Condensed Matter Emulation

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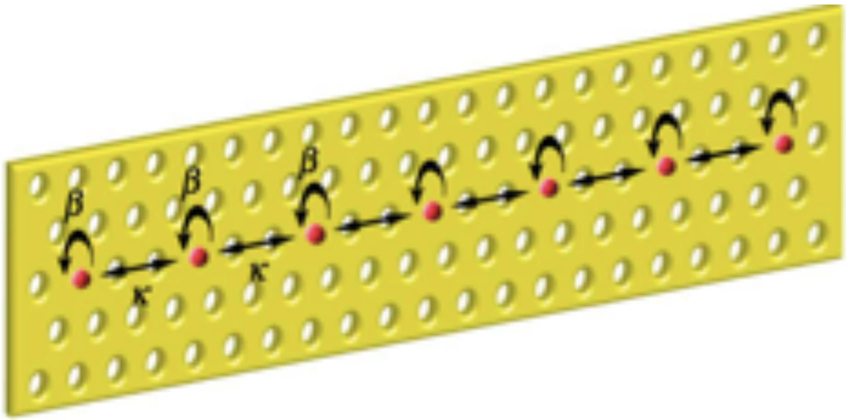
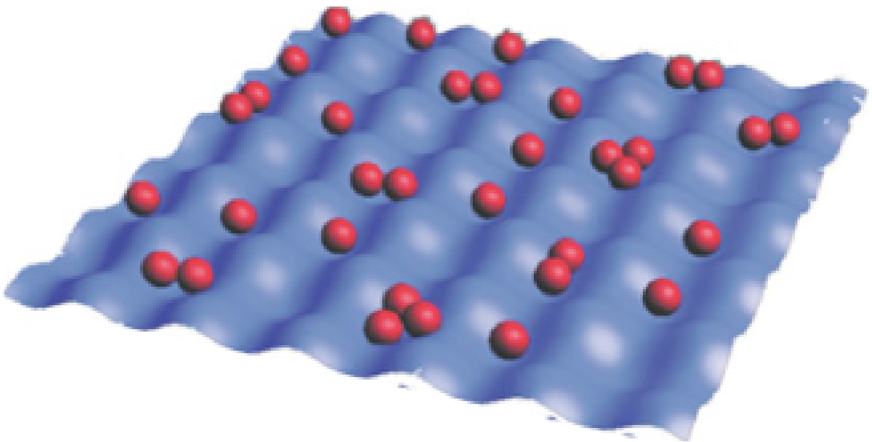
Coupled Atom Cavity Systems and Bose Hubbard Physics





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Outline

- Bose Hubbard model (cold gases)
  - Simplistic description
  - Meanfield description
  - Perturbation analysis
  - Experiments with BECs
- Coupled Atom Cavities
  - A single atom cavity
  - Hamiltonian for coupled atom cavities
  - Relation to the Bose Hubbard model
- Solid Light
  - Meanfield description
  - Perturbation analysis

Reference Material

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- Condensed matter emulation
  - *Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond*, M. Lewenstein *et al.*, Advances in Physics **56**, 243 (2007)
  - *Many-body physics with ultracold gases*, I. Bloch, J. Dalibard and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008)
- Bose Hubbard model (ultra-cold gases)
  - *Quantum phases in an optical lattice*, D. van Oosten, P. van der Straten and H.T.C Stoof, Phys. Rev. A **63**, 053601 (2001)
  - M. Greiner *et al.*, Nature **415**, 39 (2002)
- Bose Hubbard model (couple atom cavities)
  - *Quantum phase transitions of light*, A.D. Greentree *et al.*, Nature Physics **2**, 856 (2006)
  - *Strongly interacting polaritons in coupled arrays of cavities*, M.J. Hartmann *et al.*, Nature Physics **2**, 849 (2006)

Bose-Hubbard Model

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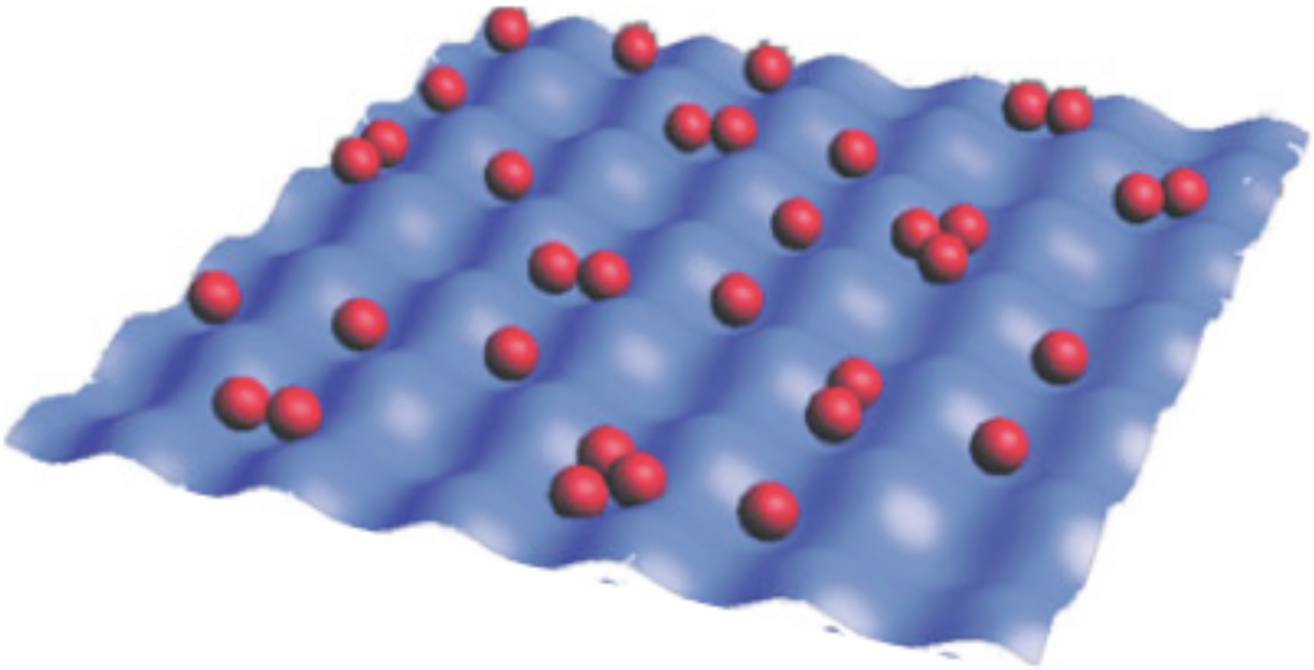

Hamiltonian

$$H = -J \sum_{i,j} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i \left(n_i - 1 \right) + \sum_i \epsilon_i n_i$$

Bose-Hubbard Model

Assumptions

- The thermal and mean interaction energies at a single site are much smaller than the energy separation to the first excited band
- The Wannier functions decay essentially within a single lattice constant
- Under these assumptions:
  - Only the lowest energy band needs to be included in our description
  - The hoping matrix elements are only significant for nearest neighbours
  - The interactions are dominated by the on-site contribution only

Hamiltonian

$$H = -J \sum_{i,j} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i \left(n_i - 1\right) + \sum_i \epsilon_i n_i$$

Superfluid state ($U = 0$)

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$$H = -J\sum_{i,j} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) + \sum_i \epsilon_i n_i$$

- The manybody ground state is simply an ideal BEC where all the atoms are in the $q = 0$ Bloch state of the lowest band

$$|\Psi_N^{U=0}\rangle = \frac{1}{\sqrt{N!}}\left(\frac{1}{\sqrt{N_L}}\sum_i a_i^\dagger\right)^N |0\rangle$$

- Hence the groundstate is a Gross-Pitaevskii type state with a condensate fraction equal to one
- However the critical temperature is significantly reduced (effective mass) as compared to the *free* case

Mott Insulator Phase ($U >> J$)

$$H = -J \sum_{i,j} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i \left(n_i - 1\right) + \sum_i \epsilon_i n_i$$

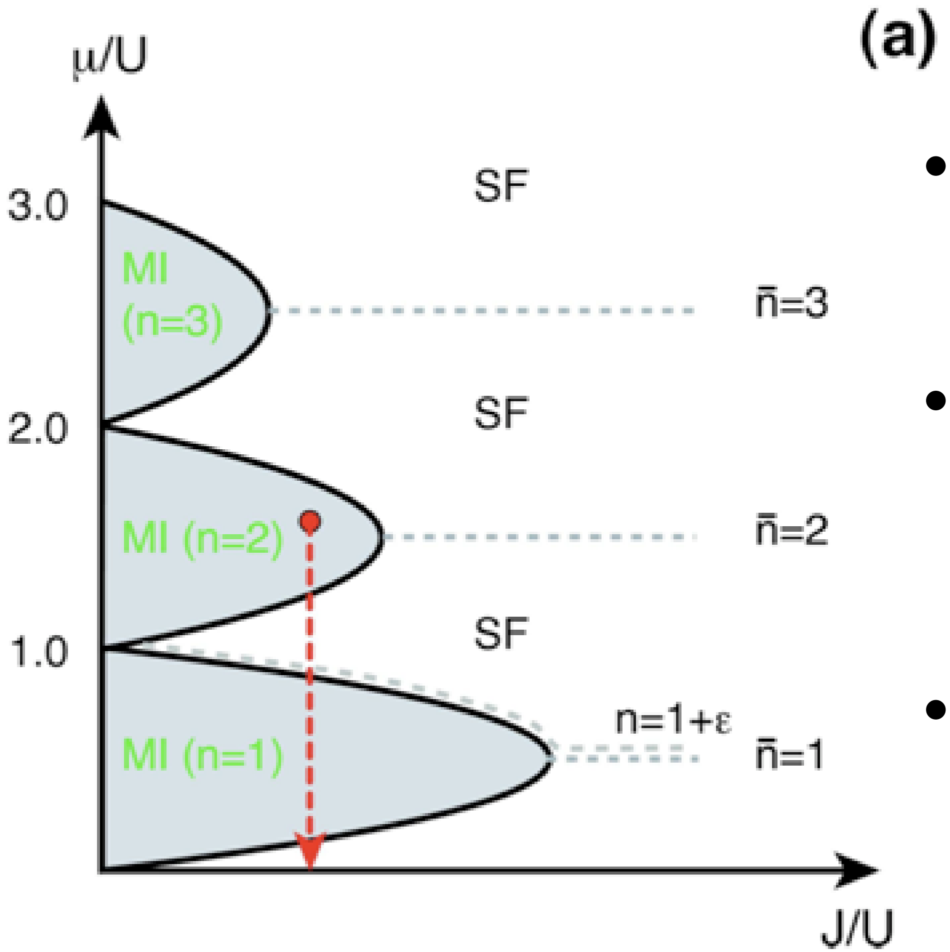
- Assume (for the moment) the number of atoms is equal to the number of lattice points

$$|\Psi^{J=0}_{N=N_L}\rangle = \left(\prod_i a_i^\dagger\right) |0\rangle$$

- With increasing J the atoms start to hop around, which involves double occupancy, increasing the energy by U. However, the ground state is no longer a simple product state
- Once J becomes of order or larger than U the gain in kinetic energy outweighs the repulsion due to double occupancy
- The atoms then undergo a transition, in the thermodynamic limit, so a superfluid state.

BHM (PhaseDiagram)

$$H = -J\sum_{i,j} a_i^{\dagger} a_j + \frac{U}{2}\sum_i n_i\,(n_i - 1) + \sum_i \epsilon_i n_i$$



- *U/J* → *0*: the KE dominate and the ground state is a delocalized superfluid
- *U/J* is large: interactions dominate and one obtains a series of MI phases with fixed integer filling ($\partial n/\partial \mu = 0$)
- The transition between the SF and MI phases is associated with a loss of long-range order

BHM (Meanfield)

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$$H = -J\sum_{i,j} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) + \sum_i \epsilon_i n_i$$

Meanfield substitution

$$a_i^\dagger a_j = \langle a_i^\dagger \rangle a_j + a_i^\dagger \langle a_j \rangle - \langle a_i^\dagger \rangle \langle a_j \rangle = \psi(a_i^\dagger + a_j) - \psi^2$$

$$\downarrow$$

$$\begin{aligned} H_{MF} &= -zJ\psi\left(a^\dagger + a\right) + \frac{U}{2}n\left(n-1\right) - \mu n + zJ\psi^2 \\ &= -\psi\left(a^\dagger + a\right) + \frac{\overline{U}}{2}n\left(n-1\right) - \overline{\mu}n + \psi^2 = H_0 + \psi V \end{aligned}$$

BHM (Meanfield Perturbation)

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$$H_{MF} = H_0 + \psi V$$

Expansion in ψ (odd powers zero)

- Denote the unperturbed energy of the state with n particles by $E_n^{(0)}$

$$E_g^{(0)} = \left\{ E_n^{(0)} |n = 0, 1, 2, 3...\rangle \right\}_{min}$$

$$E_g^{(0)} = 0, \ \text{ if } \overline{\mu} = 0$$

$$E_g^{(0)} = \frac{1}{2}\overline{U}g(g-1) - \overline{\mu}g, \ \text{ if } \overline{U}(g-1) < \overline{\mu} < \overline{U}g$$

- Second order correction

$$E_g^{(2)} = \psi^2 \sum_{n \neq g} \frac{|\langle g|V|n\rangle|^2}{E_g^{(0)} - E_n^{(0)}} = \left(\frac{g}{\overline{U}(g-1) - \overline{\mu}} + \frac{g+1}{\overline{\mu} - \overline{U}g} \right) \psi^2$$

BHM (Meanfield Critical Points)

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$$E_g = E_g^{(0)} + E_g^{(2)} + O(\psi^4)$$

Minimize energy as a function of superfluid parameter

$$\psi = 0, \quad \text{when} \quad \left(\frac{g}{\overline{U}(g-1) - \overline{\mu}} + \frac{g+1}{\overline{\mu} - \overline{U} g} \right) > 0$$

$$\psi \neq 0, \quad \text{when} \quad \left(\frac{g}{\overline{U}(g-1) - \overline{\mu}} + \frac{g+1}{\overline{\mu} - \overline{U} g} \right) < 0$$

$$\downarrow$$

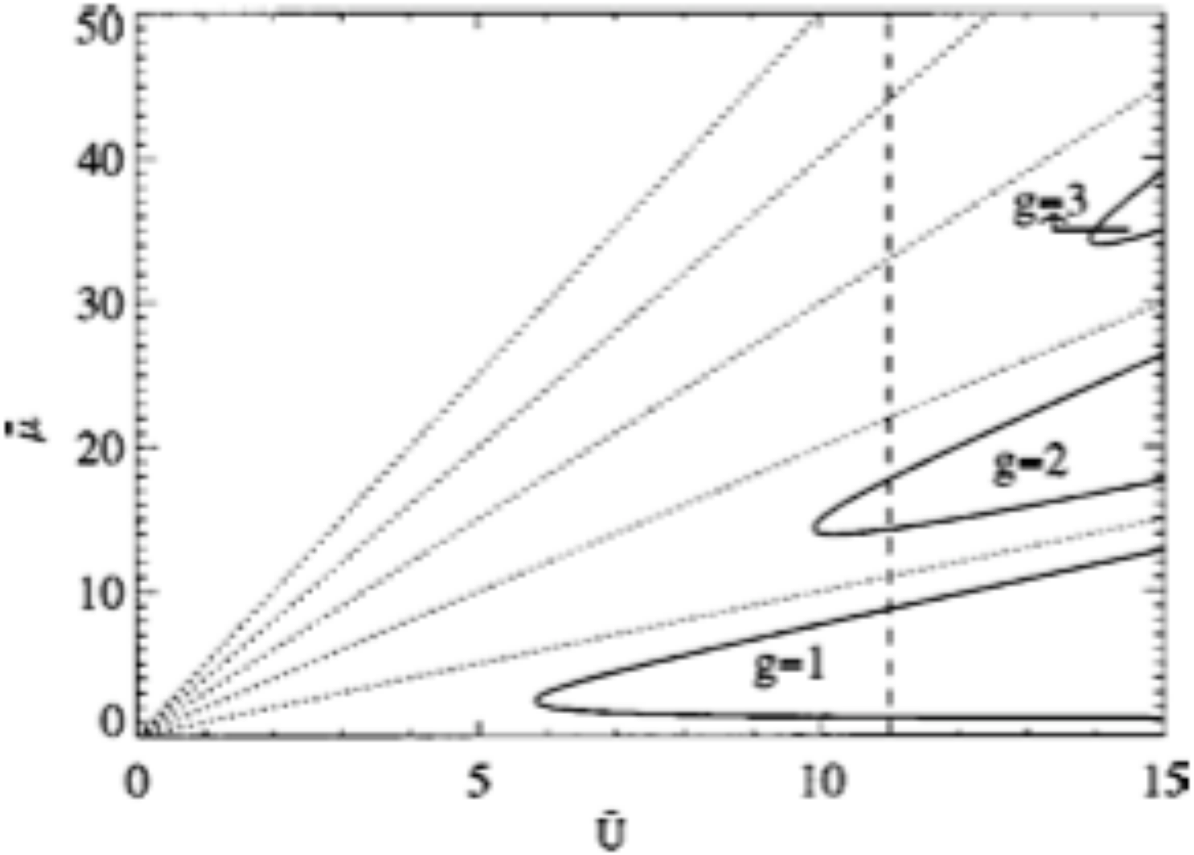
$$\overline{\mu}_{\pm} = \frac{1}{2}\left[\overline{U}(2g-1) - 1\right] \pm \frac{1}{2}\sqrt{\overline{U}^2 - 2\overline{U}(2g+1) + 1}$$

$$\overline{U}_c = 2g + 1 + \sqrt{(2g+1)^2 - 1}$$

BHM (Phase Diagram Revisited)

$$\overline{\mu}_{\pm} = \frac{1}{2}\left[\overline{U}(2g-1)-1\right] \pm \frac{1}{2}\sqrt{\overline{U}^2 - 2\overline{U}(2g+1)+1}$$

$$\overline{U}_c = 2g+1+\sqrt{(2g+1)^2-1}$$



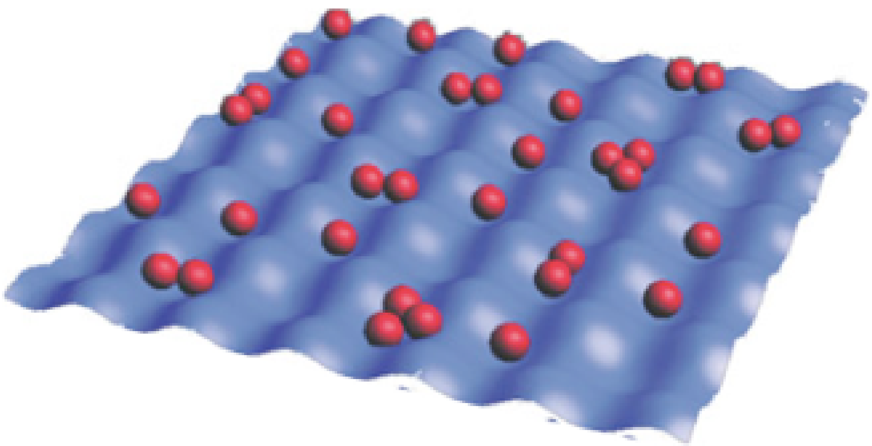
Meanfield

$$\overline{U}_c = 5.83, \quad \text{for } g = 1$$

QMC (z=4)

$$\overline{U}_c = 7.34, \quad \text{for } g = 1$$

BHM (Experiments: BECs)



Translation to experimental parameters

$$(V_0/E_r)_c = \frac{1}{4}\ln^2\left(\frac{\sqrt{2}d}{\pi a}(U/J)_c\right),$$

where E_r is the recoil energy $E_r = h^2/(2m\lambda^2)$

BHM (Experimental Results)

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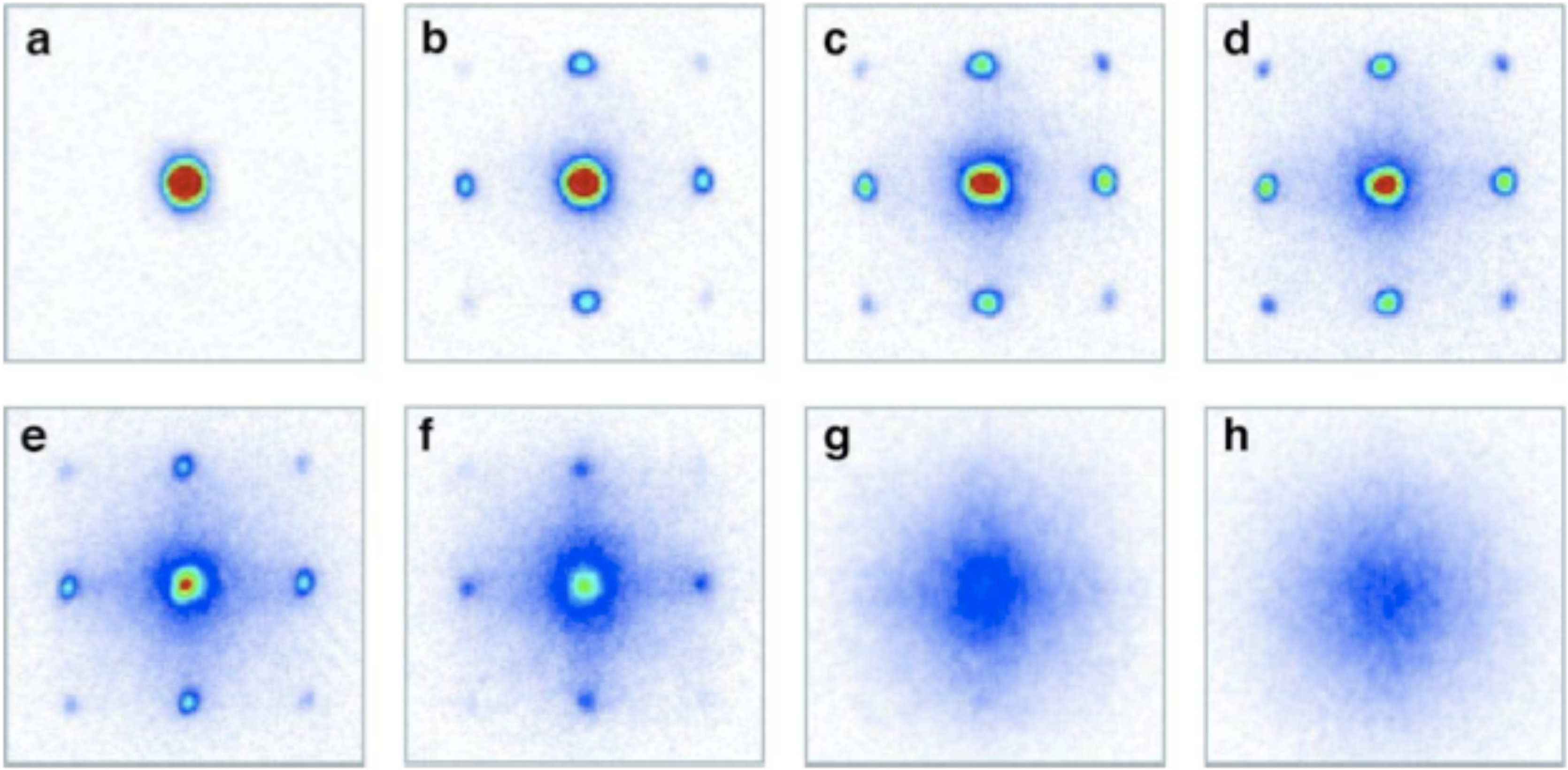

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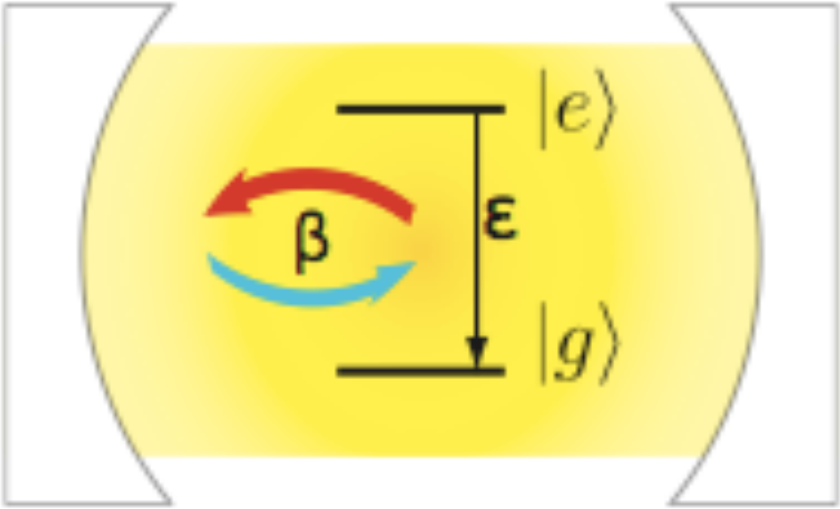
Momentum distribution

$$n(\mathbf{k}) \sim |\tilde{w}(\mathbf{k})|^2 \sum_R e^{i\mathbf{k}\cdot\mathbf{R}} G^{(1)}(\mathbf{R})$$

- MI phase: the one particle density matrix decays to zero exponentially
- SF phase: is characterized by by a momentum distribution which exhibits sharp peaks at the reciprocal lattice vectors $\boldsymbol{k} = \boldsymbol{G}$ ($\boldsymbol{G}.\boldsymbol{R} = 2\pi n$)
- The peaks in the momentum distribution initially grow because of the decrease in the spatial extent of the Wannier function $w(r)$, which results in an increase in its Fourier transform at higher momentum
- In the MI regime the remnants of the interference peaks remain as long as $G^{(1)}(\boldsymbol{R})$ extends over several lattice spacings

Single Atom Cavity

Jaynes-Cummings Model



- A two level atom interacts with a quantized cavity field
- The JC Hamiltonian consists of a single-moded quantized electromagnetic field, atomic excitation, and atom-field interaction terms

$$H^{\mathrm{JC}} = H_{\mathrm{field}} + H_{\mathrm{atom}} + H_{\mathrm{int}}$$

Jaynes-Cummings Model

$$H^{\mathrm{JC}} = H_{\mathrm{field}} + H_{\mathrm{atom}} + H_{\mathrm{int}}$$

- The Hamiltonian of the quantized free electromagnetic field for a single mode of frequency is

$$H_{\mathrm{field}} = \omega \left(a^{\dagger} a + \frac{1}{2} \right)$$

- The Hamiltonian of the atomic excitation is

$$H_{\mathrm{atom}} = \epsilon \sigma_{+} \sigma_{-}$$

- The Hamiltonian for the atom-photon interaction is derived from a classical description of a two-level transition of the electric dipole interaction (dipole approximation) and the rotating wave approximation:

$$H_{\mathrm{int}} = \beta \left(\sigma_{+} a + \sigma_{-} a^{\dagger} \right)$$

Single Atom Cavity

Jaynes-Cummings Model

$$H^{\mathrm{JC}} = \epsilon\sigma_+\sigma_- + \omega a^\dagger a + \beta\left(\sigma_+ a + \sigma_- a^\dagger\right)$$

$$\mathcal{H}^{\mathrm{JC}} = \begin{pmatrix} \boxed{0} & & & & & \\ & \epsilon & \beta & & & \\ & \beta & \omega & & & \\ & & & \epsilon+\omega & \sqrt{2}\beta & \\ & & & \sqrt{2}\beta & 2\omega & \\ & & & & & \ddots \end{pmatrix}, \quad \begin{pmatrix} |g,0\rangle \\ |e,0\rangle \\ |g,1\rangle \\ |e,1\rangle \\ |g,2\rangle \\ \vdots \end{pmatrix}$$

Single Atom Cavity

Jaynes-Cummings Model

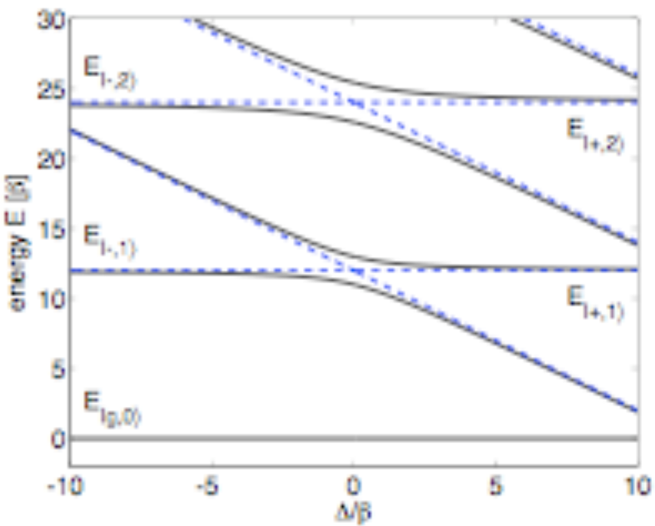
$$H^{\mathrm{JC}} = \epsilon\sigma_+\sigma_- + \omega a^\dagger a + \beta\left(\sigma_+ a + \sigma_- a^\dagger\right)$$

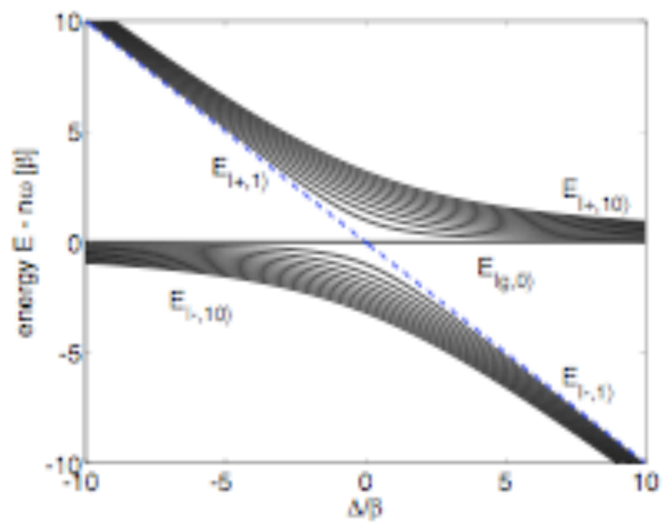
Bare basis	Dressed basis
$\|g,0\rangle$	$\|g,0\rangle$
$\|e,0\rangle$	$\|-,1\rangle$
$\|g,1\rangle$	$\|+,1\rangle$
$\|e,1\rangle$	$\|-,2\rangle$
$\|g,2\rangle$	$\|+,2\rangle$
$\vdots$	$\vdots$

$$E_{|g,0\rangle} = 0$$

$$E_{|\pm,n\rangle} = n\omega \pm \sqrt{n\beta^2 + \Delta^2/4} - \frac{\Delta}{2}$$

$$\Delta = \omega - \epsilon$$





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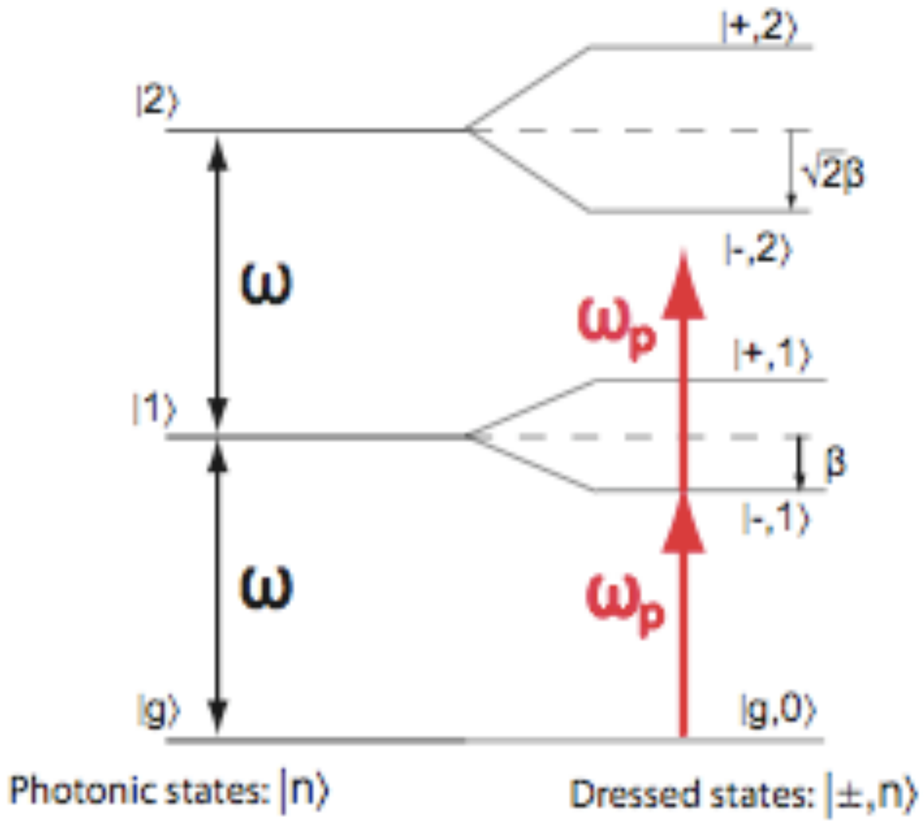
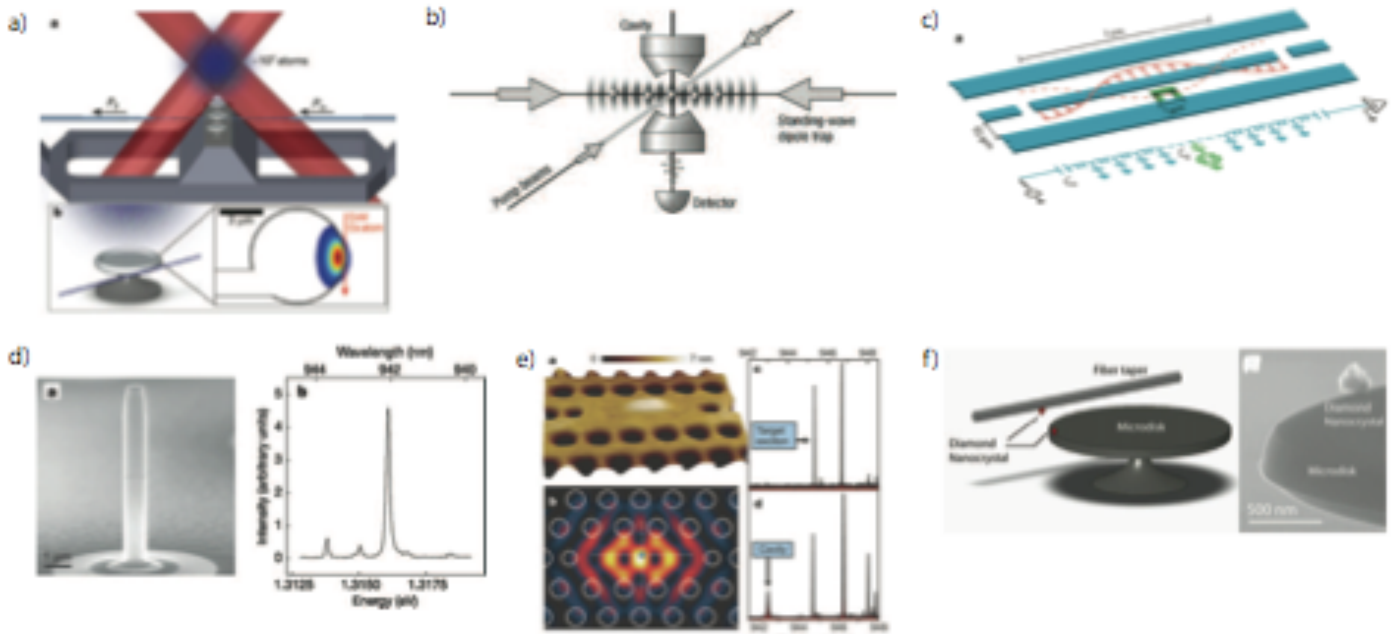
Jaynes-Cummings Model: Photon Blockade



Single Atom Cavity

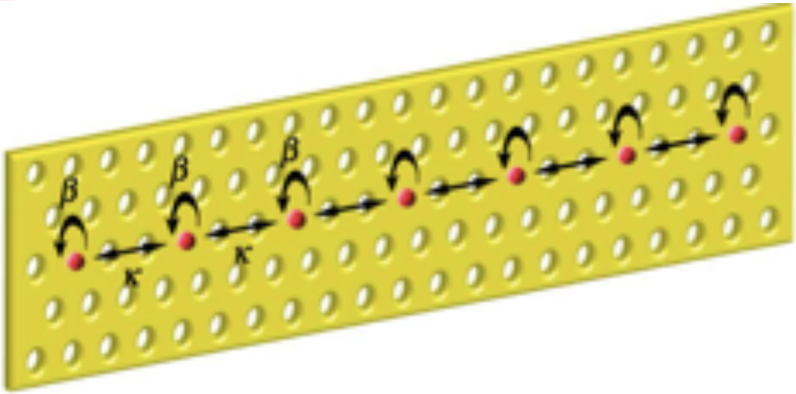
Experimental Possibilities (I)



Depictions of micro-cavities which have been coupled to high-dipole moment resonant transition systems: (a) cesium atoms in microtoroid cavity [1], (b) rubidium atom in Fabry-Perot resonators [2], (c) microstrip cavity with charge qubit [3] (d) quantum dots in Fabry-Perot resonator [4], (e) quantum dots in photonic bandgap cavity, and (f) diamond nitogen-vacancy centre in whispering gallery mode microdisk [5].

Coupled Atom Cavities

Experimental Possibilities (II)



- A possible realisation of a 1D solid-light systems. Here holes are drilled into a thin membrane and lattice defects serve as the optical cavities housing two-level atoms.

[1] **Observation of strong coupling between one atom and a monolithic microresonator**, T. Aoki *et al.*, Nature **443**, 671 (2006)

[2] **Vacuum-stimulated cooling of single atoms in three dimensions**, S. Nuβmann *et al.,* Nature Physics **1**, 122 (2005)

[3] **Superconducting quantum bits**, J. Clarke and F. Wilhelm, Nature **453**, 1031 (2008)

[4] **Strong Coupling in a single quantum dot-semiconductor microcavity system**, J.P. Reithmaier *et al.*, Nature **432**, 197 (2004)

[5] **Coherent interference effects in a nano-assembled diamond NV center cavity-QED system**, P. Barclay *et al.*, Optics Express **17**, 8081 (2009)

Coupled Atom Cavities

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JCH Hamiltonian

$$H^{\mathrm{JCH}} = \sum_i H_i^{\mathrm{JC}} - \kappa \sum_{\langle i,j \rangle} a_i^\dagger a_j - \sum_i \mu_i \left(\sigma_+^i \sigma_-^i + a_i^\dagger a_i \right)$$

Meanfield Hamiltonian

$$a_i^\dagger a_j = \langle a_i^\dagger \rangle a_j + a_i^\dagger \langle a_j \rangle - \langle a_i^\dagger \rangle \langle a_j \rangle = \psi (a_i^\dagger + a_j) - \psi^2$$

$$\downarrow$$

$$H^{\mathrm{JCH}} = H^{\mathrm{JC}} - z\kappa\psi(a^\dagger + a) + z\kappa\psi^2 - \mu \left(\sigma_+ \sigma_- + a^\dagger a \right)$$

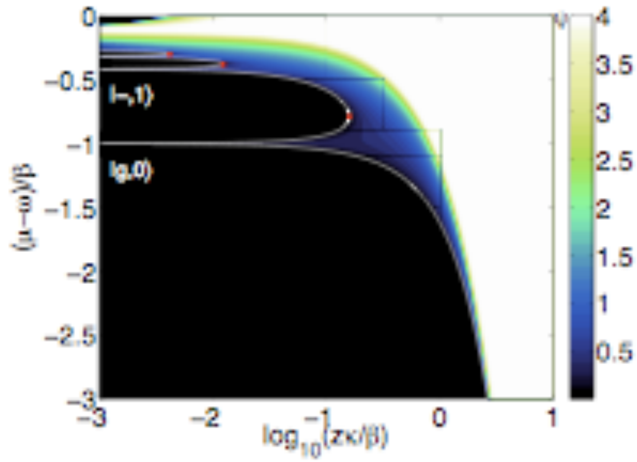
Finding the Groundstate

$$\mathcal{H}^{\mathrm{MF}} = z\kappa|\psi|^2 I +$$

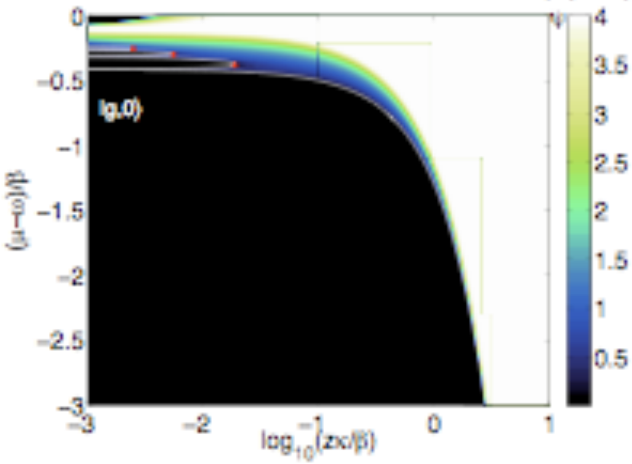
$$\begin{pmatrix}
0 & & -z\kappa\psi & & & & & \\
 & \epsilon-\mu & \beta & -z\kappa\psi & & & & \\
-z\kappa\psi & \beta & \omega-\mu & & -\sqrt{2}z\kappa\psi & & & \\
 & -z\kappa\psi & & \epsilon+\omega-2\mu & \sqrt{2}\beta & -\sqrt{2}z\kappa\psi & & \\
 & & -\sqrt{2}z\kappa\psi & \sqrt{2}\beta & 2(\omega-\mu) & & -\sqrt{3}z\kappa\psi & \\
 & & & -\sqrt{2}z\kappa\psi & & \epsilon+2\omega-3\mu & \sqrt{3}\beta & \ddots \\
 & & & & -\sqrt{3}z\kappa\psi & \sqrt{3}\beta & 3(\omega-\mu) & \\
 & & & & & \ddots & & \ddots
\end{pmatrix}$$

Coupled Atom Cavities

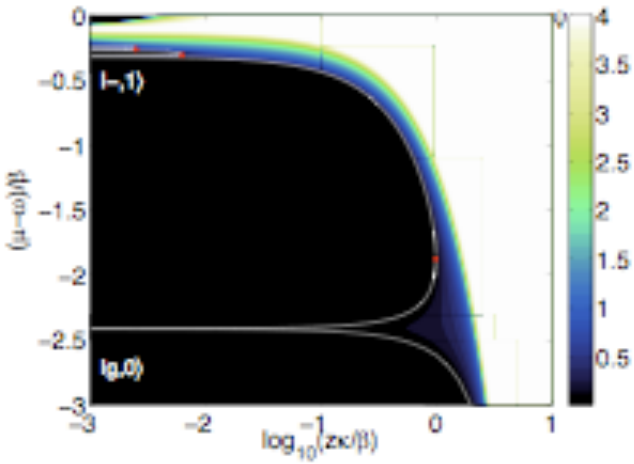
Meanfield Solution



(a) $\Delta/\beta = 0$



(b) $\Delta/\beta = -2$



(c) $\Delta/\beta = 2$