Condensed Matter Emulation

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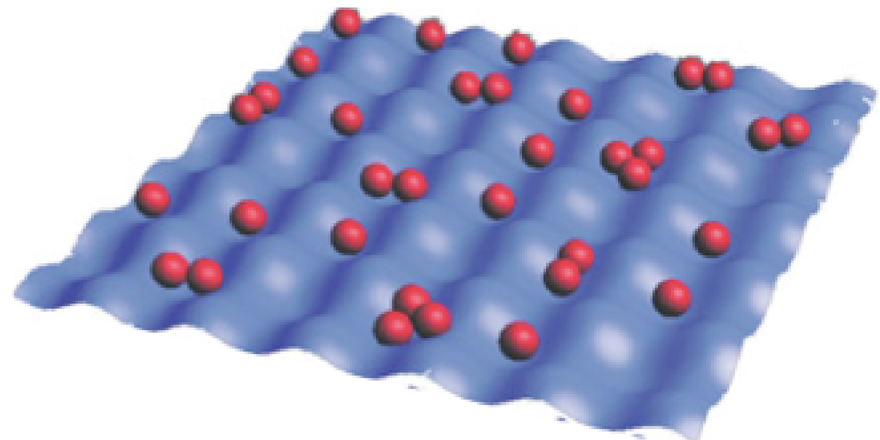
Condensed Matter

- Disordered
- Unknown interactions
- Little control



Cold Atoms

- Tunable dispersion
- Tunable interactions
- “Perfect” control
- Clean or controlled disorder
- Engineered Hamiltonians

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These 4 Lectures

- Lecture 1:
  - Introduction to emulation
  - The integer/fractional quantum Hall effect (solid state)
- Lecture 2:
  - Emulation of the quantum Hall effect (ultra-cold atoms)
- Lecture 3:
  - Coupled Atom Cavity (CAC) systems
  - Bose-Hubbard emulation (ultra-cold gases and CAC systems)
- Lecture 4:
  - Fractional Quantum Hall Effect (CAC systems)
  - Unconventional superconductivity?

Reference Material

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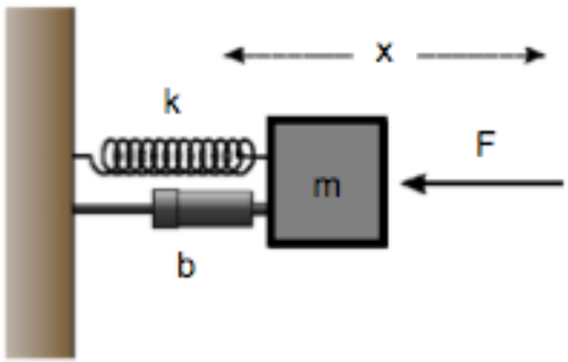
- Condensed matter emulation
  - *Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond*, M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen and U. Sen, Advances in Physics **56**, 243 (2007)
  - *Many-body physics with ultracold gases*, I. Bloch, J. Dalibard and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008)
- Integer/Fractional Quantum Hall effect
  - *Introduction to the fractional quantum Hall effect*, S. M. Girvin, http://www.bourbaphy.fr/girvin.ps

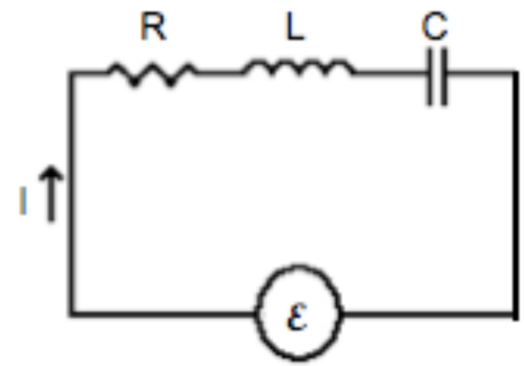
Equivalence of Physical Systems

Driven damped oscillator



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F\cos(\omega t)$$

RLC circuit



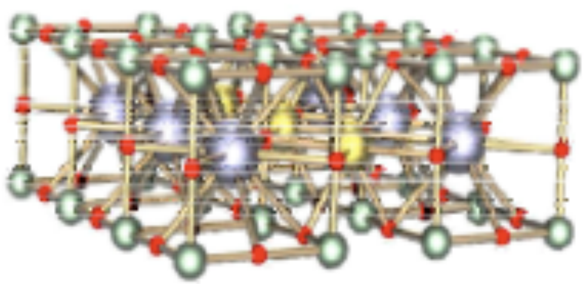
$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}x = \varepsilon\cos(\omega t)$$

ANALOGOUS MECHANICAL & ELECTRICAL QUANTITIES

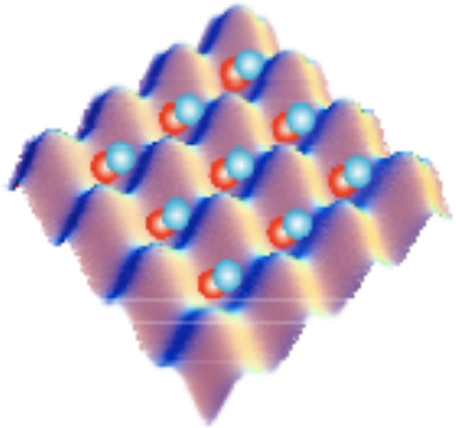
Mechanical		Electrical	
x	Displacement	q	Charge
$\dot{x}$ (v)	Velocity	$\dot{q}$ (I)	Current
m	Mass	L	Inductance
b	Friction	R	Resistance
1/k	Mechanical Compliance	C	Capacitance
F	Amplitude of impressed force	$\mathcal{E}$	Amplitude of impressed emf

Equivalence of Physical Systems

YBCO superconductor



Optical Lattice



$$H = -t_{e(a)} \sum_{\langle i,j \rangle,\sigma} \left(c^{\dagger}_{i,\sigma} c_{j,\sigma} + \text{h.c.} \right) + U_{C(s)} \sum_{i} n_{i,+1} n_{i,-1}$$

ANALOGOUS CONDENSED MATTER AND OPTICAL LATTICE QUANTITIES

	Condensed Matter		Atom-Optical
Carriers	Electron/Holes		Fermionic atoms
e^-e^-	Coulomb charge coupling	s	S-wave scattering length
m_e	Electron mass	m_a	Atomic mass
U_c	Coulomb Interaction	U_s	S-wave Interaction
t_e	Electronic tunneling energy	t_a	Atomic tunneling energy
Lattice	Atomic ions		Optical standing waves
a, b, c	Lattice Constants	$(\lambda_x, \lambda_y, \lambda_z)/2$	Optical wavelength
V_{ion}	Binding energy	V_{lat}	Lattice depth

The Quantum Hall Effect

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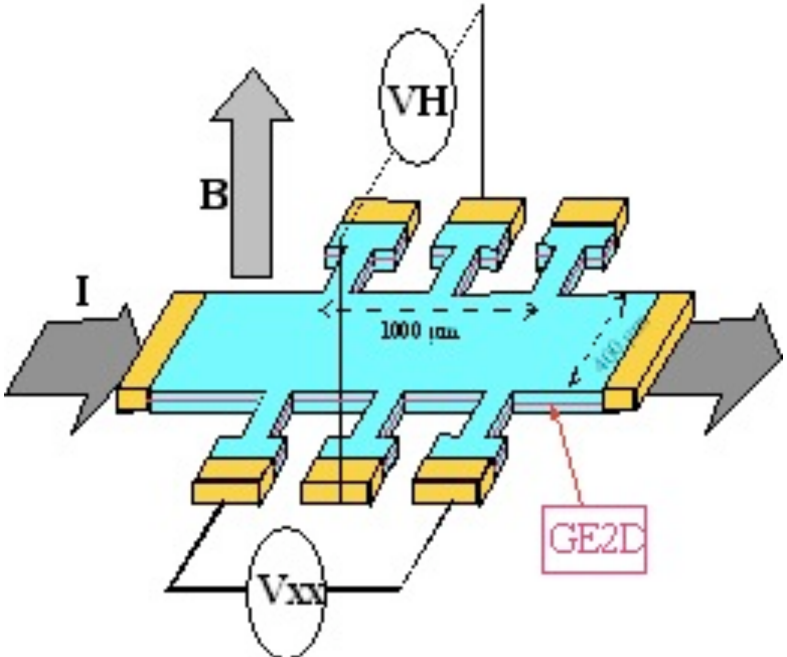
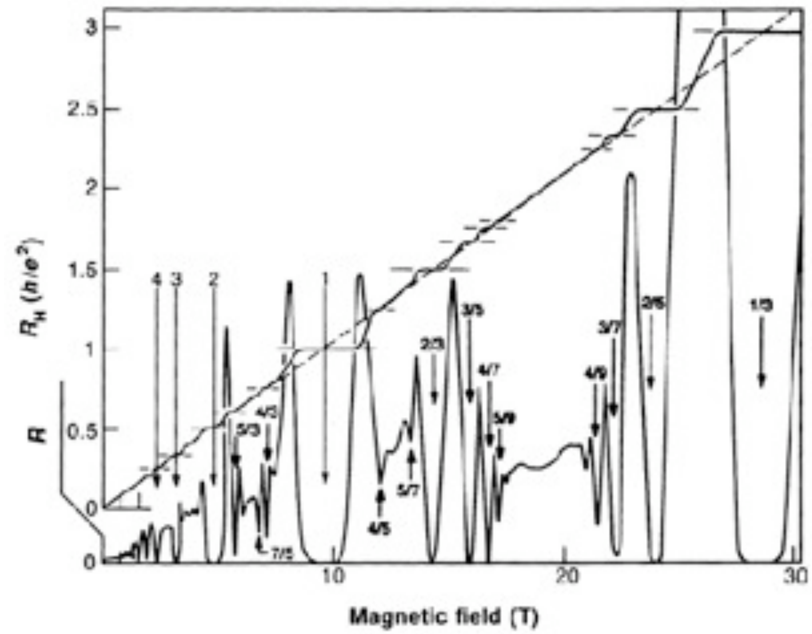
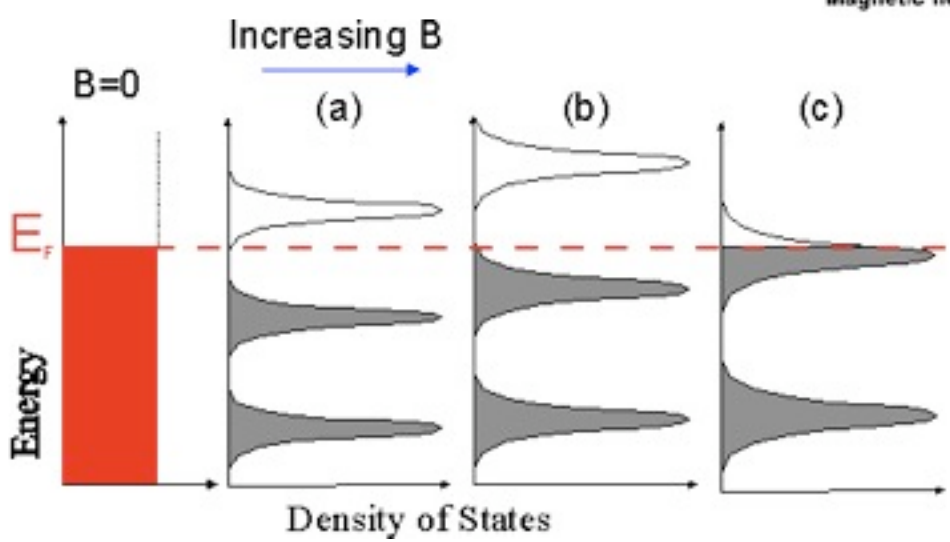
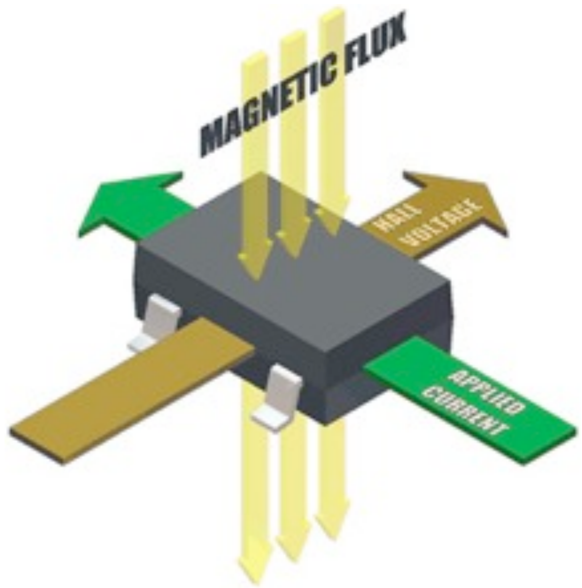


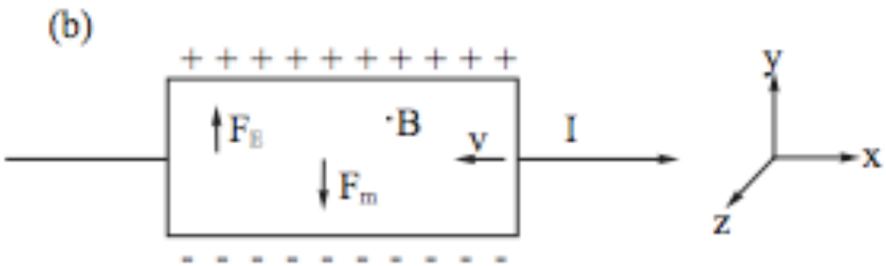




The Hall Effect



$$\mathbf{J} = -ne\mathbf{v}$$



Force on carrier

$$\mathbf{F} = -e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) = \mathbf{F_E} + \mathbf{F_M}$$

Equilibrium

$$\mathbf{F_E} = -\mathbf{F_M}$$

y-component

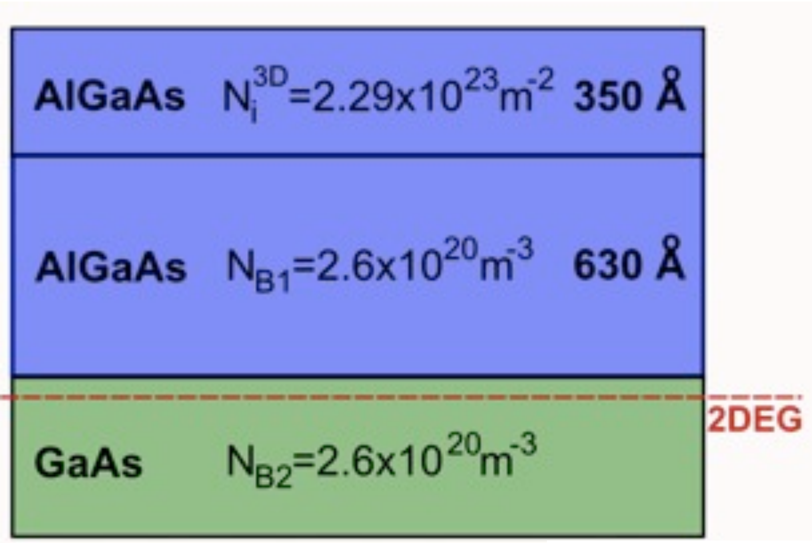
$$F_y = ev_x B_z - eE_y = 0$$

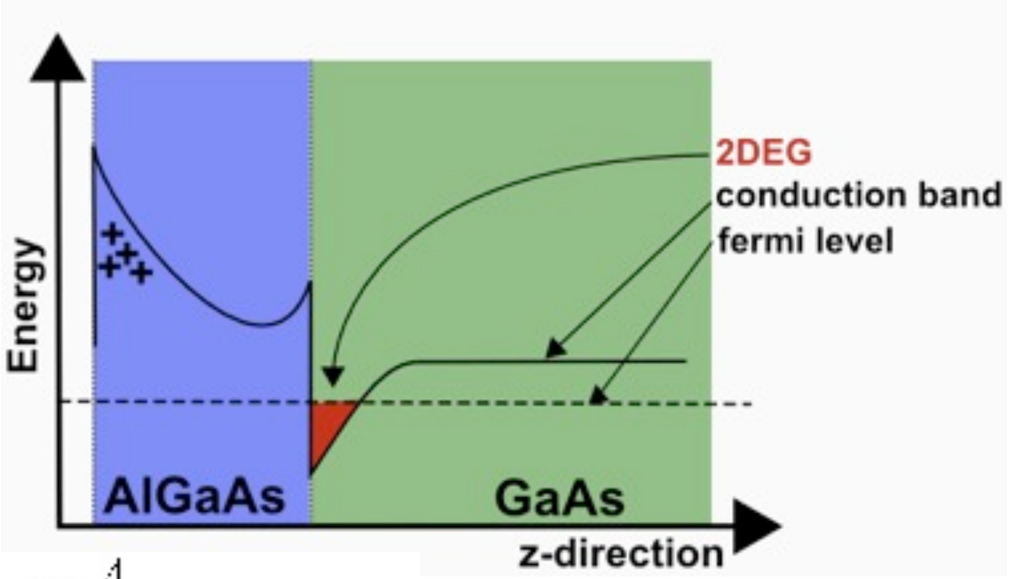
$$E_y = -\frac{J_x B_z}{ne}$$

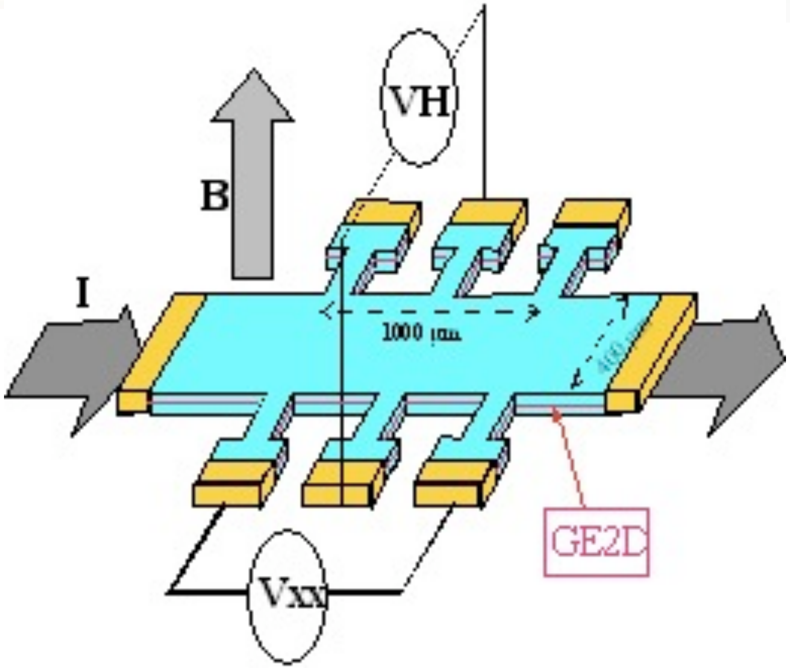
Hall coefficient

$$R_H = \frac{E_y}{J_x B_z} = -\frac{1}{ne}$$

Experimental Setup (QHE)







Experimental Observation (IQHE)

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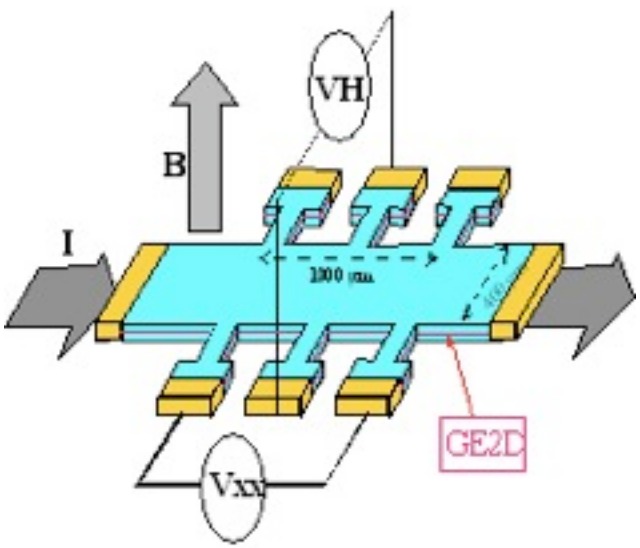
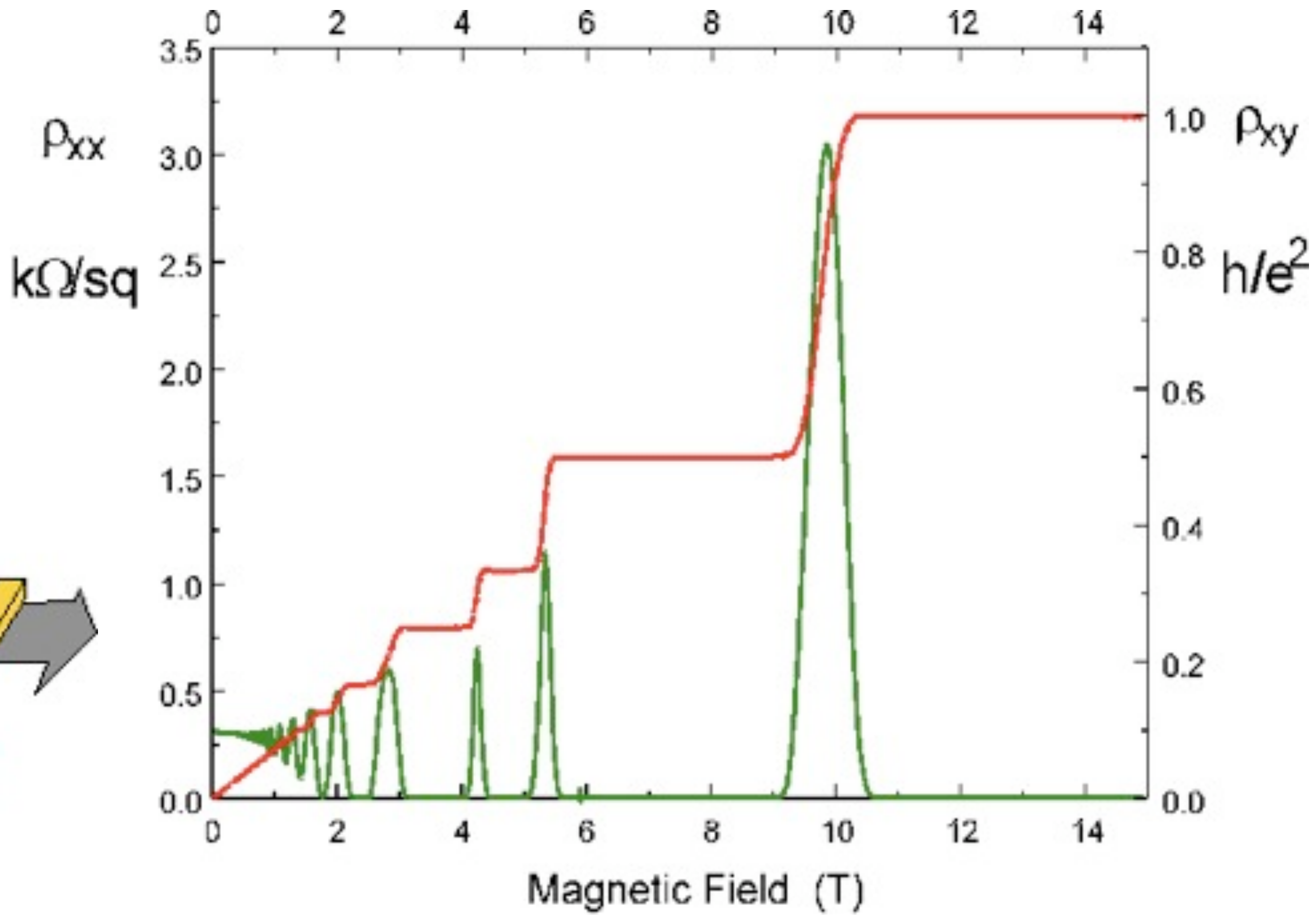
$$\frac{1}{\rho_{xy}} = \sigma_{xy} = \nu \frac{e^2}{h}, \quad \nu = 1, 2, 3... \ \text{(Integer)}$$





Electron in Magnetic Field

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Cyclotron frequency

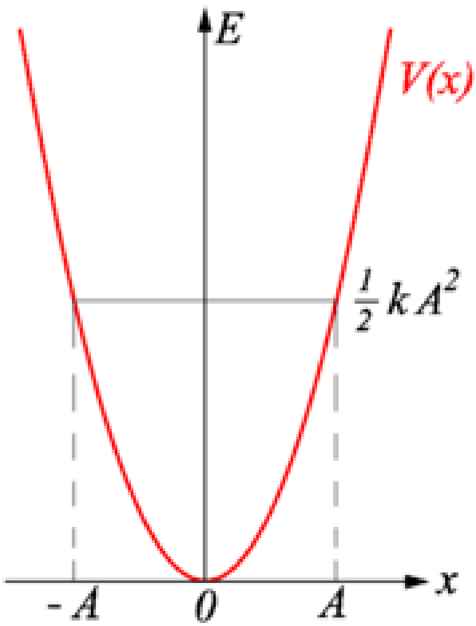
$$\omega_c = \left| \frac{eB}{m} \right|$$

Cyclotron radius

$$R_c = \frac{v}{\omega_c} = \frac{\sqrt{2mE}}{|eB|}$$

Harmonic oscillator analogy

$$V(x) = \frac{1}{2} m\omega^2 x^2$$

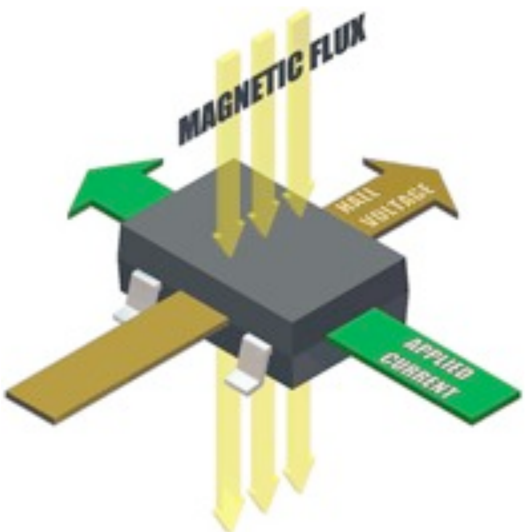


Quantum Treatment

Schrodinger Equation

$$\left[\frac{1}{2m}\left(i\hbar\nabla - q\mathbf{A}(\mathbf{r},t)\right)^2 + q\phi(\mathbf{r},t)\right]\psi(\mathbf{r},t) = i\hbar\frac{d}{dt}\psi(\mathbf{r},t)$$

$$\mathbf{B} = \nabla \times \mathbf{A} = B_z\hat{k}$$



Choice of gauge

$$\mathbf{A} = \hat{j}B_z x \quad \text{(Landau gauge)}$$

$$\mathbf{A} = -\hat{i}B_z y/2 + \hat{j}B_z x/2 \quad \text{(Symmetric gauge)}$$

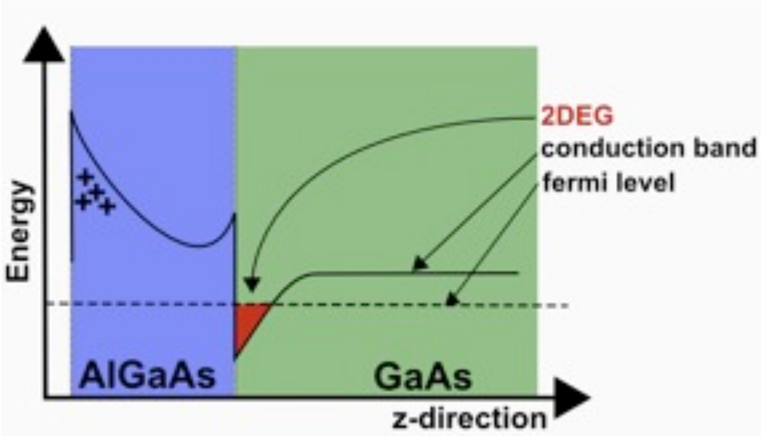
Physical results independent of gauge

We choose Landau gauge

Landau Gauge

Schrodinger Equation (Landau gauge)

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{ie\hbar B_z}{m}\frac{\partial}{\partial y} + \frac{(eB_z x)^2}{2m} + V(z)\right]\psi(\mathbf{r}) = E\psi(\mathbf{r})$$



Drop z-dependence (2DEG)

↓

Vector potential independent of y

↓

Plane wave solutions for y-direction: 1D Schrodinger Equation

1D Schrodinger Equation

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$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{1}{2}m\omega_c^2\left(x+\frac{\hbar k}{eB}\right)^2\right]u(x)=\epsilon u(x)$$

Schrodinger Equation for 1D harmonic oscillator

Vertex of parabolic potential displaced by $-\hbar k/eB$

Energy eigenvalues

$$\epsilon_{nk}=(n-1/2)\hbar\omega_c,\ \ \text{where } n=1,2,3,\ldots$$

Wavefunction

$$\psi_{nk}(x,y)\propto H_{n-1}\left(\frac{x-x_k}{l_b}\right)e^{-\frac{(x-x_k)^2}{2l_b^2}}e^{iky},\ \ \text{where } l_b=\sqrt{\hbar/|eB_z|}$$

The eigenvalues (Landau levels) depend on *n* but not *k*

Landau Levels DOS

B=0: 2D electron DOS is a constant

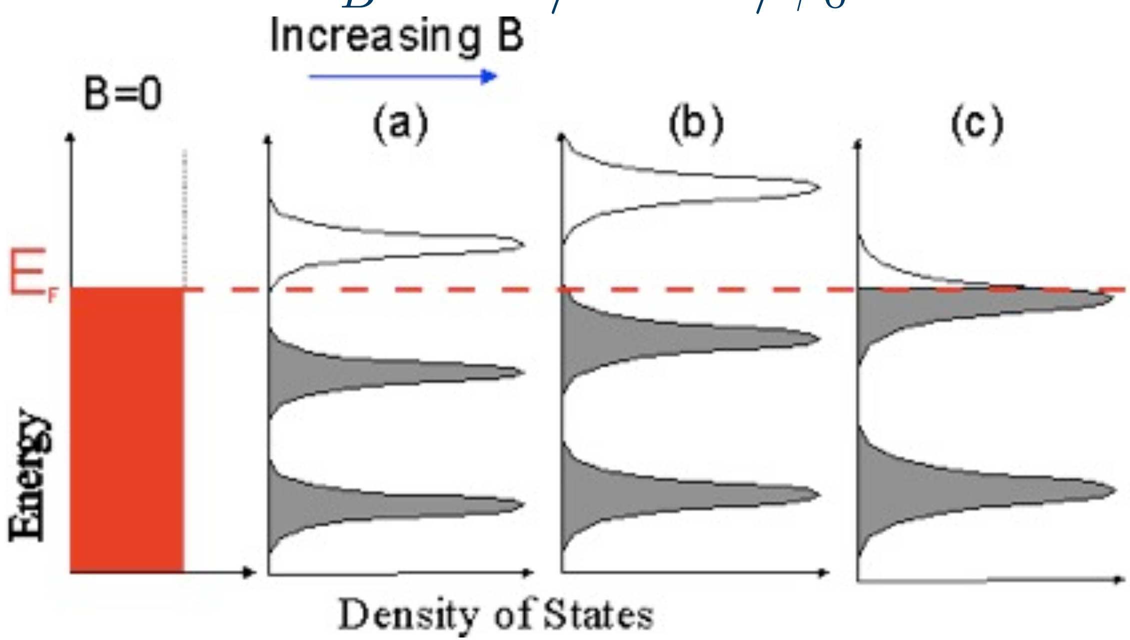
|B|>0: 2D electron DOS is a series of delta functions

↓

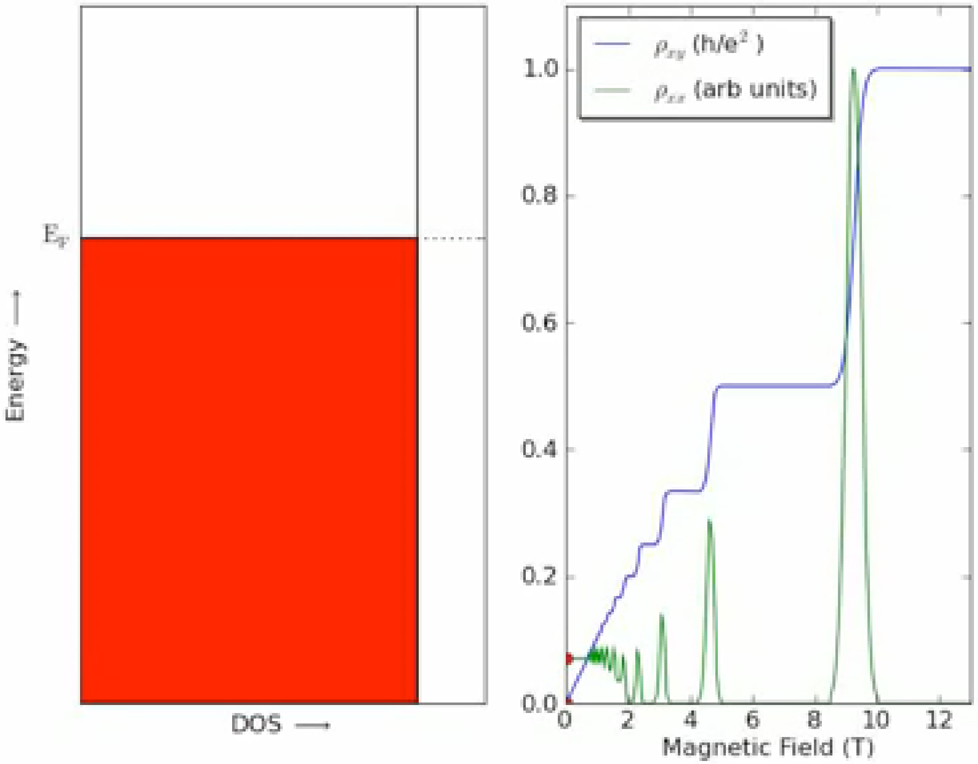
Landau Levels (LLs)

Number of states in each LL per unit area

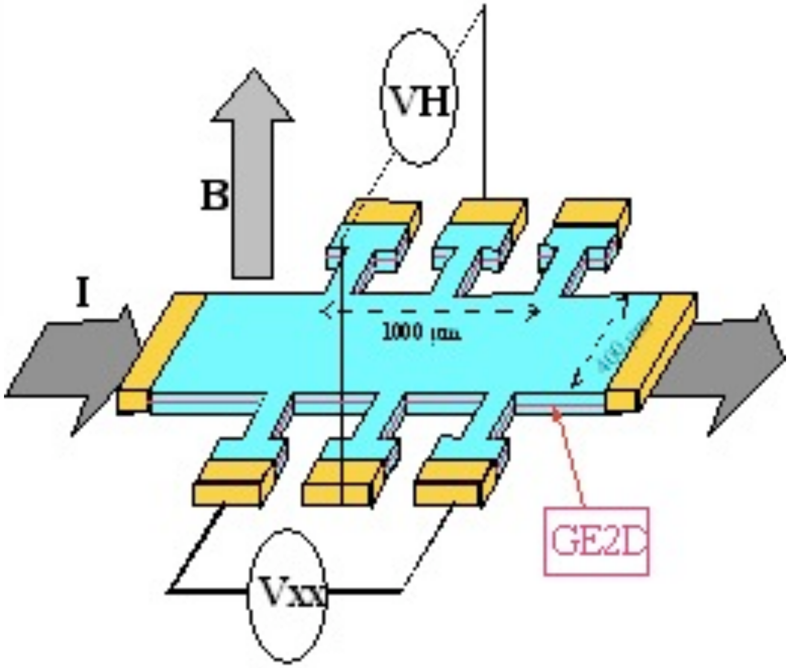
$$n_B = eB/h = B/\phi_0$$



IQHE



E_F
Energy
DOS
ρ_{xy} (h/e^2)
ρ_{xx} (arb units)
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
12
Magnetic Field (T)



VH
B
I
1000 μm
Vxx
GE2D

Landau Levels: Transport

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Number of occupied LLs

$$\nu = \frac{n_{2D}}{n_B} = 2\pi l_b^2 n_{2D}$$

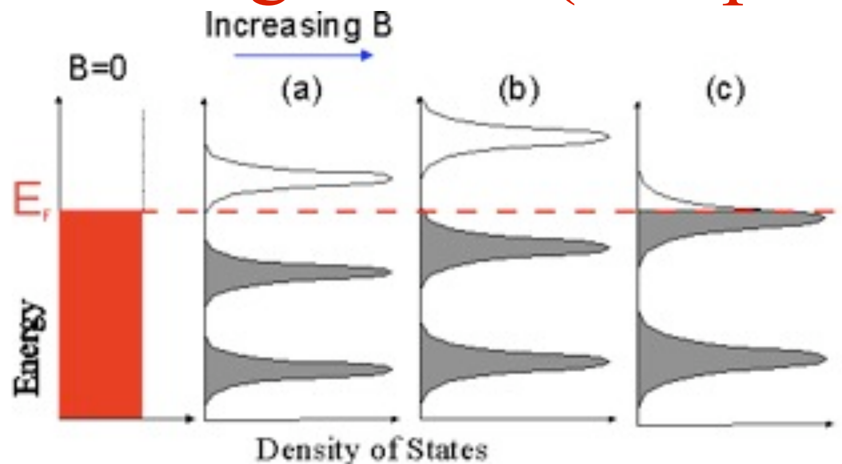
Between LLs (n integer)

$$B_n = \frac{h n_{2D}}{en}$$

$$\downarrow$$

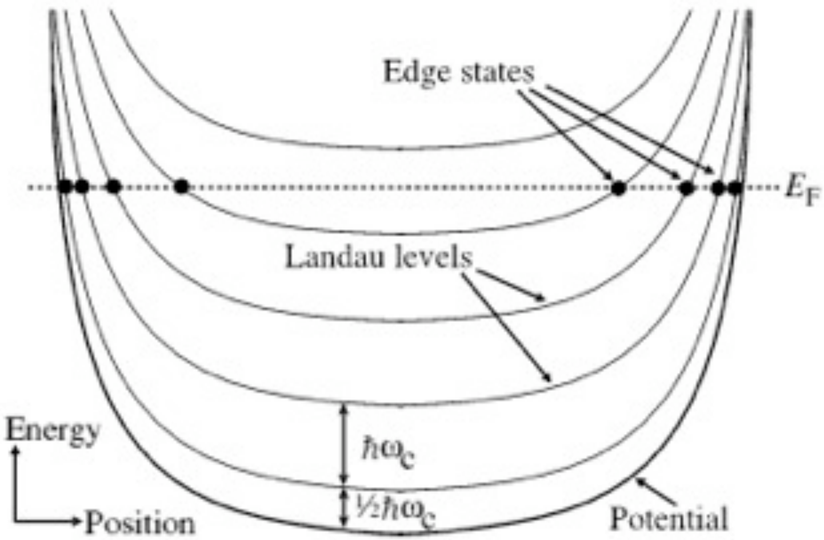
Fermi energy between LLs: low DOS (incompressible)

Within LL high DOS (compressible)

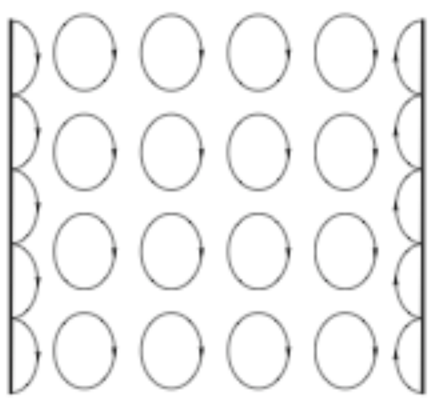


Confinement

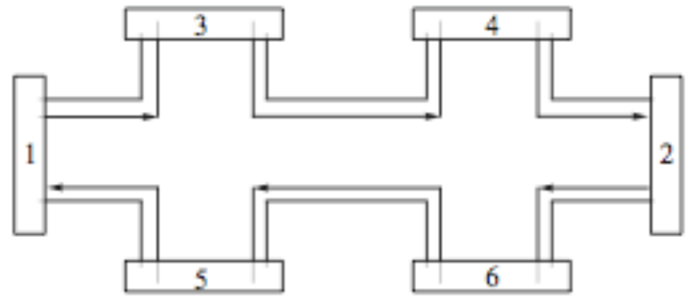
LLs: confined geometry



(b)



Hall bar schematic



Fermi energy between LLs

↓

Edge state transport

$$I = -\frac{Ne^2}{h}V$$

$$\rho_{xy} = \frac{V_5 - V_3}{I} = -\frac{V_1}{I} = \frac{h}{Ne^2}$$

Assumptions: No disorder

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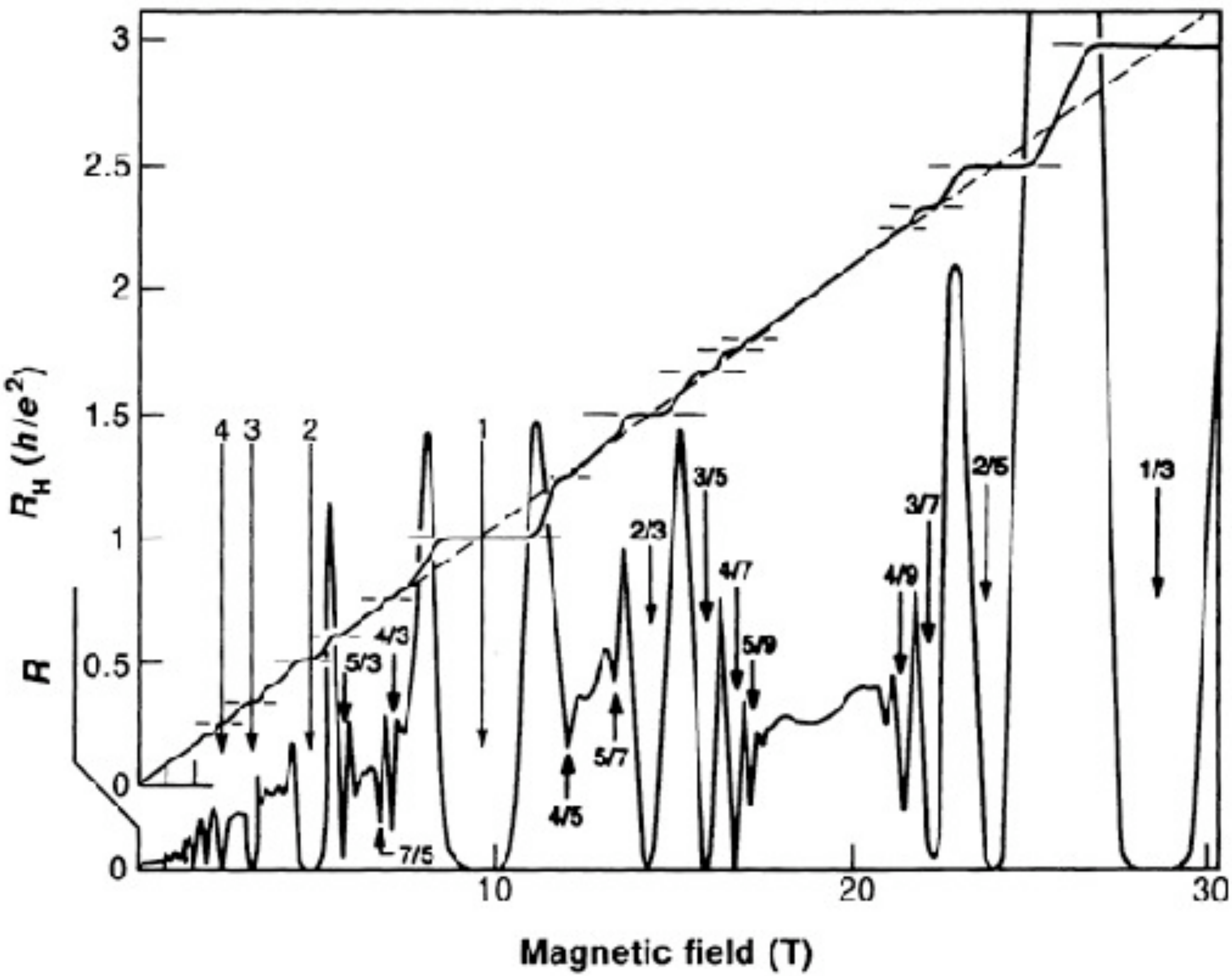
- Scattering between edge states in the same edge
  - Is forward hence no effect (exception high currents)
- Scattering between opposing edges
  - Very very weak if Fermi energy is between Landau levels
- Surely Fermi energy adjusts to always be in a LL: Why are plateaus wide?
  - Disorder important: localized states between LLs in bulk
  - Finite DOS between LLs
  - Do not contribute to electrical properties (localized)

The Surprise (FQHE)



Theoretical Model of FQHE

- Controlled by Coulomb repulsion between electrons
  - Ignore disorder
  - Discover the nature of the *special* many-body correlated state
- Consider symmetric gauge (remember results are gauge independent)

$$\mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B}$$

  - Preserves rotational symmetry
- Consider only Lowest Landau Level (LLL): No interactions

$$\phi_m = \frac{1}{\sqrt{2\pi l_b^2 2^m m!}} z^m e^{-\frac{1}{4}|z|^2}, \quad \text{where } z = (x+iy)/l_b$$

- All the states are degenerate: can have any linear combination

$$\Psi(x,y) = f(z) e^{-\frac{1}{4}|z|^2} \qquad f(z) = \prod_{j=1}^{N} (z - Z_j)$$

The LLL Many-Body State

$$\psi[z] = f[z]e^{-\frac{1}{4}\sum_j |z_j|^2}$$

f is a polynomial representing the Slater determinant with all states occupied

2 particles

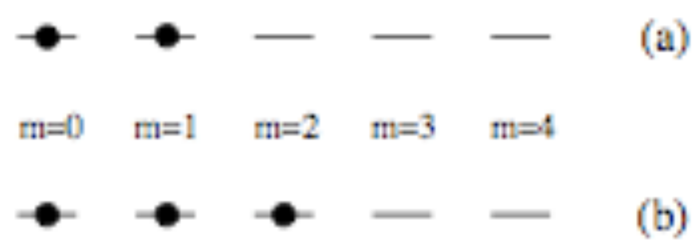
$$f[z] = \begin{vmatrix} (z_1)^0 & (z_2)^0 \\ (z_1)^1 & (z_2)^1 \end{vmatrix} = (z_1)^0(z_2)^1 - (z_2)^0(z_1)^1 = (z_2 - z_1)$$

3 particles

$$f[z] = \begin{vmatrix} (z_1)^0 & (z_2)^0 & (z_3)^0 \\ (z_1)^1 & (z_2)^1 & (z_3)^1 \\ (z_1)^2 & (z_2)^2 & (z_3)^2 \end{vmatrix} = -\prod_{i<j}^{3}(z_i - z_j)$$

N particles

$$f_N[z] = \prod_{i<j}^{N}(z_i - z_j)$$



Lauglin Variational Ansatz

$$f_N^m[z] = \prod_{i<j}^{N} (z_i - z_j)^m \qquad \nu = 1/m$$

To be analytic *m* must be an integer

To preserve antisymmetry *m* must be odd

$$\nu = \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$$

In the plasma analogy the electron density is

$$n = \frac{1}{m} \frac{1}{2\pi l_b^2}$$

Other wave-functions developed to describe more general states in the hierarchy of rational filling factors at which quantized Hall plateaus were observed

Plasma Analogy (I)

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$$|\Psi[z]|^2 = \prod_{i<j}^{N} |z_i - z_j|^{2m} e^{-\frac{1}{2}\sum_{j=1}^{N} |z_j|^2} = e^{-\beta U}$$

$$\beta = \frac{2}{m} \qquad U = m^2 \sum_{i<j} (-\ln|z_i - z_j|) + \frac{m}{4}\sum_k |z_k|^2$$

2D system

$$\int d\mathbf{s}\cdot\mathbf{E} = 2\pi Q \qquad \mathbf{E}(\mathbf{r}) = \frac{Q\hat{r}}{r} \qquad \phi(\mathbf{r}) = Q\left(-\ln\frac{r}{r_0}\right)$$

- Hence, potential energy among a group of objects with charge *m* is

$$U_0 = m^2 \sum_{i<j} (\ln|z_i - z_j|)$$

- Second term in U (Poissons Equation)

$$-\nabla^2 \frac{1}{4}|z|^2 = -\frac{1}{l_b^2} = 2\pi\rho_B$$

Plasma Analogy (II)

$$U = m^2 \sum_{i<j} (-\ln |z_i - z_j|) + \frac{m}{4} \sum_k |z_k|^2$$

Potential energy of interaction among a group of objects with charge *m*

Energy of charge *m* objects interacting with negative background

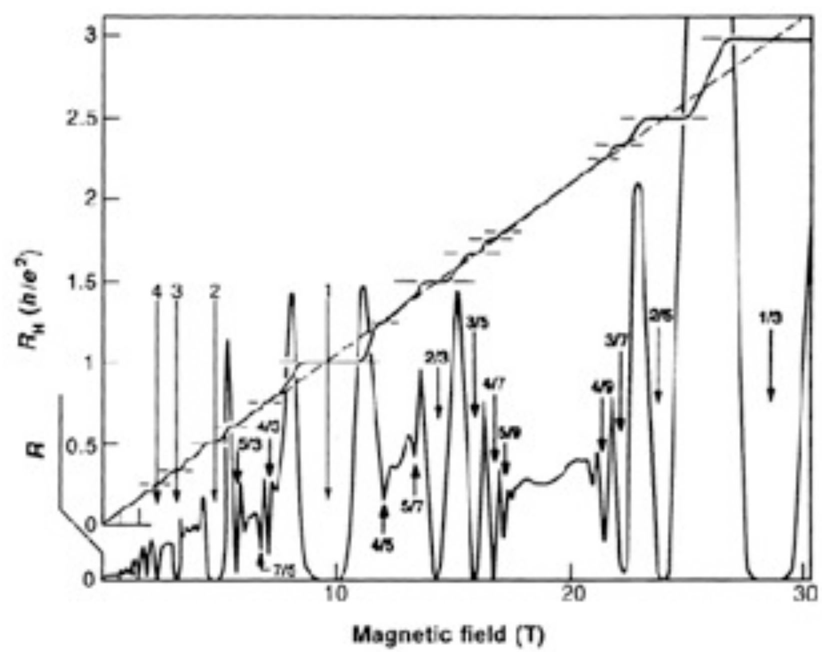
$2\pi l_b^2$ → Area containing one flux quantum → Background charge density B/ϕ_0

Neutrality → $nm + \rho_B = 0$ → $n = \frac{1}{m} \frac{1}{2\pi l_b^2}$

For a filled LL, with *m =1*, this is the correct answer for the density, since every single-particle state is occupied and there is one state per flux quantum

Excitation Gap?

- Every pair of particles has a relative angular momentum greater than or equal to *m*
- Because the relative angular momentum of a pair can change only in discrete (even integer) units it turns out that a hard core, repulsion, model has an excitation gap
- For example for $m = 3$, any excitation out of the Laughlin ground state weakens the nearly ideal correlations by forcing at least one pair of particles to have relative angular momentum *1* instead of *3*.
- This costs energy: hence a gap



Importance

- Two Nobel Prizes
- IQHE 1985 (Klaus von Klitzing);
- FQHE 1998 (Robert Laughlin, Horst Stormer and Daniel Tsui)
- The value of the resistance at the plateaus only depends on fundamental *constants* of physics: electric charge (e) and Planck's constant (h)
- It is accurate to 1 part in 100000000
- The IQHE is used as the primary resistance standard (although 1 klitzing (h/e^2) is 25,813 Ohms)
- Next Lecture we will examine how to emulate the fractional regime in BECs.