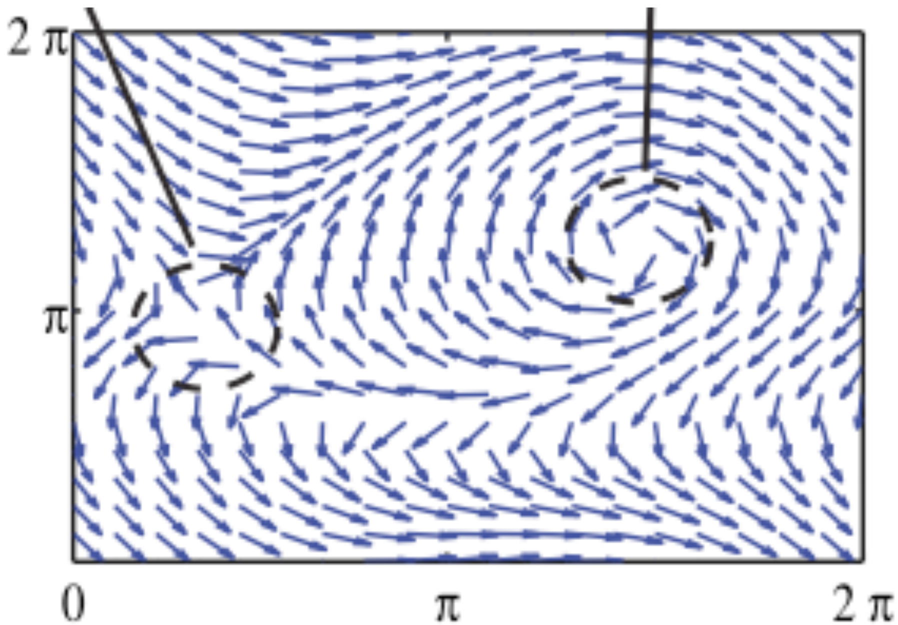
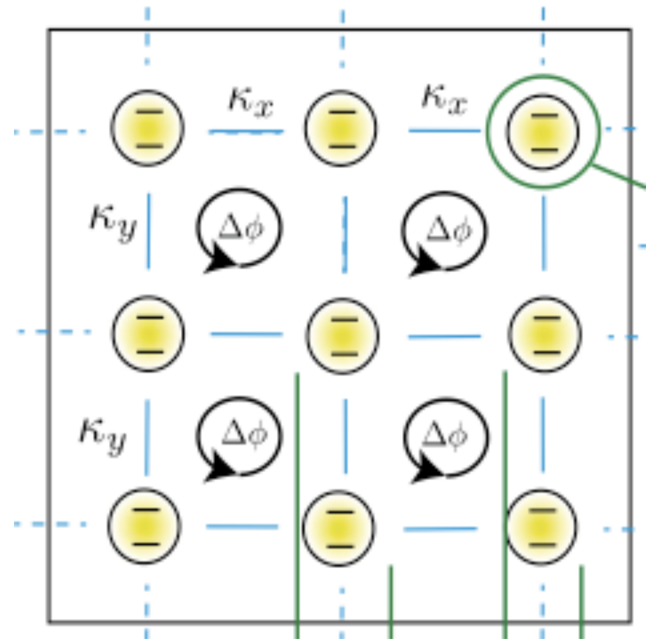
Fractional Quantum Hall Effect in Lattice Models





Andy Martin
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Outline

- Bose-Hubbard Model
  - Synthetic magnetic field
  - Laughlin ansatz
  - Exact diagonalisation
- Jaynes Cummings Bose-Hubbard Model
  - Synthetic magnetic field
  - Laughlin ansatz
  - Exact diagonalisation & Chern number
- Stuff I didn't discuss
  - Anderson localisation
  - Quantum metamaterials
  - Unconventional superconductivity: dipolar interactions

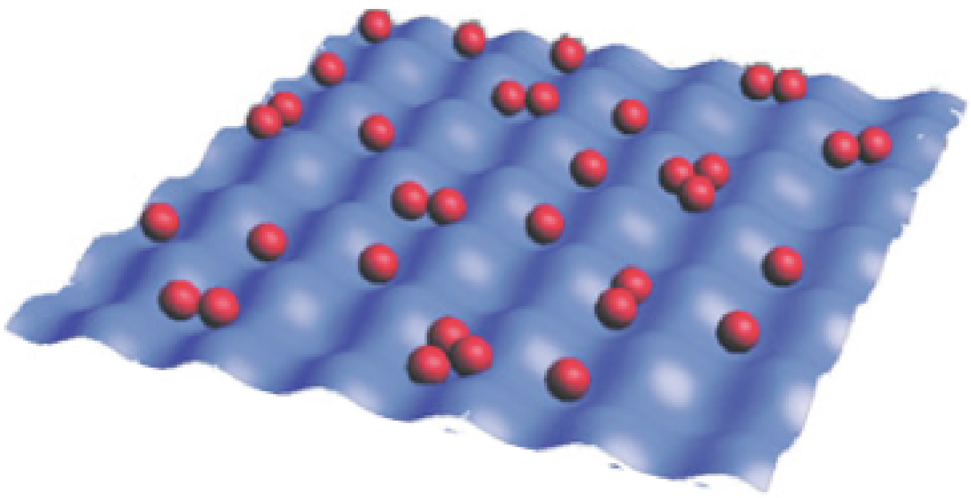
Reference Material

- Condensed matter emulation
  - *Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond*, M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen and U. Sen, Advances in Physics **56**, 243 (2007)
  - *Many-body physics with ultracold gases*, I. Bloch, J. Dalibard and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008)
- Ultracold gases
  - *Fractional quantum Hall states of atoms in optical lattices*, A.S. Sorensen, E. Demler and M.D. Lukin, Phys. Rev. Lett. **94**, 086803 (2005)
  - *Fractional quantum Hall effect in optical lattices*, M. Hafezi, A.S. Sorensen, E. Demler and M.D. Lukin, Phys. Rev. A **76**, 023613 (2005)
- Coupled atom cavities
  - *Fractional quantum Hall physics in Jaynes-Cummings-Hubbard lattices* , A.L.C. Hayward, A.M. Martin, A.D. Greentree, Phys. Rev. Lett. **108**, 223602 (2012)

Bose-Hubbard System

Reminder

$$H = -J\sum_{\{j,k\}}(\hat{a}_j^\dagger \hat{a}_k + \hat{a}_k^\dagger \hat{a}_j) + U\sum_j n_j(n_j - 1)$$



Bose-Hubbard System

Time varying quadrupolar potential

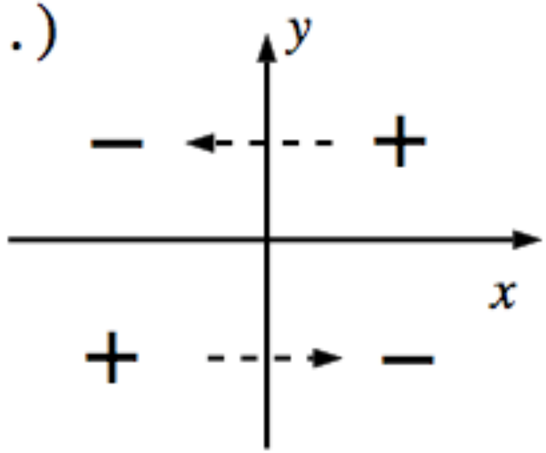
$$V(t) = V_{\mathrm{qp}} \sin(\omega t)\hat{x}\,\hat{y}$$

Tunnelling in x and y turned on at specific times

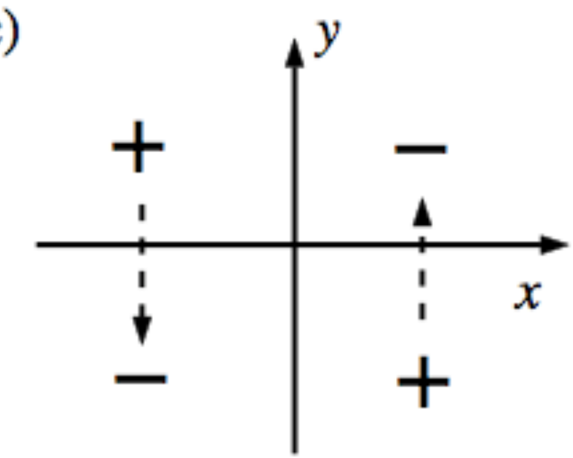
$$t = t_0 n \; (n = 0, 1, 2, \ldots)$$

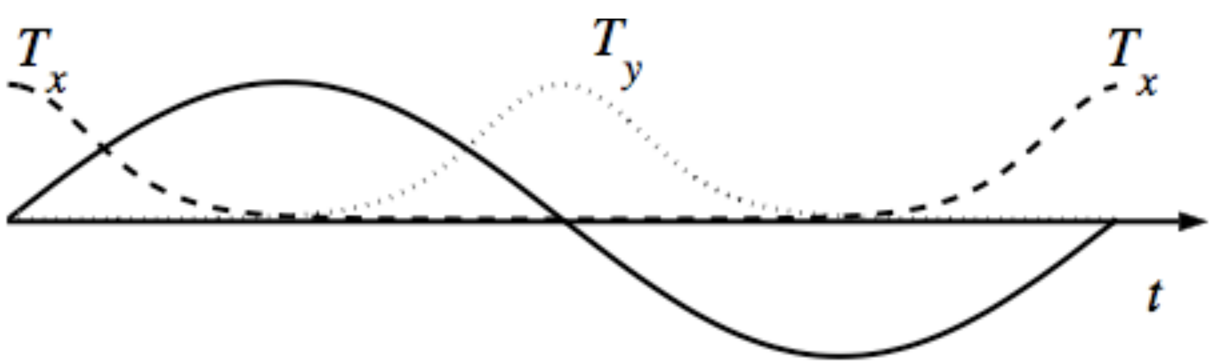
$$t = t_0(n + 1/2)$$

$$t_0 = 2\pi/\omega$$



(c)

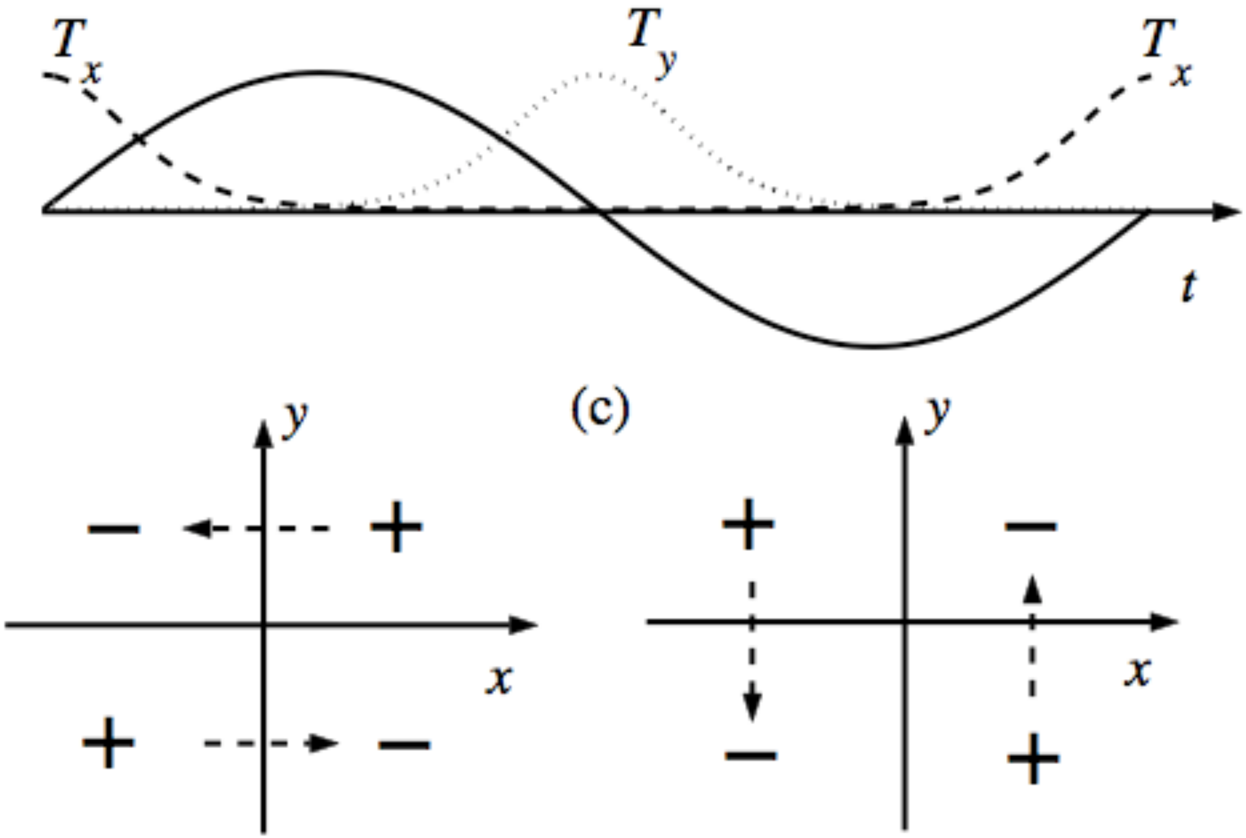




Bose-Hubbard System

Synthetic magnetic field

$$H_{\text{eff}} \approx -J\sum_{x,y} \hat{a}^{\dagger}_{x+1,y}\hat{a}_{x,y} + \hat{a}^{\dagger}_{x,y+1}\hat{a}_{x,y}e^{i2\pi\alpha x} + \text{H.c.}$$



Tunneling in the x direction is followed by a positive potential in the first and third quadrant, and hence atoms will experience a lower potential by moving in the direction of the dashed arrows.

Tunneling in the y direction and opposite sign of the potential. When combined, the dashed lines make a circular cyclotron motion

Bose-Hubbard System

Synthetic magnetic field: Symmtric Gauge

$$H = -J\sum_{x,y} \hat{a}^{\dagger}_{x+1,y}\hat{a}_{x,y}e^{-i\pi\alpha y} + \hat{a}^{\dagger}_{x,y+1}\hat{a}_{x,y}e^{i\pi\alpha x} + \text{H.c.}$$

$$+ U\sum_{x,y} \hat{n}_{x,y}(\hat{n}_{x,y} - 1),$$

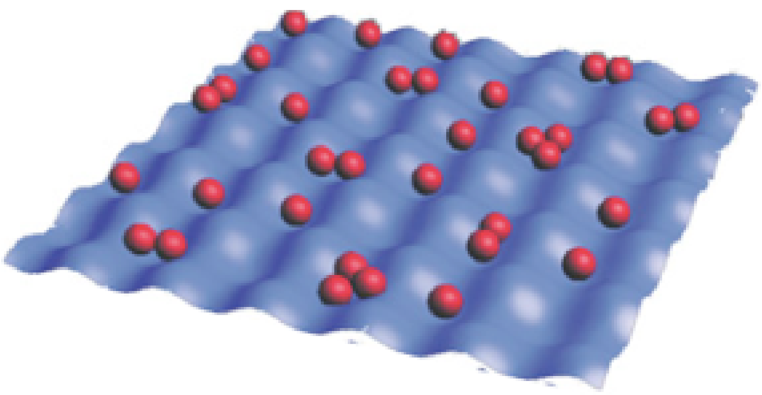
$$\vec{A} = \frac{B}{2}(-y, x, 0)$$

$$-J\sum_{\langle ij\rangle} a_i^{\dagger} a_j \exp\left(\frac{2\pi i}{\Phi_0}\int_i^j \vec{A}\cdot d\vec{l}\right)$$

Energy Scales

$$\hbar\omega_c = 4\pi J\alpha$$

$$U$$



Laughlin Ansatz

$$\Psi(z_1,z_2,\ldots,z_N)=\prod_{j>k}^{N}(z_j-z_k)^m\prod_{j=1}^{N}e^{-y_i^2/2},$$

$$\Downarrow$$

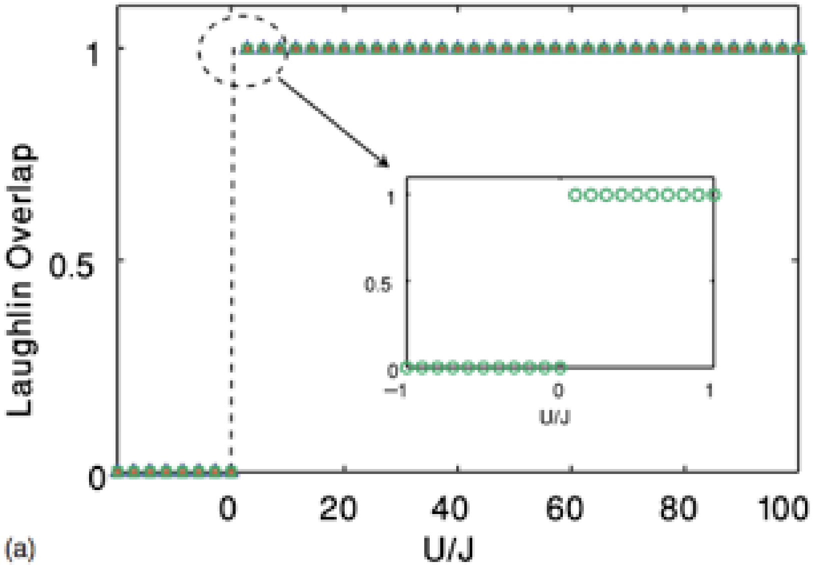
$$\Psi(z_1,z_2,\ldots,z_N)=f_{rel}(z_1,z_2,\ldots,z_N)F_{\mathrm{c.m.}}(Z)\exp\left(-\sum_i y_i^2/2\right)$$

$$f_{rel}=\prod_{i<j}\vartheta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}\left(\frac{z_i-z_j}{L_x}\middle|\, i\frac{L_y}{L_x}\right)^2$$

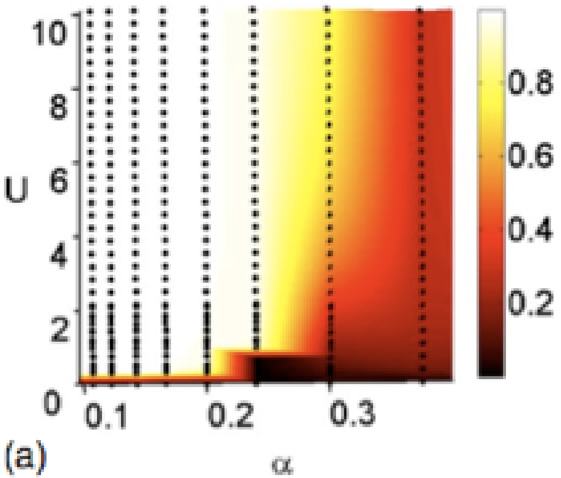
$$F_{\mathrm{c.m.}}(Z)=\vartheta\begin{bmatrix}l/2+(N_\phi-2)/4\\ -(N_\phi-2)/2\end{bmatrix}\left(\frac{2\sum_i z_i}{L_x}\middle|\, 2i\frac{L_y}{L_x}\right)$$

Bose-Hubbard System

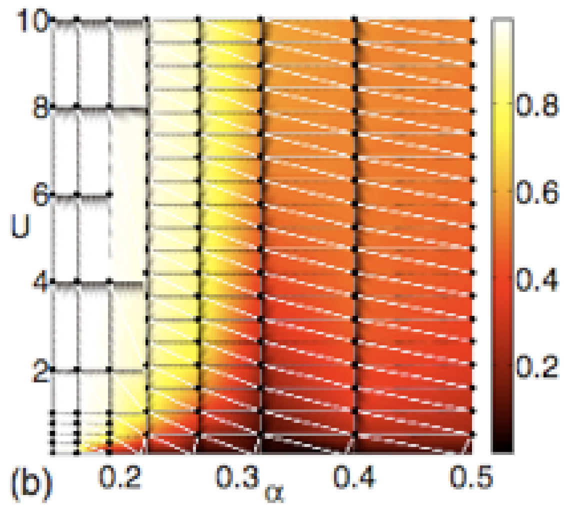
Overlap



(a)



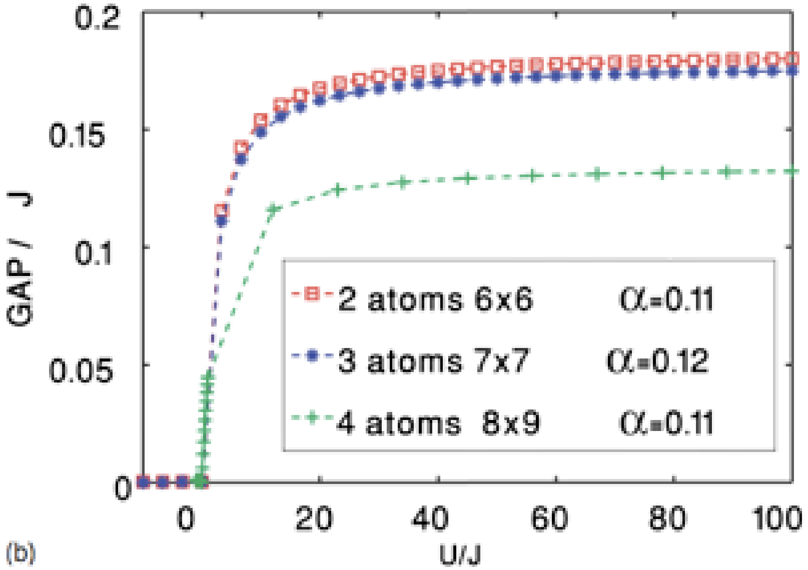
(a)

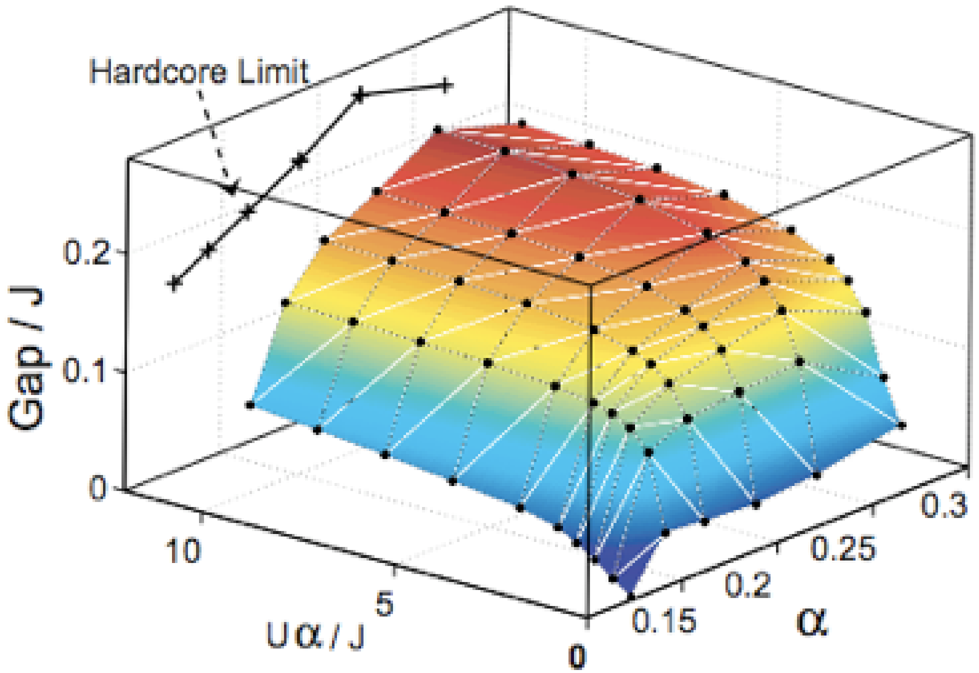


(b)

Bose-Hubbard System

Energy Gap





Jaynes Cummings

Model

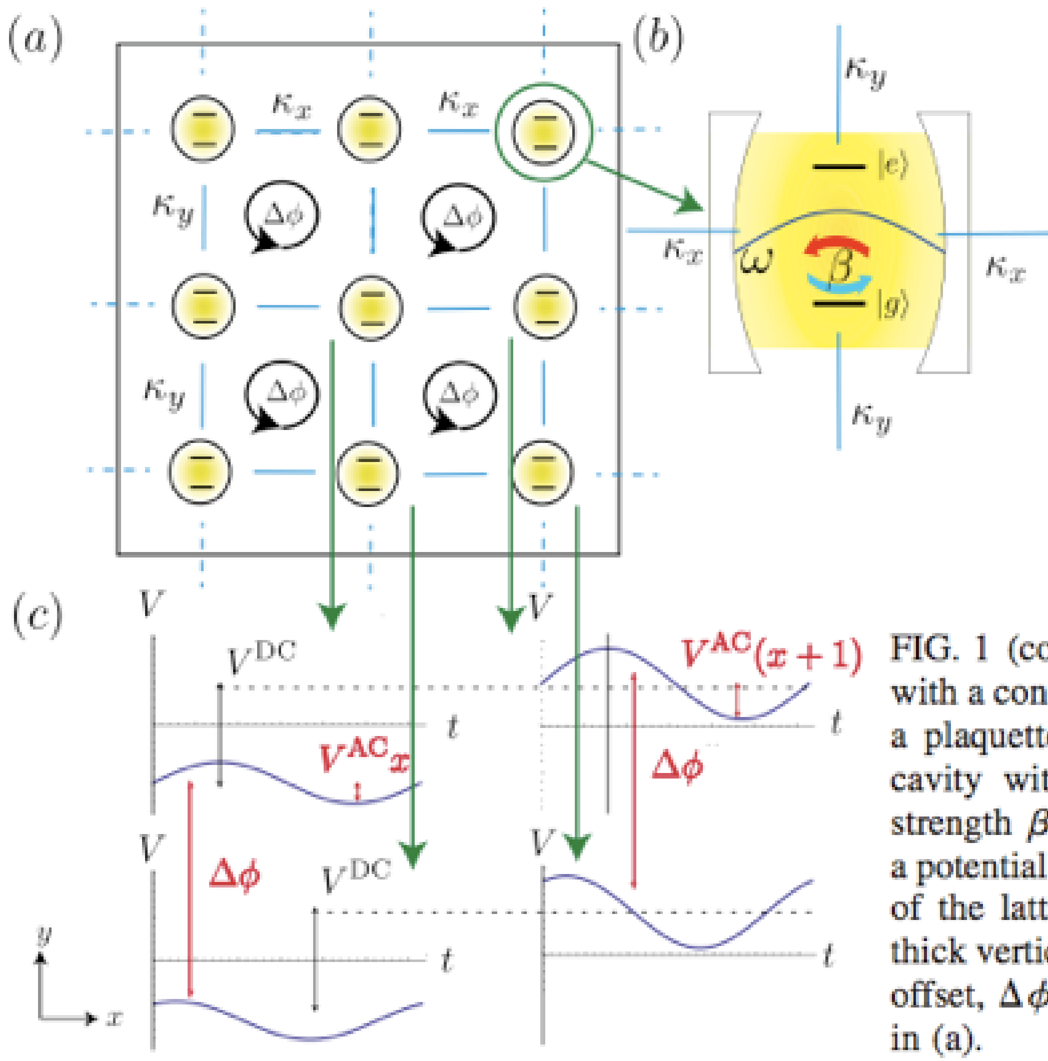


FIG. 1 (color online). (a) Schematic of a square JCH lattice with a constant effective magnetic field. Photons moving around a plaquette acquire a phase $\Delta\phi$. (b) A single-mode photonic cavity with frequency ω coupled to a two-level atom with strength β. (c) Scheme for breaking TRS in photonic cavities: a potential $V = [V^{DC} + V^{AC} \sin(\omega^{rf} t + \Delta\phi y)]x$ (x and y in units of the lattice spacing) is applied to the cavities (indicated by thick vertical green arrows) by dynamically tuning ω. The phase offset, $\Delta\phi$, along y results in the synthetic magnetic field seen in (a).

Jaynes Cummings

Laughlin

$$\Psi_L(\bar{z}) \propto F_{\rm cm}(Z) f_{\rm rel}(\bar{z}) \prod_i \psi_i^L$$

$$F_{\rm cm}(Z) = \theta\begin{bmatrix} N_p/q + (N_\phi - 2)/2q \\ -(N_\phi - 2)/q \end{bmatrix}\left(q\frac{Z}{L_x} \,\middle|\, iq\frac{L_y}{L_x}\right)$$

$$f_{\rm rel}(\bar{z}) = \prod_{i<j}^{N_p} \theta_1\left(\frac{z_i - z_j}{L_x} \,\middle|\, i\frac{L_y}{L_x}\right)^q,$$

Jaynes Cummings

Results

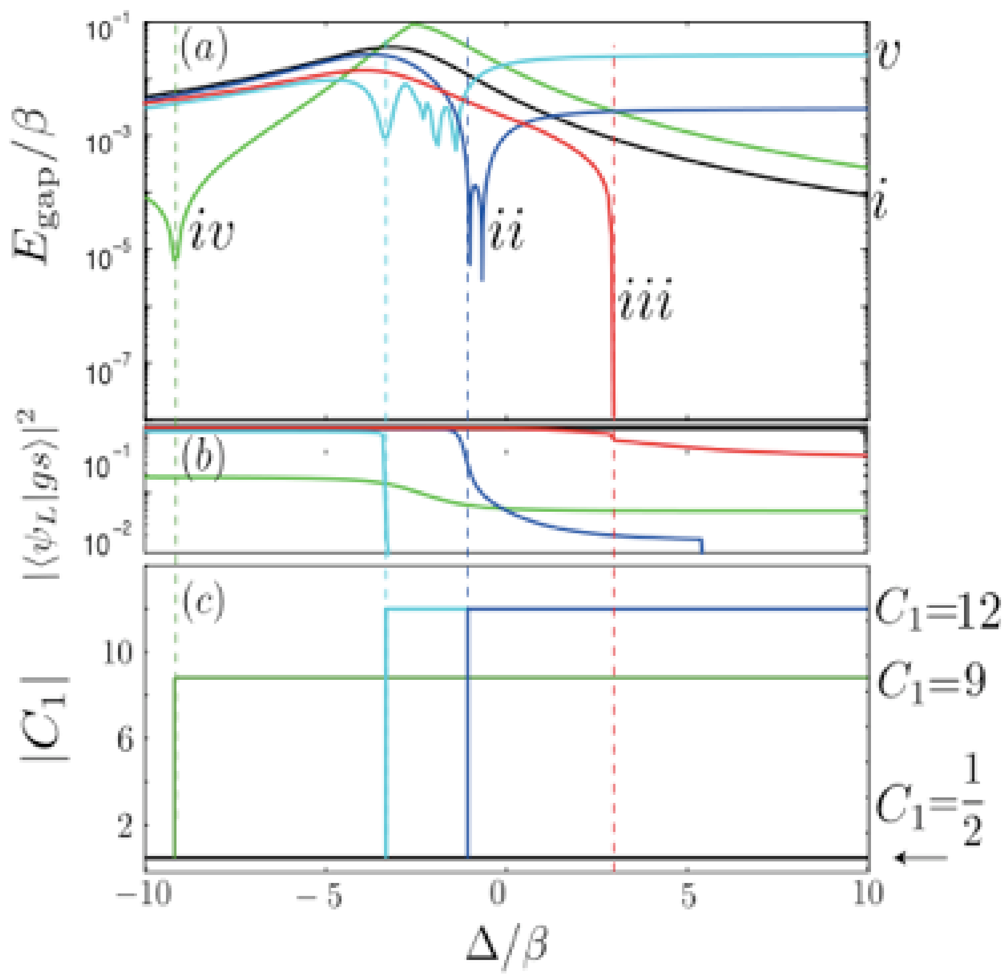
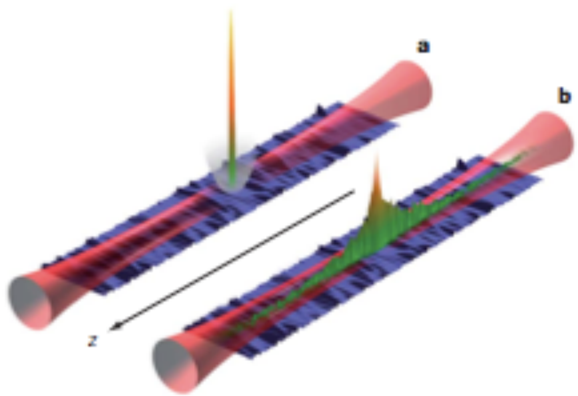


TABLE I. Results for systems of size $L_x \times L_y$ sites with N_p excitations. All systems have $C_1 = 1/2$ below the transition strength $\Delta = \Delta_c$. Also shown are the Hilbert space dimensions [36] Dim(H) and the Laughlin overlap.

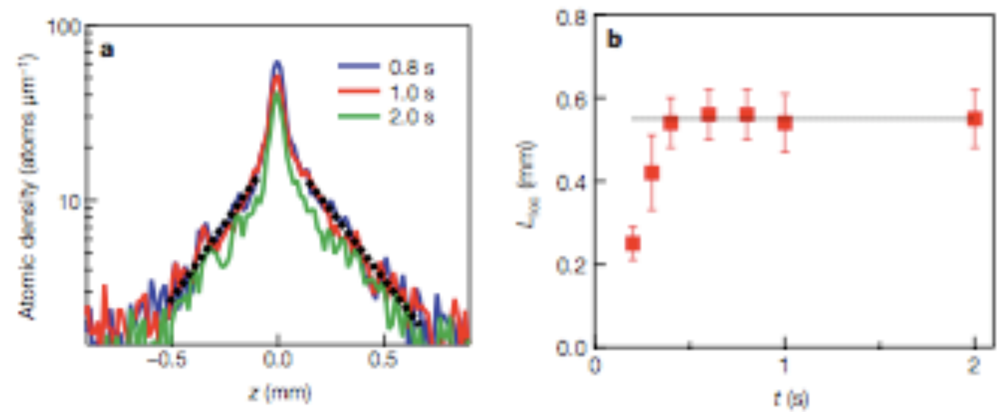
	$L_x \times L_y$	N_p	Dim(H)	α	Laughlin overlap	Transition(Δ_c)
i	4×4	2	512	0.25	0.89	
ii	5×5	2	1250	0.16	0.99	-1.1
iii	6×6	2	2592	0.11	0.99	2.5
iv	4×4	3	5472	0.37	0.29	-9.1
v	5×5	3	20 850	0.24	0.98	-3.8
vi	6×6	3	62 232	0.17	0.99	

Anderson Localization (BEC)

Setup



Results



$$L_{loc} = \frac{2\hbar^4 k_{max}^2}{\pi m^2 V_R^2 \sigma_R \left(1 - k_{max}\sigma_R\right)}$$

- Signature: exponential decay of the wavefunction

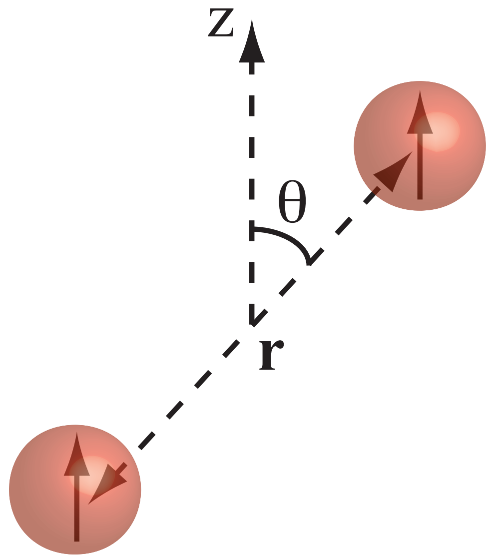
- Anderson Localization
  - *Direct observation of Anderson localization of matter waves in controlled disorder*, J. Billy *et al.*, Nature **453**, 891 (2008)

Long-Range Dipolar Interaction

$$U_{dd}(\mathbf{r}) = \frac{C_{dd}}{4\pi}\left[\frac{1-3\cos^2\theta}{r^3}\right]$$

Long-Range

Anisotropic



Magnetic dipole-dipole interaction:

Atoms

^{52}Cr (Bosons)

Tilman Pfau (Stuttgart)

Electrostatic dipole-dipole interaction:

Polar Molecules

^{39}K^{87}Rb (Bosons)

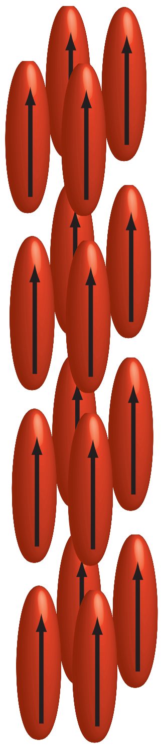
^{40}K^{87}Rb (Fermions)

Debbie Jin (JILA)

Geometry Dependent Interaction

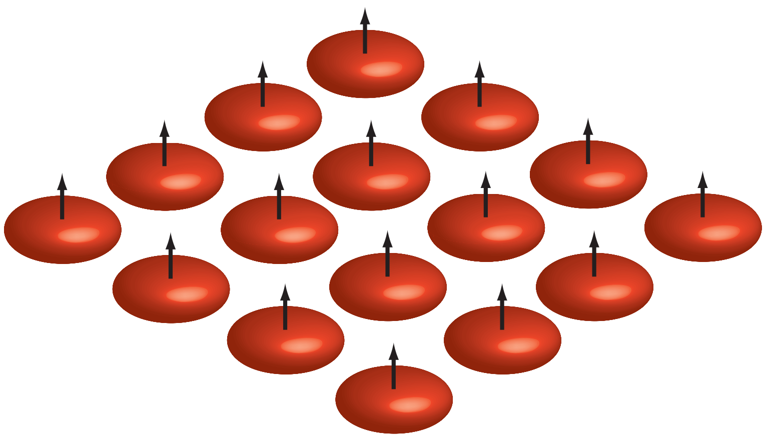
Prolate:

net top-to-tail
attractive interaction



Oblate:

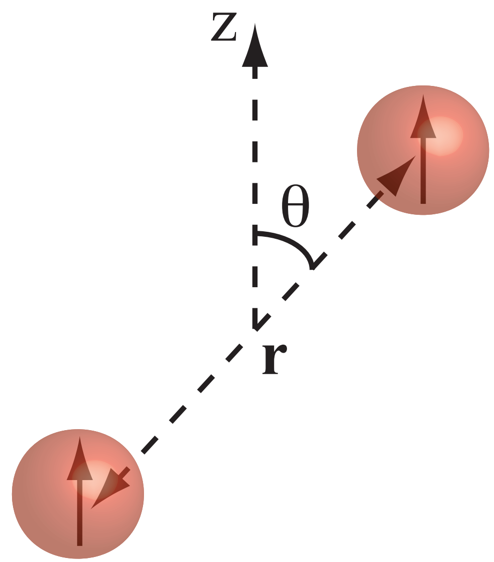
net side-by-side
repulsive interaction



Dipolar vs s-wave Interactions

long-range: $U_{dd}(\mathbf{r}) = \frac{C_{dd}}{4\pi}\left[\frac{1-3\cos^2\theta}{r^3}\right]$

short-range: $U_\mathrm{s}(\mathbf{r}-\mathbf{r}') = g\delta(\mathbf{r}-\mathbf{r}')$



$$\boxed{\varepsilon_\mathrm{dd} \equiv \frac{C_\mathrm{dd}}{3g}}$$

$\varepsilon_{dd} > 1 \rightarrow \text{collapse}$

Magnetic dipole-dipole:

^{87}Rb	$\varepsilon_{dd} = 0.007$	^{52}Cr	$\varepsilon_{dd} = 0.16$
^{23}Na	$\varepsilon_{dd} = 0.004$	Feshbach	$\varepsilon_{dd} = 4.0$

A. Griesmaier *et al.*, PRL **94**, 160401 (2005)

2D Dipolar BEC (Excitations)

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Integrate over plane

$$0 = \left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + \frac{m\omega_z^2 z^2}{2} + (g + g_d)\psi_0^2(z) - \mu \right] \psi_0$$

$g_d = 2C_{dd}/3$

Thomas-Fermi

$$n_0(z) = n_0 \left(1 - \frac{z^2}{L^2} \right), \quad \text{where } n_0 = \frac{\mu}{g + g_d}, \quad \text{and } L = \left(2\mu/m\omega_z^2 \right)^{1/2}$$

Linearize DGPE

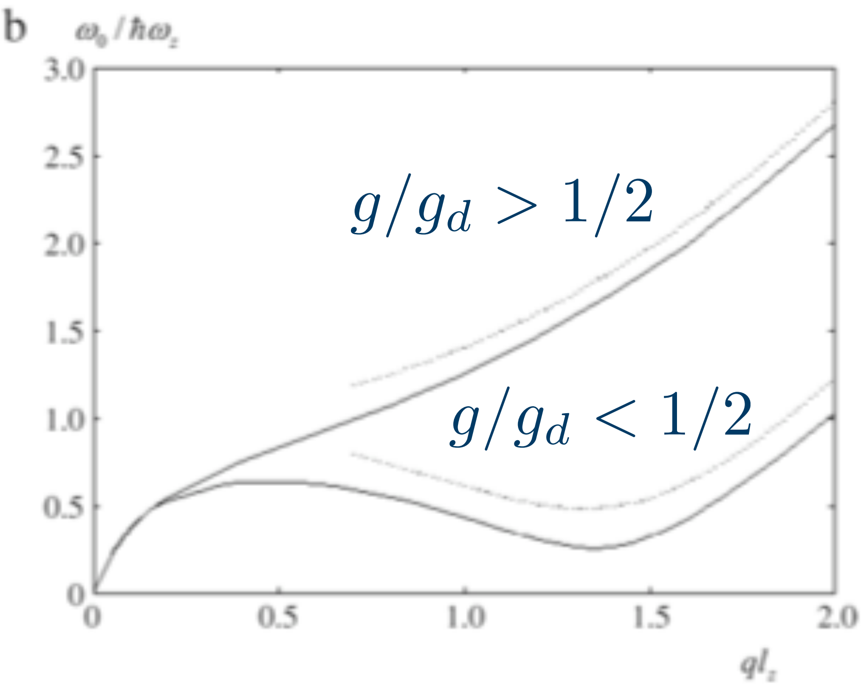
$\downarrow$

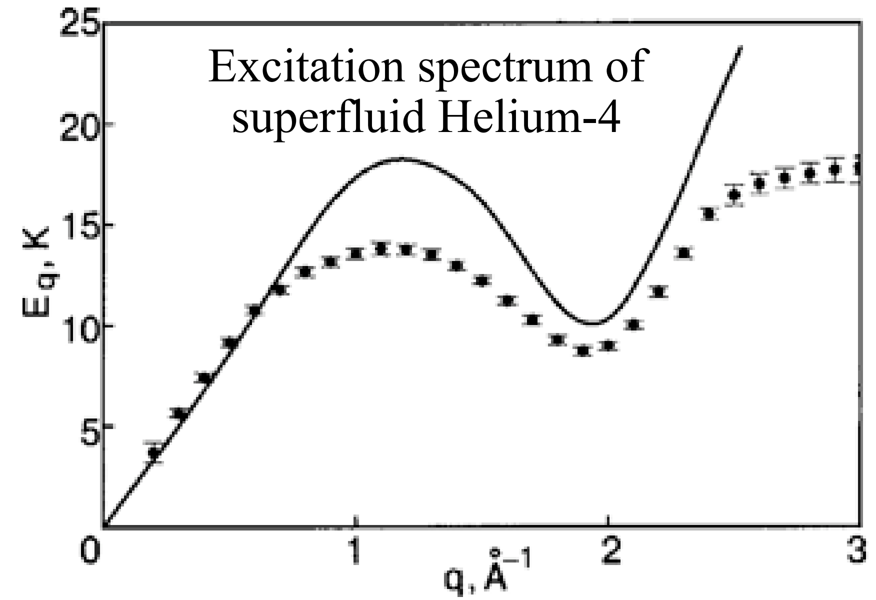
Non-Local BDGE

$\swarrow$

Momentum dependence of effective coupling strength

Dipolar BEC (Roton)





- Dipolar BEC is the first example of a weakly interacting gas offering the possibility of obtaining a roton-maxon, upto now only observed in the more complicated physics of He
- Moreover, the roton-maxon is tunable → can induce instability

Dipolar Fermi systems

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Dipolar Interaction Partially Attractive

$$\langle L=1, M=0|1-3\cos^2\theta|L=1, M=0\rangle = -\frac{4\pi}{5} < 0$$

$$\Downarrow$$

BCS Pairing?

- In a single-component Fermi gas the Pauli principle allows pairing with only odd angular momentum L of the relative motion of two particles in a Cooper pair.
- On the other hand, the anisotropy of the dipole-dipole interaction results in coupling between different angular momenta.
- Hence the problem of superfluid pairing requires consideration of all odd angular momentum L.

Properties of the BCS State

Critical Temperature

$$T_c = 1.44 E_F \exp\left\{-\frac{\pi^2 E_F}{3nC_{dd}}\right\} \qquad E_F \approx (\hbar^2/2m)(6\pi^3 n)^{2/3}$$

$$T_c \approx 100nK$$

Order Parameter

$$\Delta(p_F\hat{\mathbf{p}}) \approx 2.5T_c\sqrt{1-\frac{T}{T_c}}\sqrt{2}\sin\left(\frac{\pi}{2}\cos\theta_{\mathbf{p}}\right)$$

Properties of the BCS Gap

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- The anisotropy of the gap provides the major difference of the properties of the superfluid dipolar Fermi gas from the more conventional two component fermionic gases, with s-wave pairing
- As a consequence excitations with momenta in the direction of the dipoles acquire the largest gap. In contrast eigenenergies of excitations with momenta perpendicular to the dipoles remain unchanged
- The nodes in the gap lead to significant changes in the properties of the specific heat, i.e. well below the transition the single particle contribution to the specific heat is proportional to T^2.
- This also has a consequence for how disorder effects the critical temperature.

Unconventional Superconductivity

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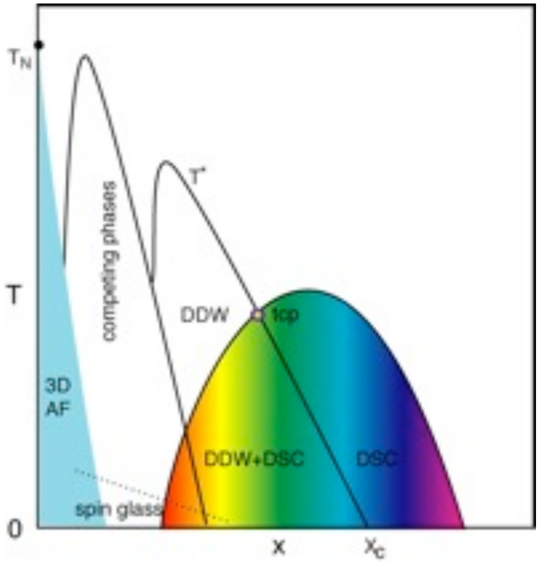

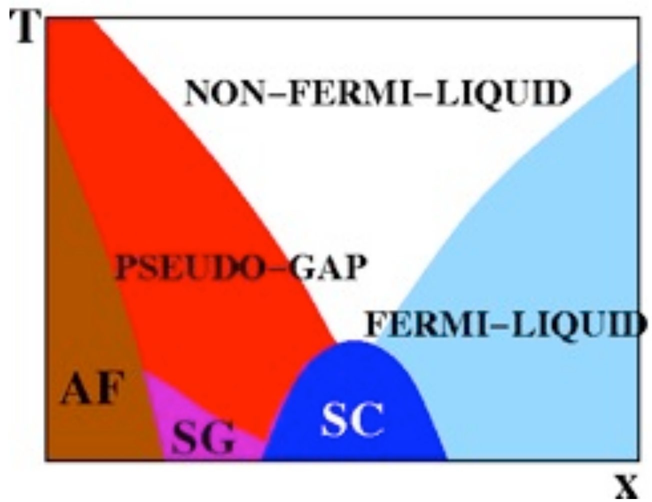

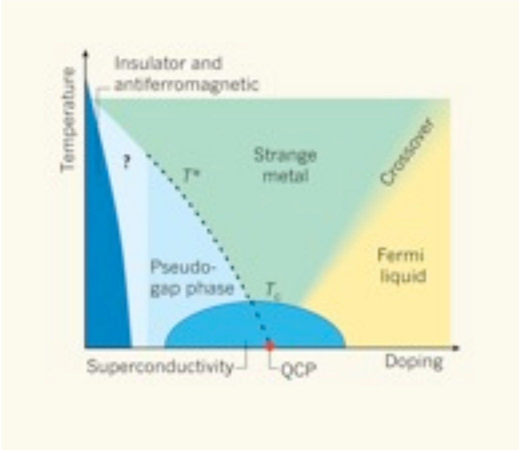

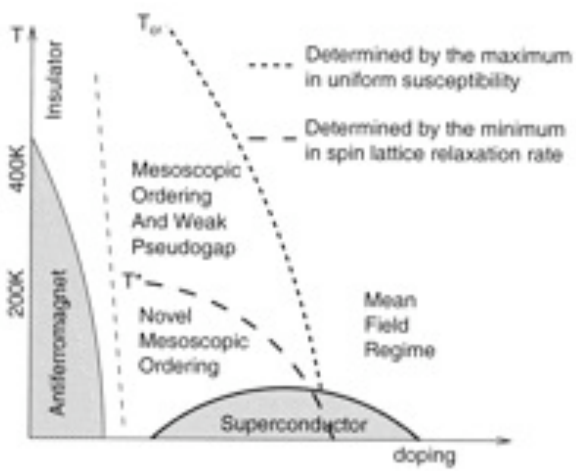

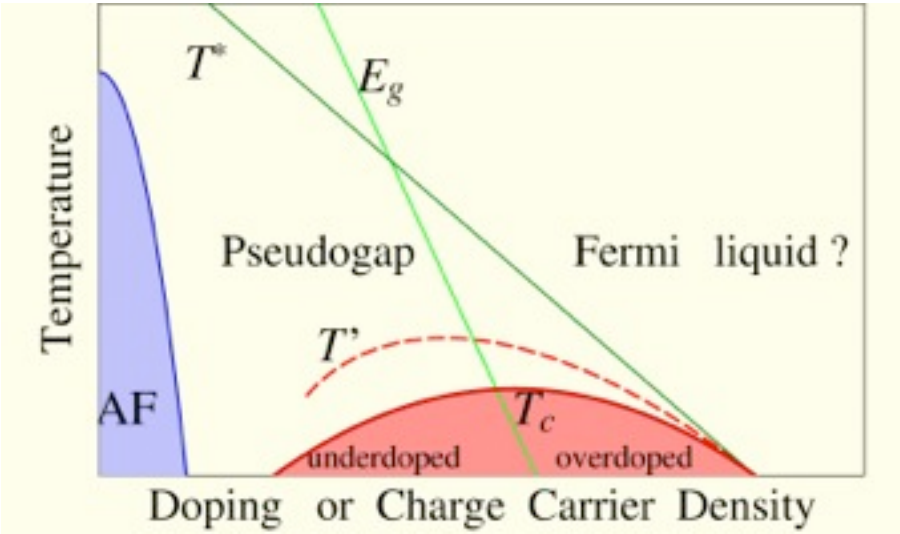

Why the Interest (money)?

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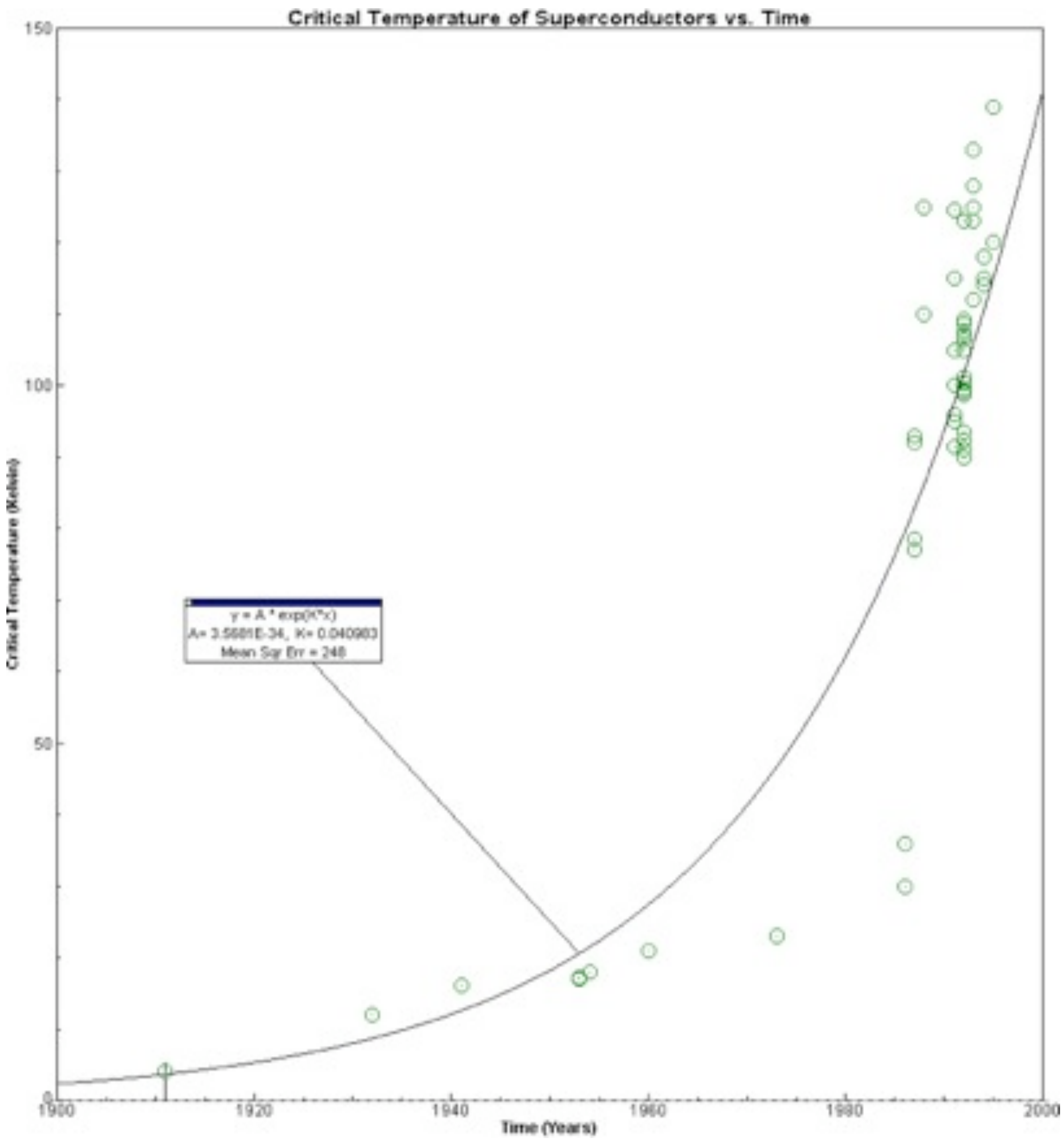
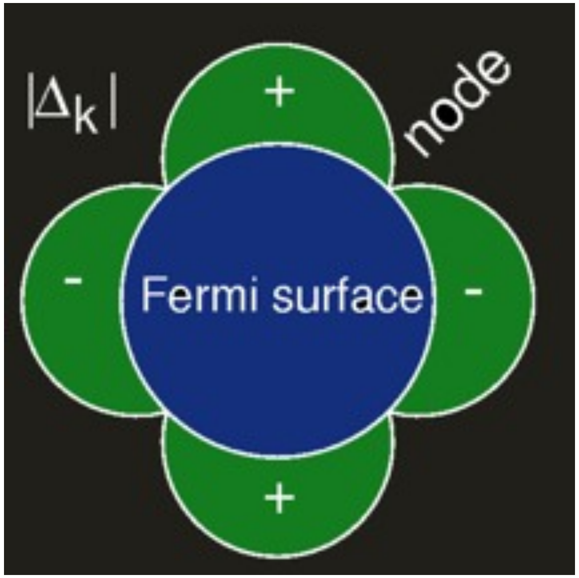
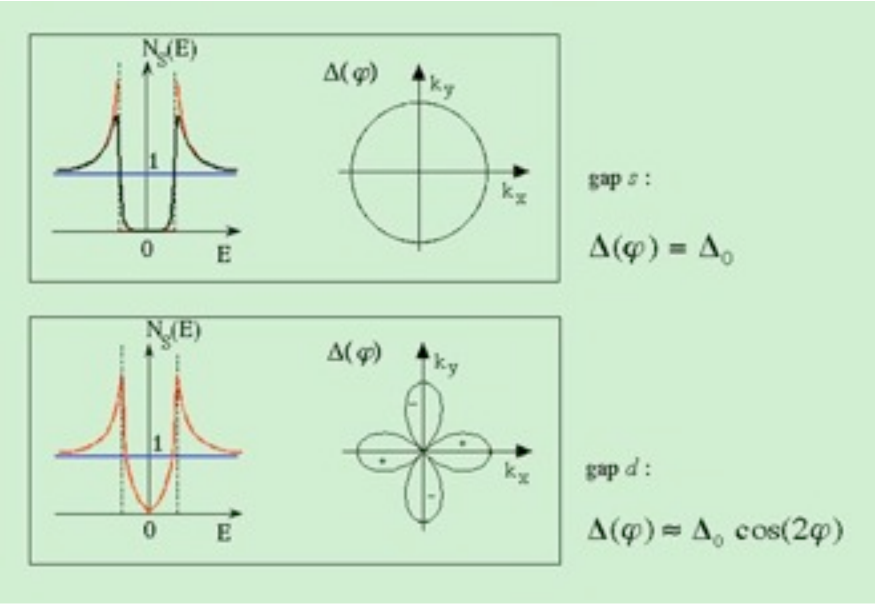


What can we agree on?

None Trivial Mechanism

Anisotropic Gap (d-wave)





How can DFG Help?

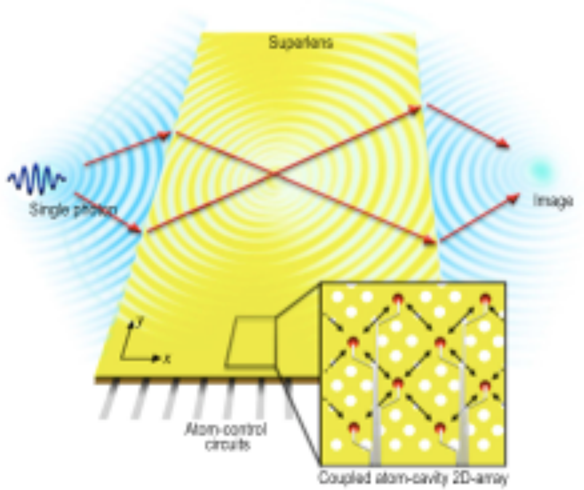
Controllable System with Anistropic Order Parameter

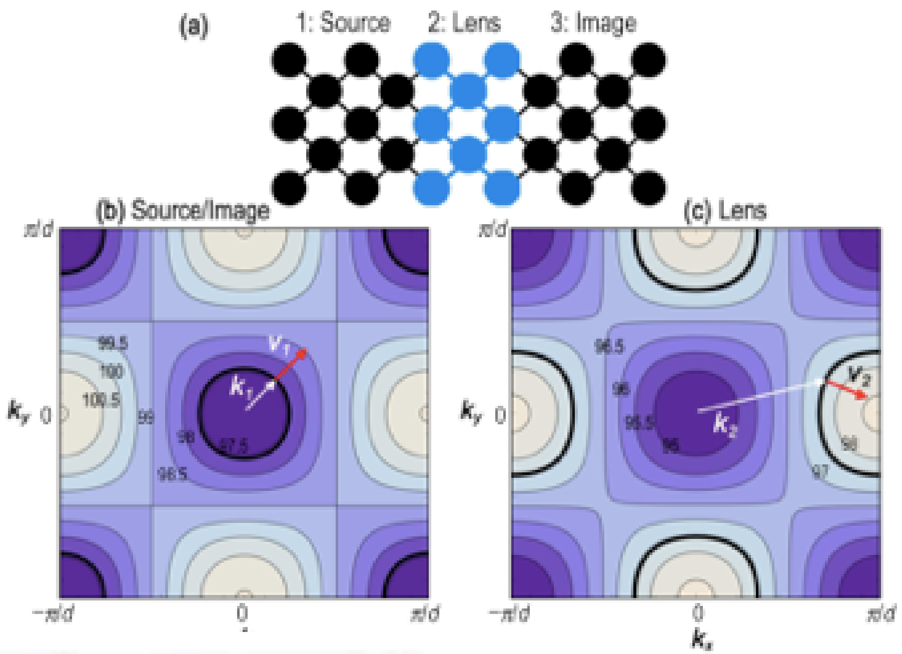
- Test or understanding of the effects of disorder on such states?
- How do interfaces effect the local symmetry?
- What does a vortex state look like?
- With these questions DFG offer no hiding places for theorists

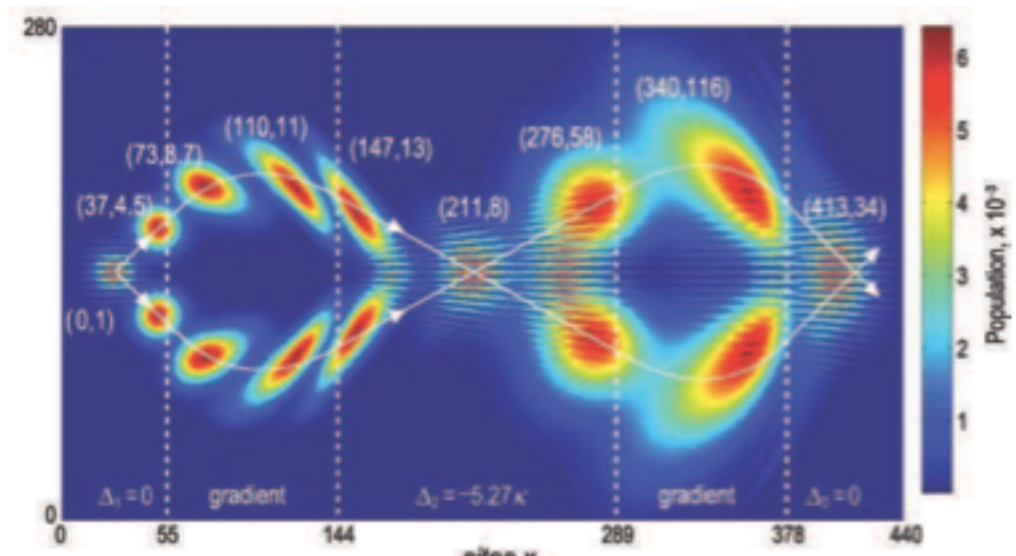
More Speculative

- Does coupling between planes significantly effect transition temperature for DFG? $\rightarrow$ Possible links to mechanisms for high temperature superconductivity.
- Maybe more applicable to strontium ruthenate (p-wave)
- Also its fun:)

Quantum Metamaterials







Contact

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