

Optical pumping time τ_p

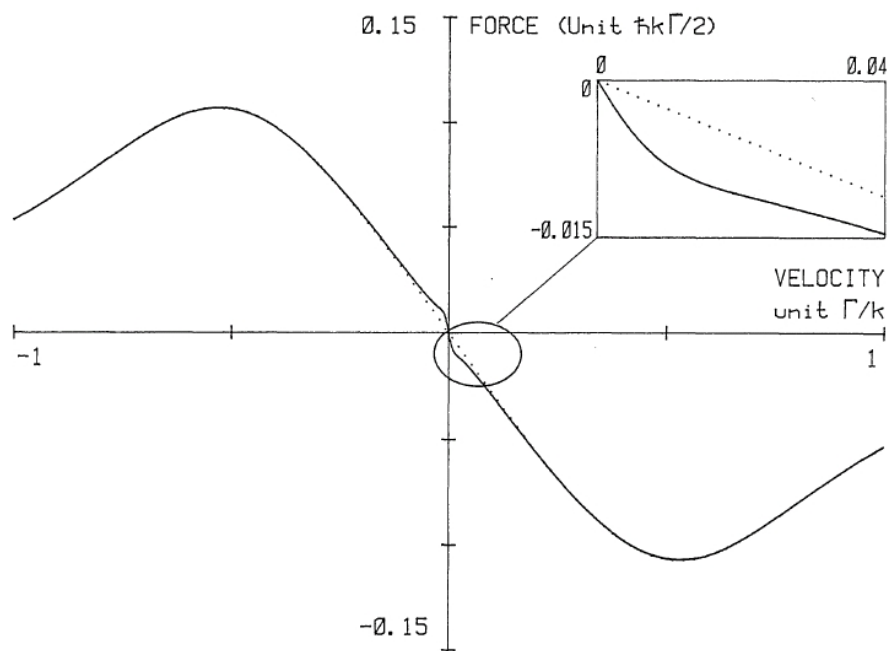
$$\langle F_{\perp} \rangle_{\lambda} = \frac{-\alpha v}{1 + v^2/v_{cr}^2} \quad kv_{cr} = 1/2\tau_p \quad \alpha = -3\hbar k^2 \frac{\delta}{\Gamma} \quad \text{Independent on } \Omega!!!$$

$$D = \frac{3}{4} \hbar^2 k^2 \frac{\delta^2}{\Gamma} s_0 \quad k_B T = \frac{D}{\alpha} \approx \frac{\hbar \Omega^2}{8|\delta|}$$

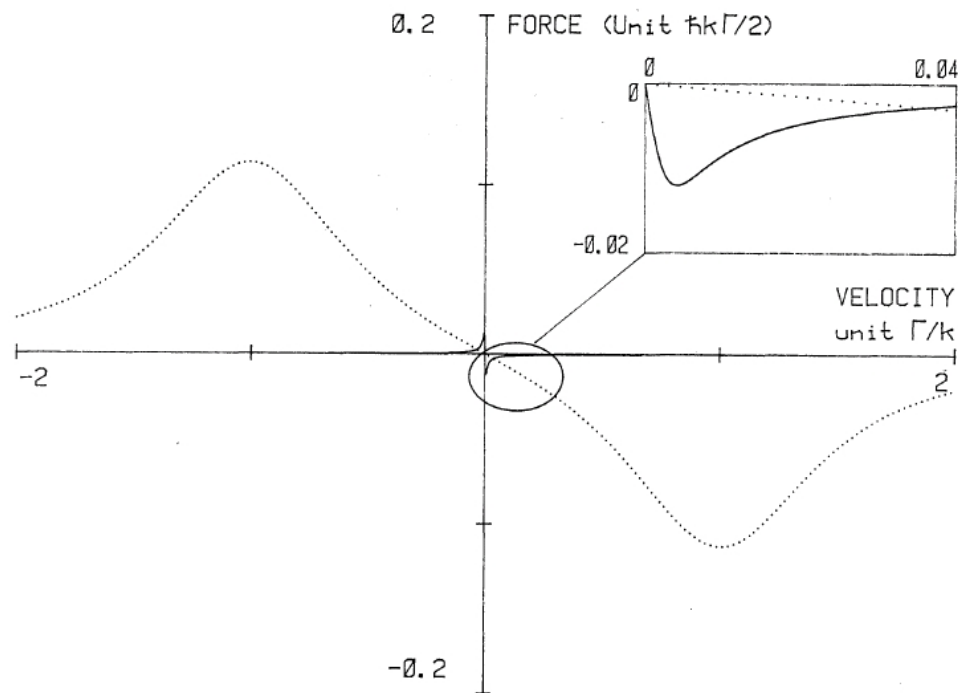


J. Dalibard et al, JOSA (1989)

$\sigma^+ - \sigma^-$



Lin \perp lin



J. Dalibard et al, JOSA (1989)

“Lin – 45° lin” magneto-optical force (Grimm, Sidorov et al, 1992-93)

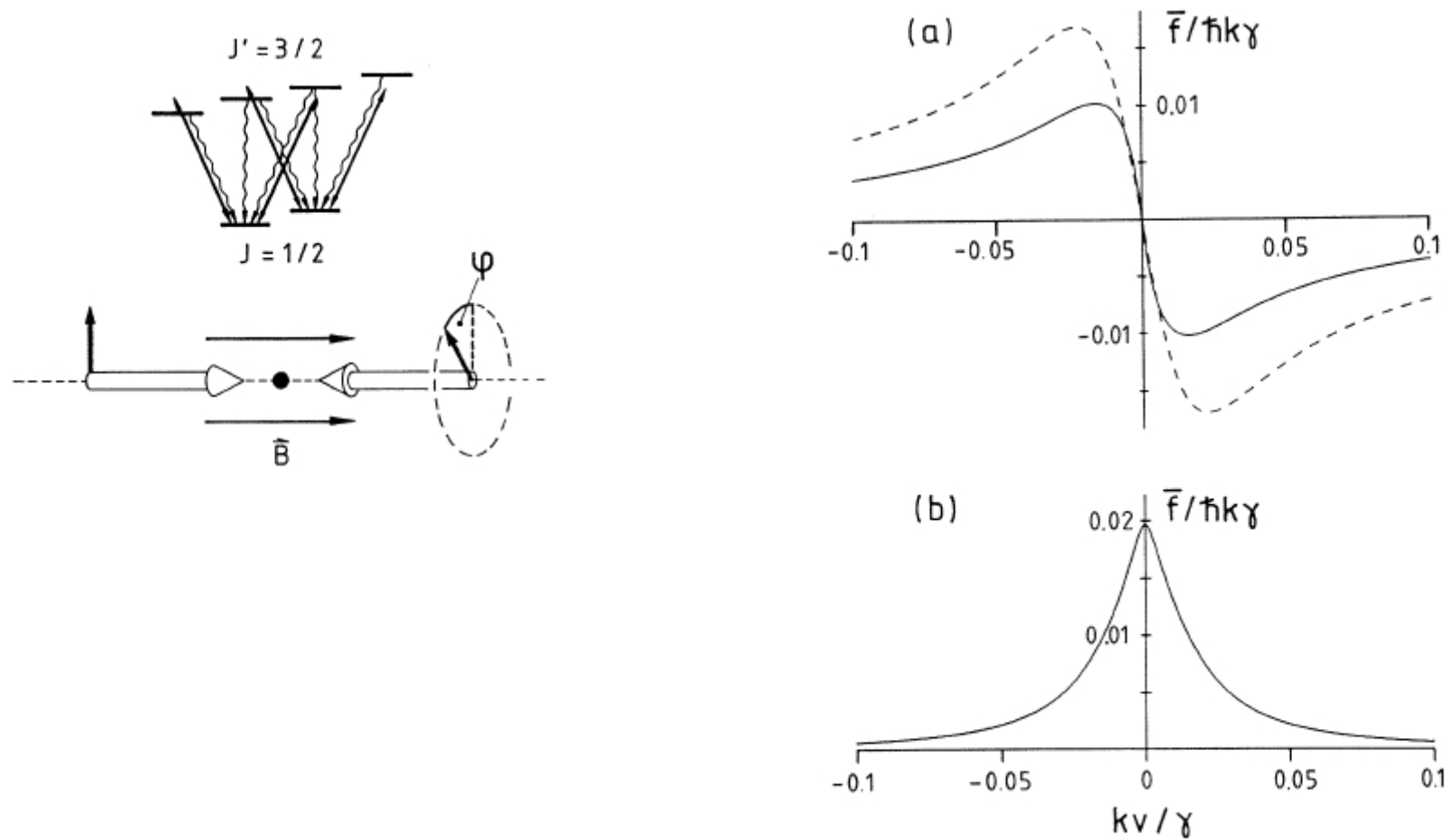


Fig. 2. Two elementary cases of the sub-Doppler radiation force $\bar{f}(v)$. (a) The pure polarization gradient cooling force at $\omega_L=0$, $\Delta=-\gamma$, and $G_0=0.2$ for $\varphi=90^\circ$ (dashed curve) and $\varphi=45^\circ$ (solid curve). (b) The pure sub-Doppler magneto-optical force at $\omega_L=\gamma$, $\Delta_0=0$, $G_0=0.2$, and $\varphi=45^\circ$.

“Lin – 45° lin” magneto-optical trap (Salomon et al, 1992)

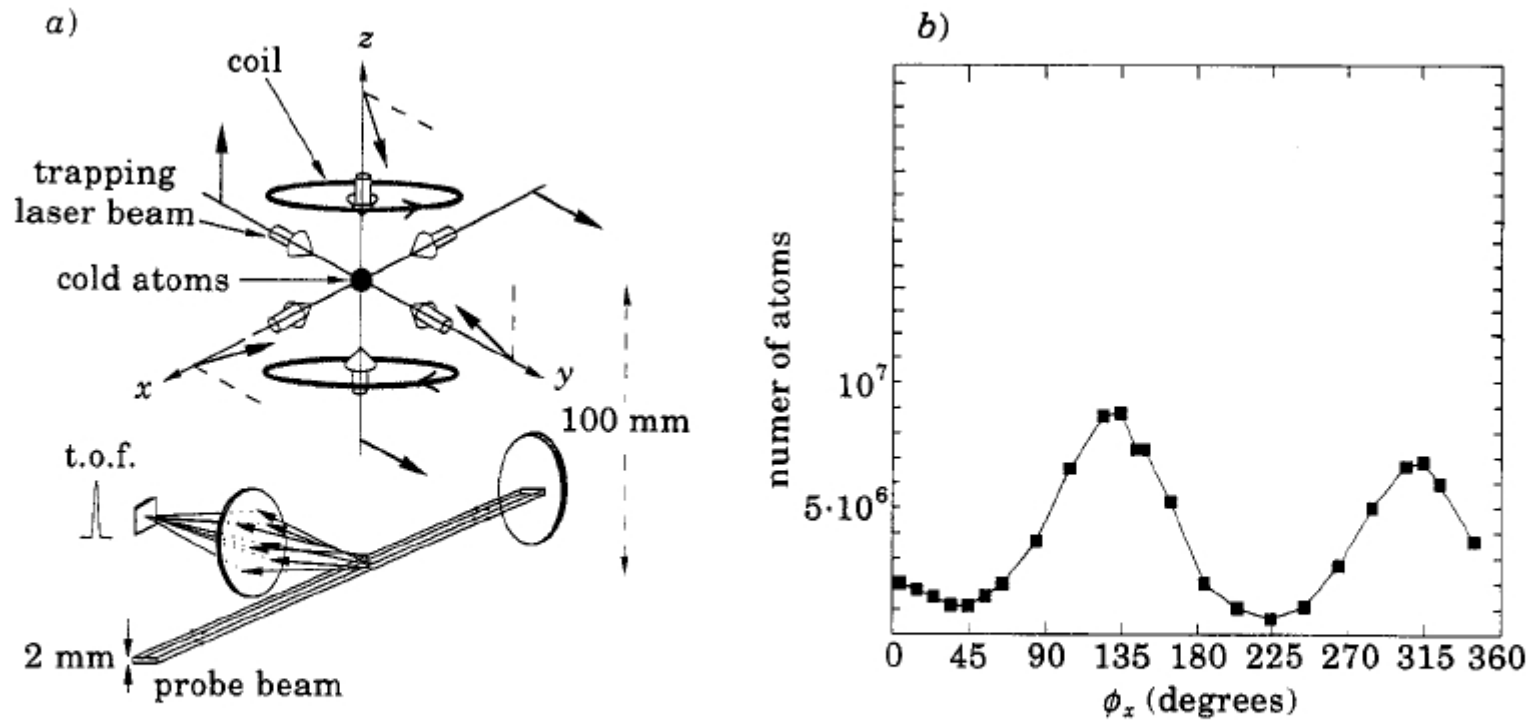
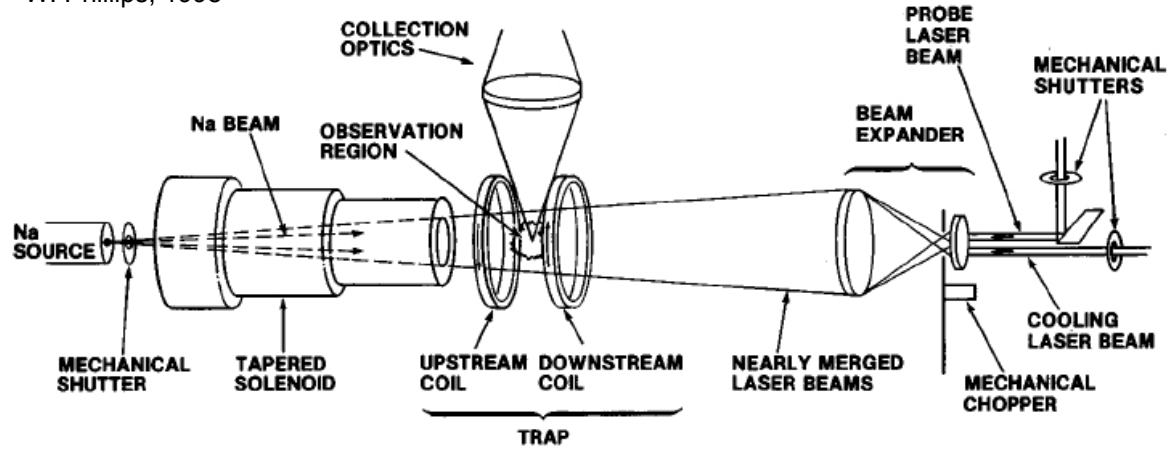


Fig. 1. – a) Experimental set-up and polarization configuration. The temperature of the trapped atoms is measured by a time-of-flight technique. b) Number of trapped atoms as a function of the angle ϕ_x between the polarization vectors in the Ox -arm of the trap. The locations of the minima and extrema demonstrate the role of the new magneto-optical force of ref. [4] in this trap.

W. Phillips, 1998



$$N_{tr} = \frac{d^2}{\sigma_c} \left(\frac{v_c}{v_{mp}} \right)^4$$

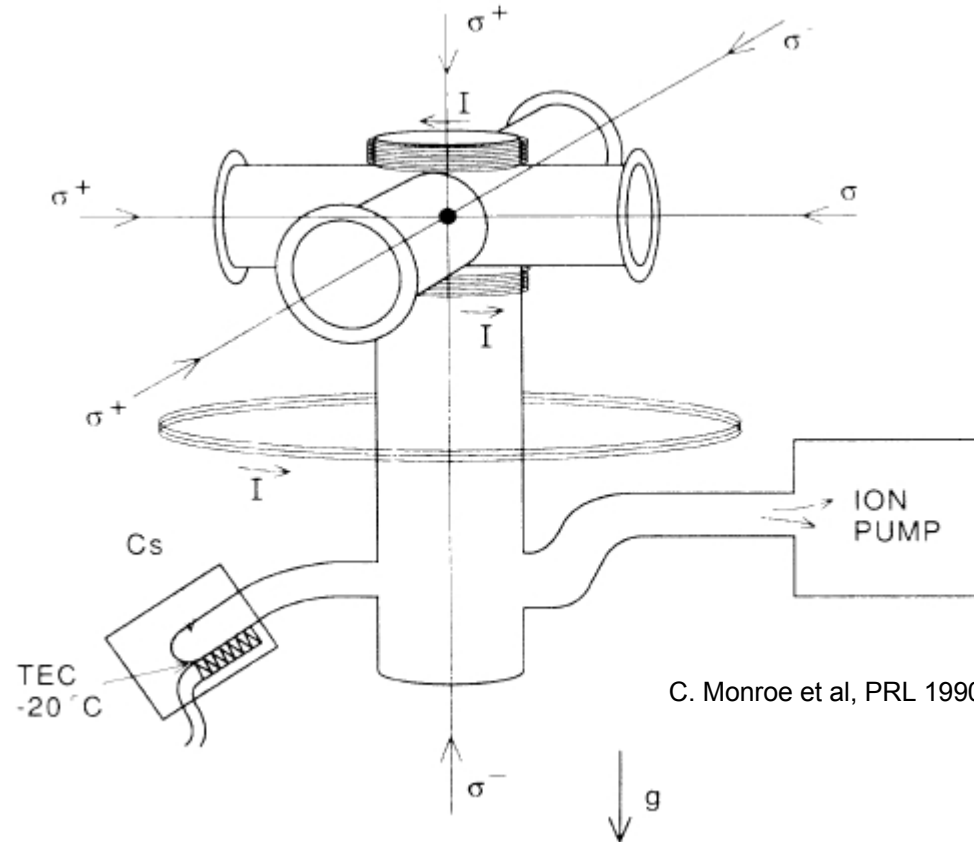
σ – collision rate, d - diameter

$$v_{cr}^2 \sim d$$

$$N_{tr} \sim d^4$$

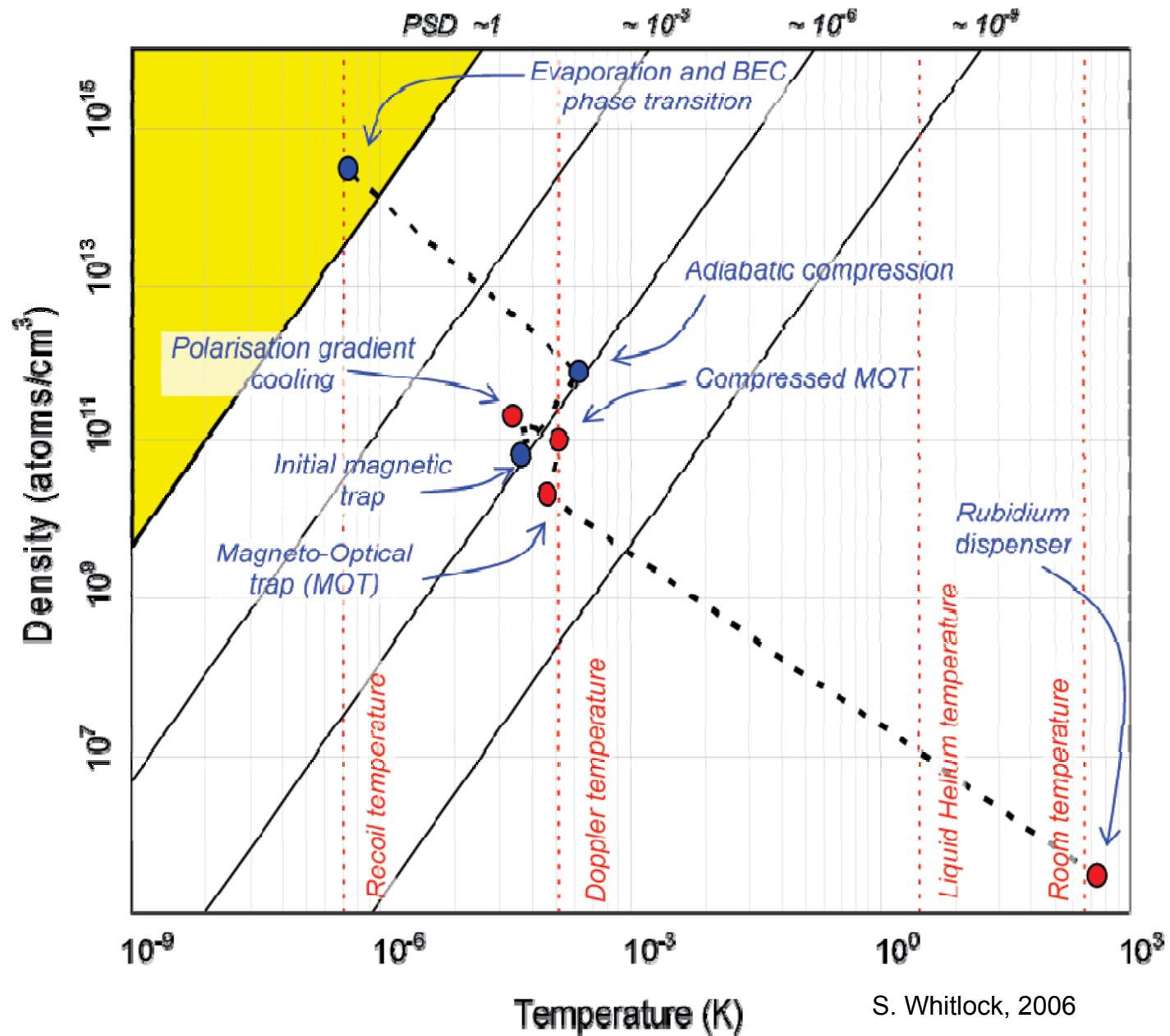
Gibble et al, 1992:

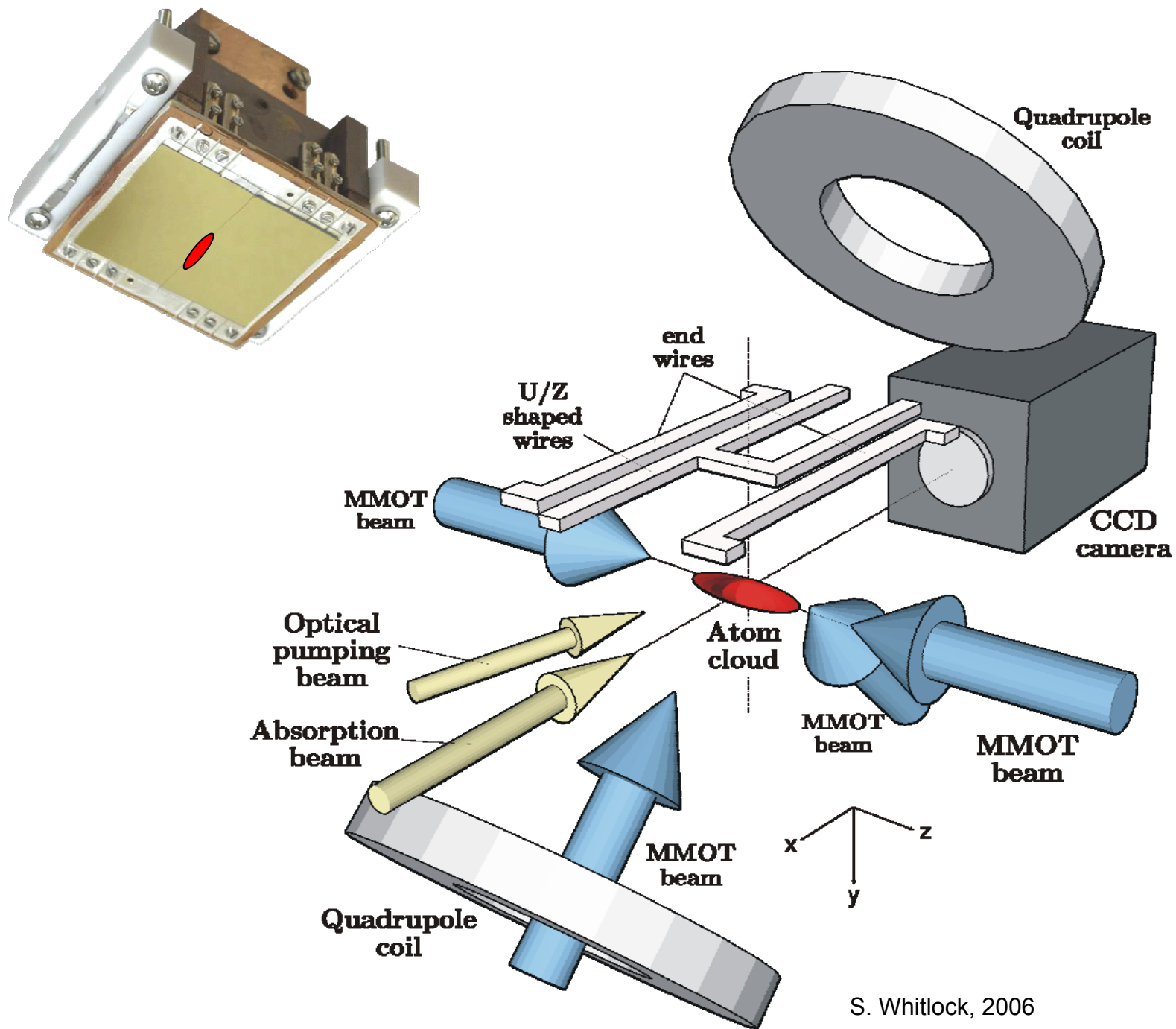
$$d = 5.5 \text{ cm } N_{tr} = 3.6 \times 10^{10} \text{ atoms}$$

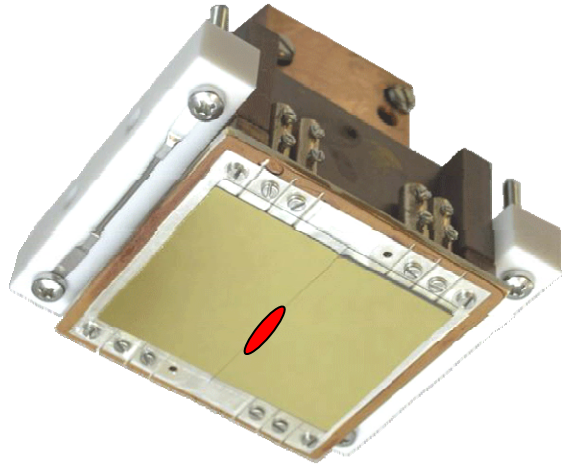


C. Monroe et al, PRL 1990

Bose-Einstein condensation on atom chip







Laser cooling of atoms on atom chip

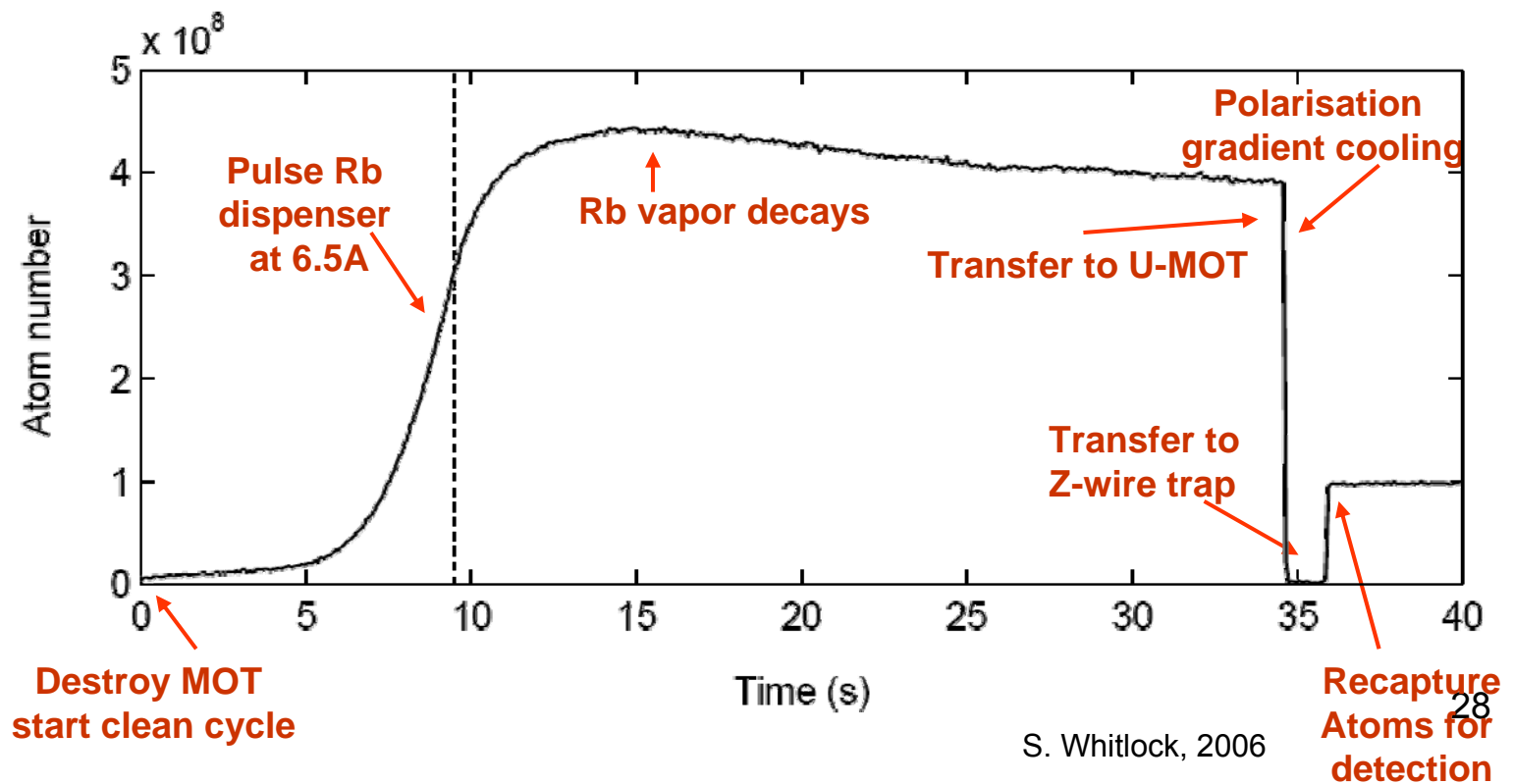
0 s – MMOT, 2×10^6 atoms

0-10 s – Rb dispenser on 6.5 A

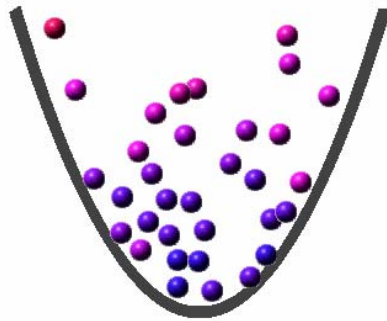
10 - 35 s – Wait for vacuum 1×10^{-11} Torr

35 s – Compressed U-wire MMOT

35 s – Z-wire magnetic trap



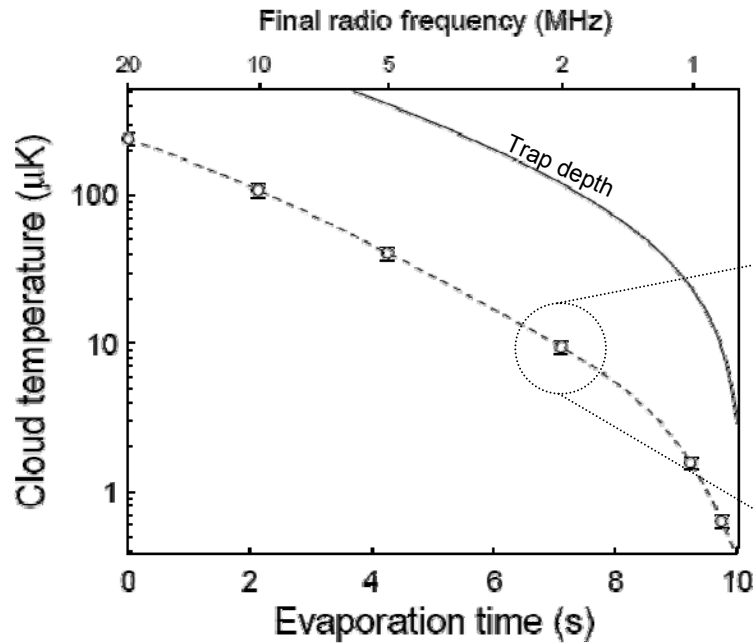
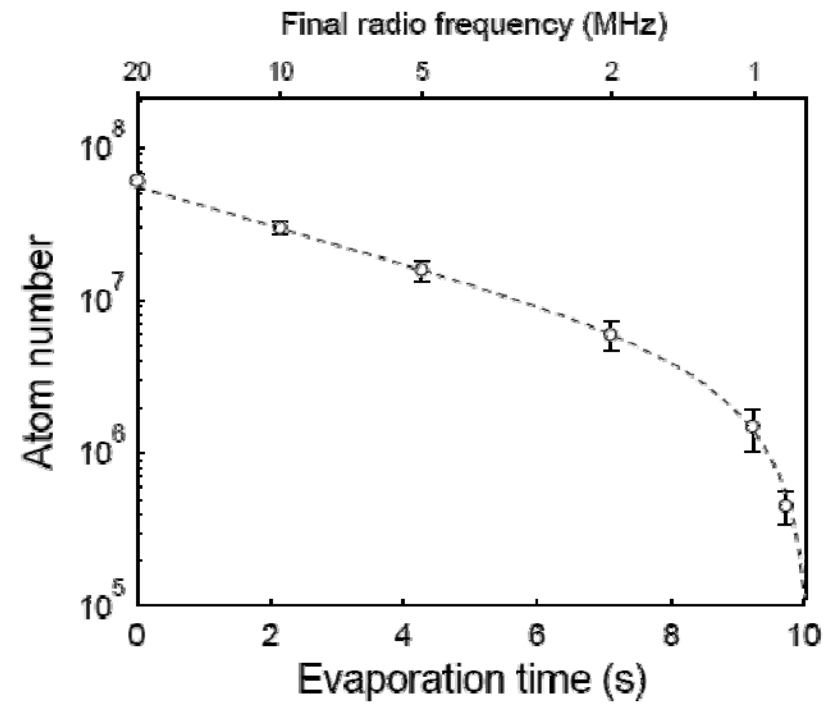
Evaporative cooling of atoms



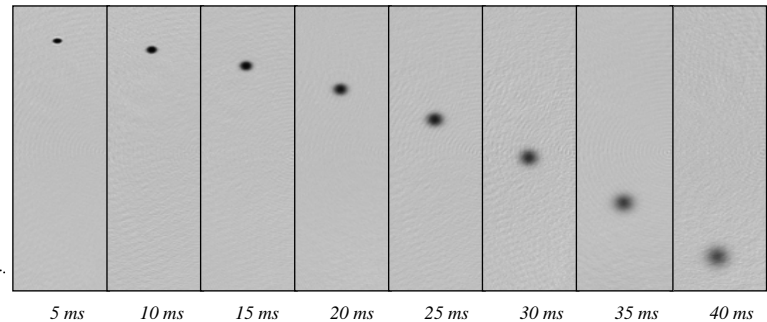
Red – high energy
Blue – low energy



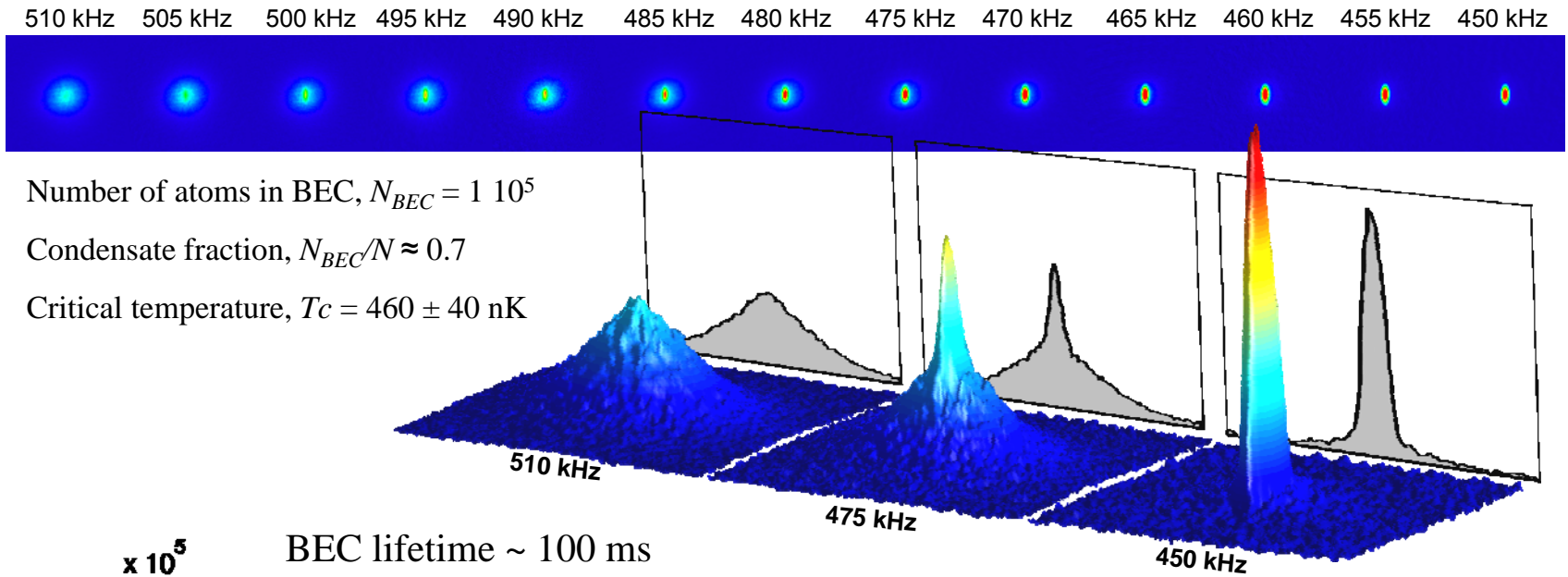
Physics 2000 website
<http://www.colorado.edu/physics/2000/index.pl>



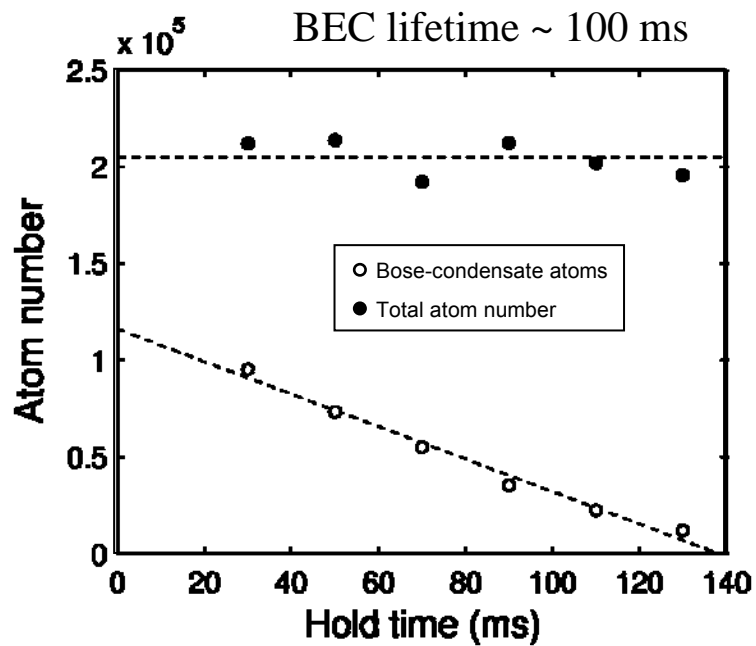
Ballistic expansion



Bose-Einstein condensation



S. Whitlock, 2006

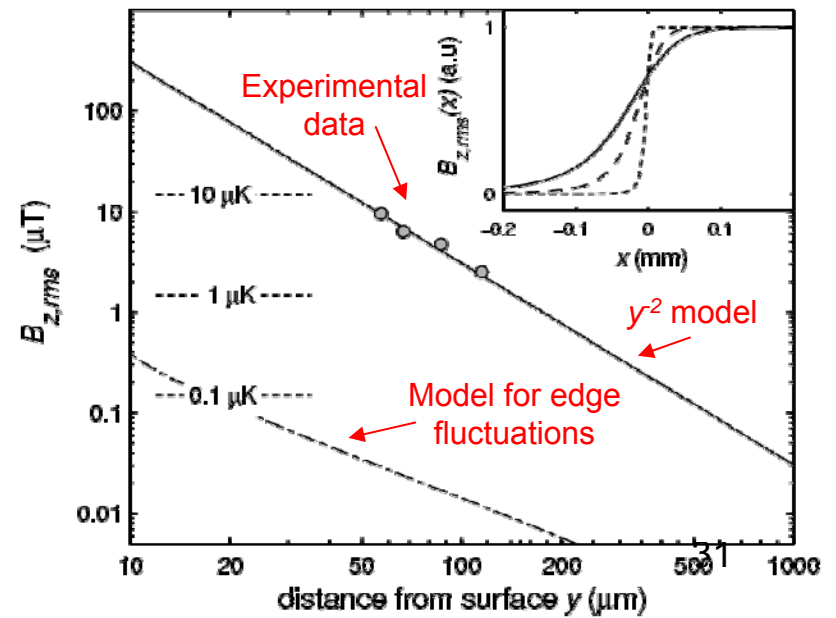
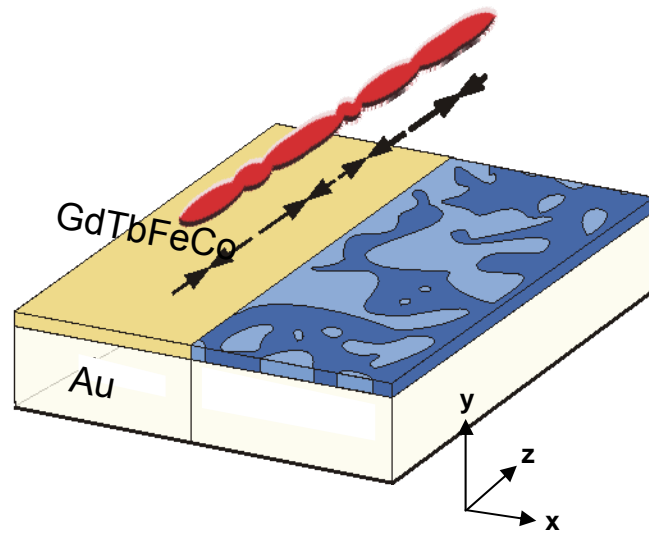


The BEC decays over 100 ms, while the total atom number remains approximately constant.

Lifetime limited by in-trap heating?

Can be extended to ~ 1 s by using a 'RF shield'

Fragmentation of cold cloud above magnetic film



S. Whitlock, 2006

Radiofrequency spectroscopy

Relax axial confinement. The thermal cloud spans 5 mm.

Sweep RF from 2 MHz to the final cut-off frequency, ν_f ,

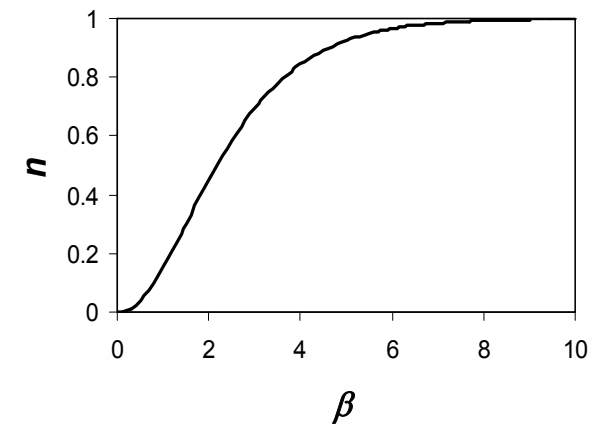
Fit cloud density to the truncated Boltzmann distribution.

Truncated Boltzmann distribution

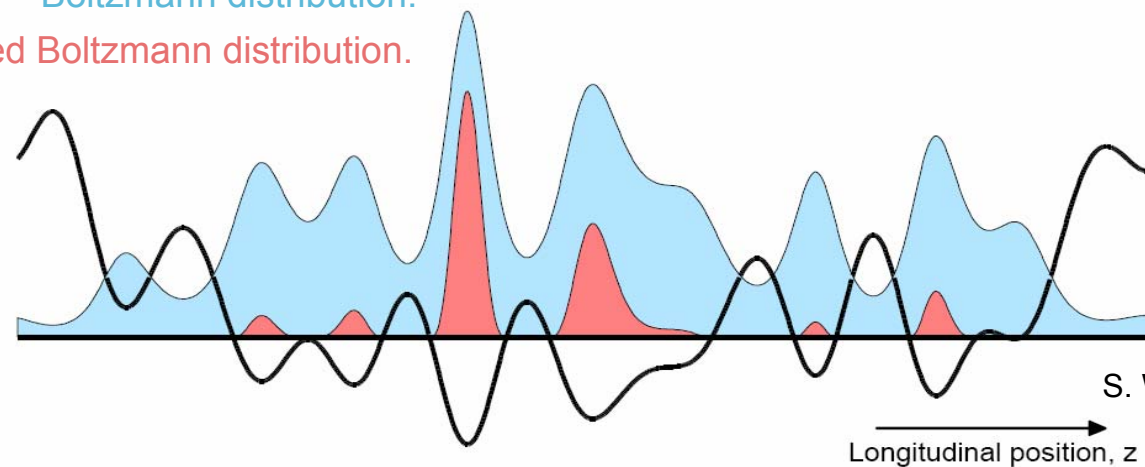
$$n(z, \beta) = n_\infty(z) [\text{erf}(\sqrt{\beta}) - 2\sqrt{\beta/\pi} e^{-\beta} (1 + 2\beta/3)],$$

Spatially dependent truncation parameter

$$\beta(z, \nu_f) = (h\nu_f - g_F \mu_B |B_z(z)|) / k_B T,$$



Boltzmann distribution.
Truncated Boltzmann distribution.



S. Whitlock, 2006

Radiofrequency spectroscopy

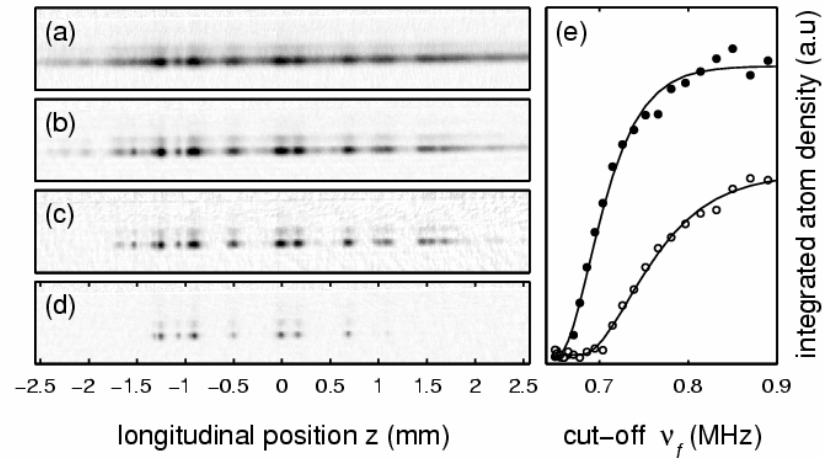
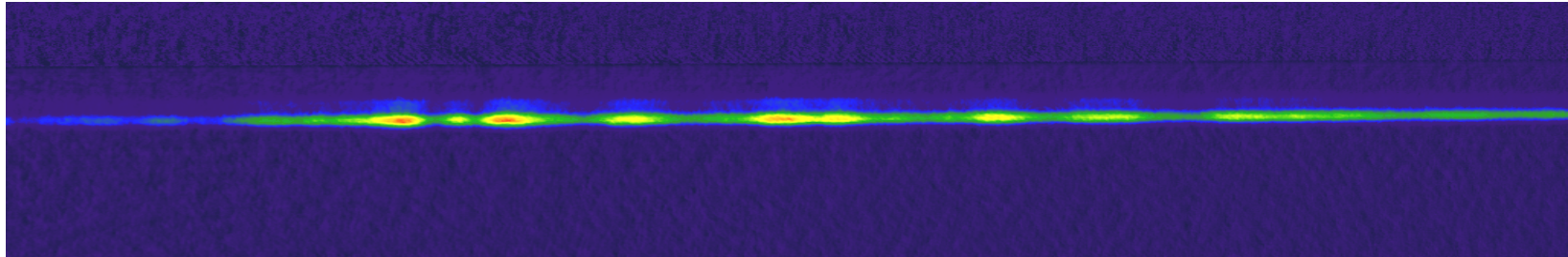
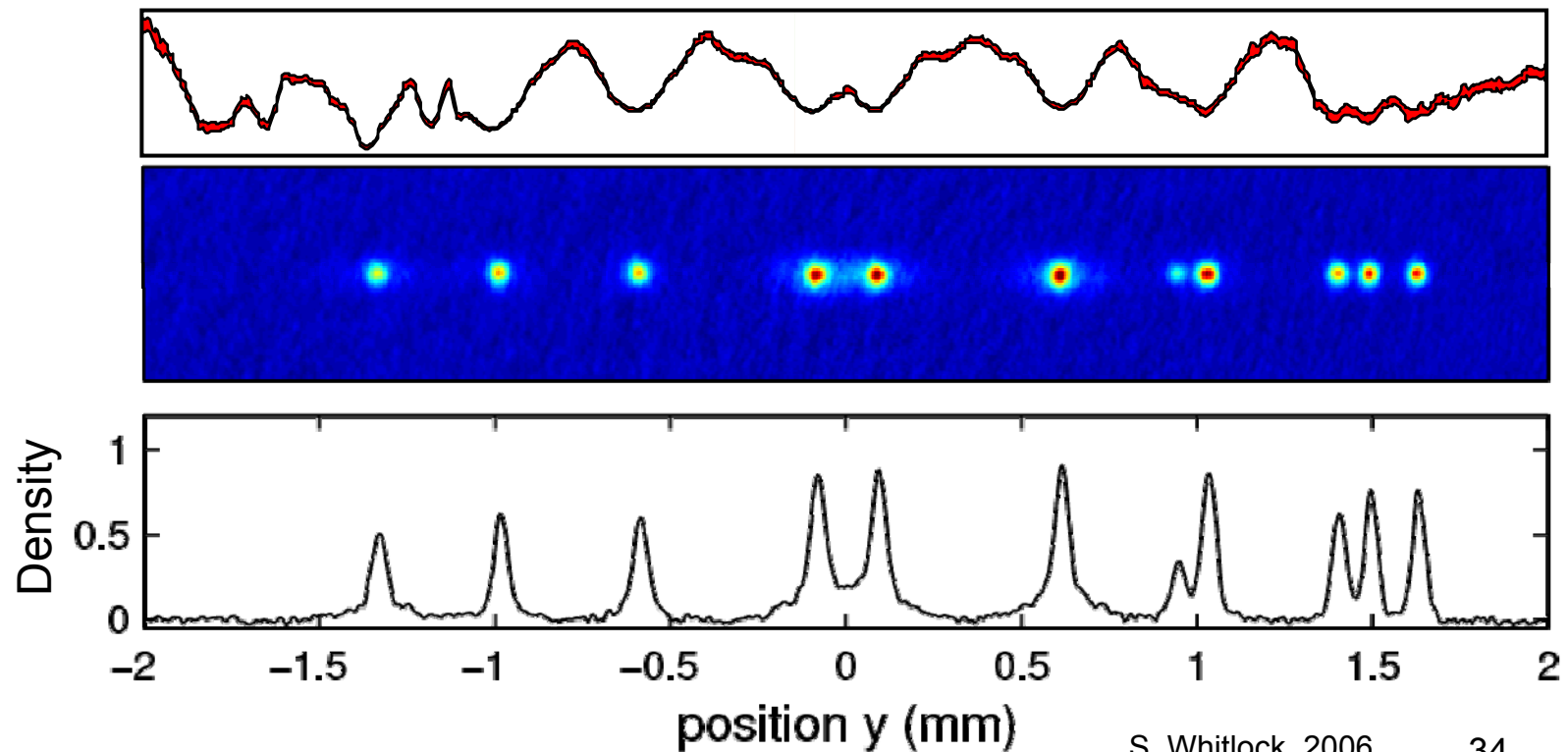


FIG. 2: Absorption images of the atomic density in the magnetic film microtrap located $y_0 = 67 \mu\text{m}$ from the surface. As the RF cut-off ν_f is decreased, the structure of the potential is revealed. (a) $\nu_f = 1238$ kHz, (b) $\nu_f = 890$ kHz, (c) $\nu_f = 766$ kHz, (d) $\nu_f = 695$ kHz and (e) integrated atomic density vs. ν_f for two longitudinal positions, $z = 0.69$ mm (solid circles) and $z = 1.00$ mm (open circles), fitted to the truncated thermal distribution function (Eq. 1).

Multiple Bose-Einstein condensates

Independently tune end wire currents to remove linear and quadratic components of the longitudinal trap potential - minimise the energy difference between each potential well.

RF evaporatively cool the elongated thermal cloud to the BEC transition to simultaneously produce 11 spatially separated Bose-Einstein condensates



Summary

1. Laser cooling allow to produce large ensembles of ultracold atoms
2. Cooling is done by the dissipative, radiation pressure force (spontaneous emission plays a major role)
3. Doppler limit ($100\div 200 \mu\text{K}$) for a two (or $J=0\rightarrow J=1$) atom
4. Sub-Doppler temperatures via the polarisation-gradient cooling (recoil energy – ultimate limit)
5. Magneto-optical trap
6. Laser and evaporative cooling on atom chip