Nonlinear Quantum Interferometry with Bose Condensed Atoms

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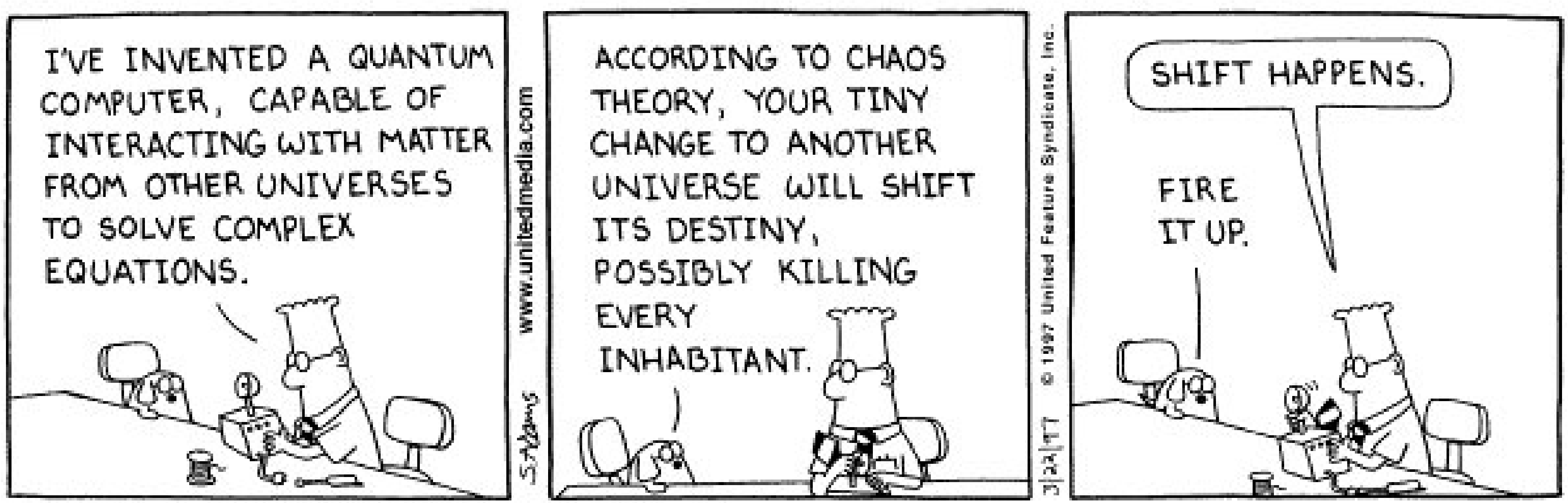
Second Quantum Revolution

- from fundamental science to practical technology

- The first quantum revolution develops the fundamental theory for understanding what already exists in our nature.
- The emergence of quantum technology is not just a way to understand what already exists, but also a way to engineer our surroundings for our own needs from science to technology. This is the second quantum revolution.

Dowling and Milburn, *Philosophical Transactions of the Royal Society of London A*, **361** (2003) 1655-1674

The Second Revolution Will Allow Us to Manipulate the Quantum World!



This page is taken from Dowling's PPT.

Outline

- **Introduction**
- Measurement and quantum mechanics
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- **Matter-wave interferometry**
- Macroscopic quantum coherence of atomic BECs
- Atomic matter-wave interference and nonlinear excitations
- Bose-Josephson junctions
- **Many-body quantum interferometry**
- Quantum spin squeezing and many-particle entanglement
- High-precision interferometry via spin squeezed state
- High-precision interferometry via NOON state
- **Summary and open problems**

1. Introduction

1.1. Measurement and quantum mechanics

- Measurement is a physical process, and the accuracy to which measurements can be performed is governed by the laws of physics.
- Systems at small scales are governed by the laws of quantum mechanics, which place limits on the accuracy to which measurements can be performed.
- The Heisenberg uncertainty relation imposes an intrinsic uncertainty on the values of measurement results of complementary observables such as position and momentum.
- In principle, every measurement apparatus is itself a quantum system. Therefore, the uncertainty relations together with other quantum constraints on the speed of evolution impose limits on how accurately we can measure quantities.

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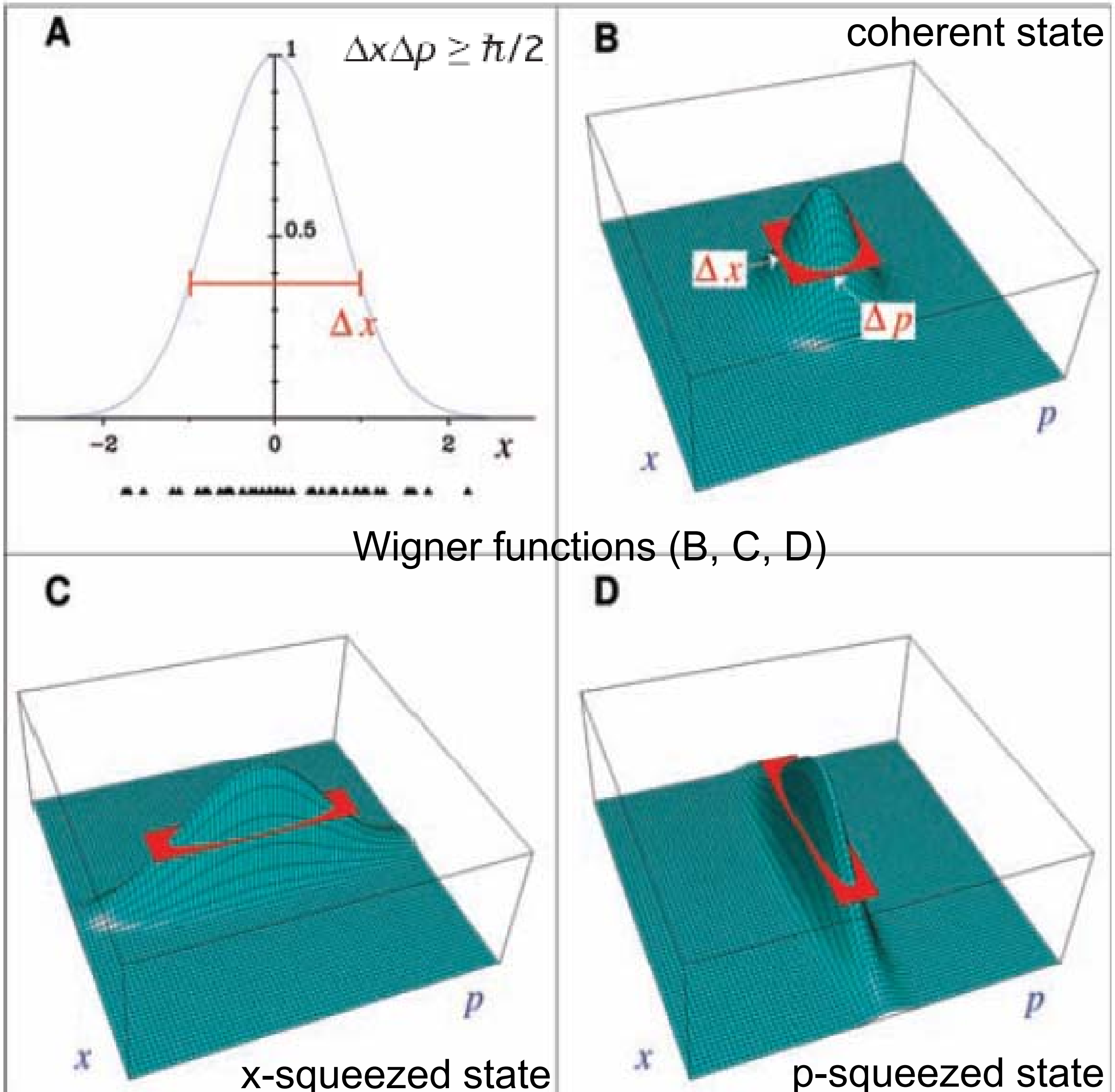
[1] Special Issue: Fundamentals of Measurement, Science (19 November 2004).

[2] V. Giovannetti, S. Lloyd, and L. Maccone, Quantum-Enhanced Measurements: Beating the Standard Quantum Limit, Science **306**, 1330 (2004).

[3] V. Giovannetti, S. Lloyd and L. Maccone, Advances in quantum metrology, Nature Photonics **5**, 222 (2011).

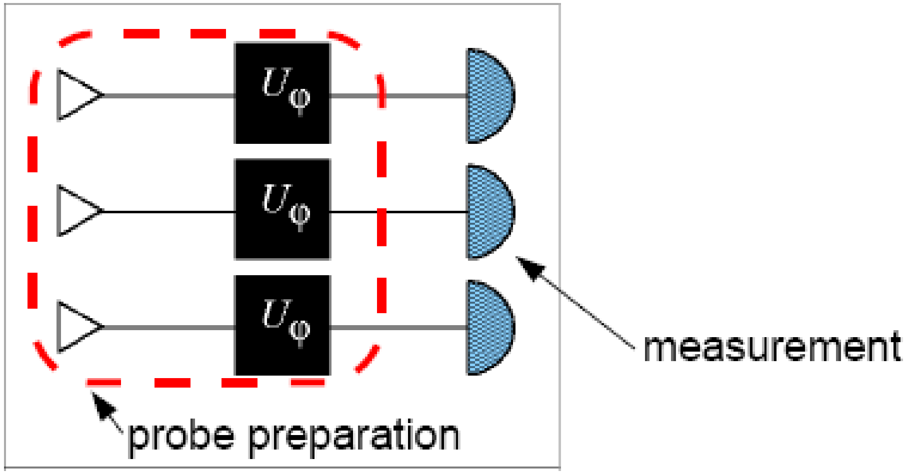


Wigner functions (B, C, D)

The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa.

--Heisenberg, 1927

To increase precision, prepare and repeat the measurement ν times,



$\Rightarrow$ the uncertainty reduces

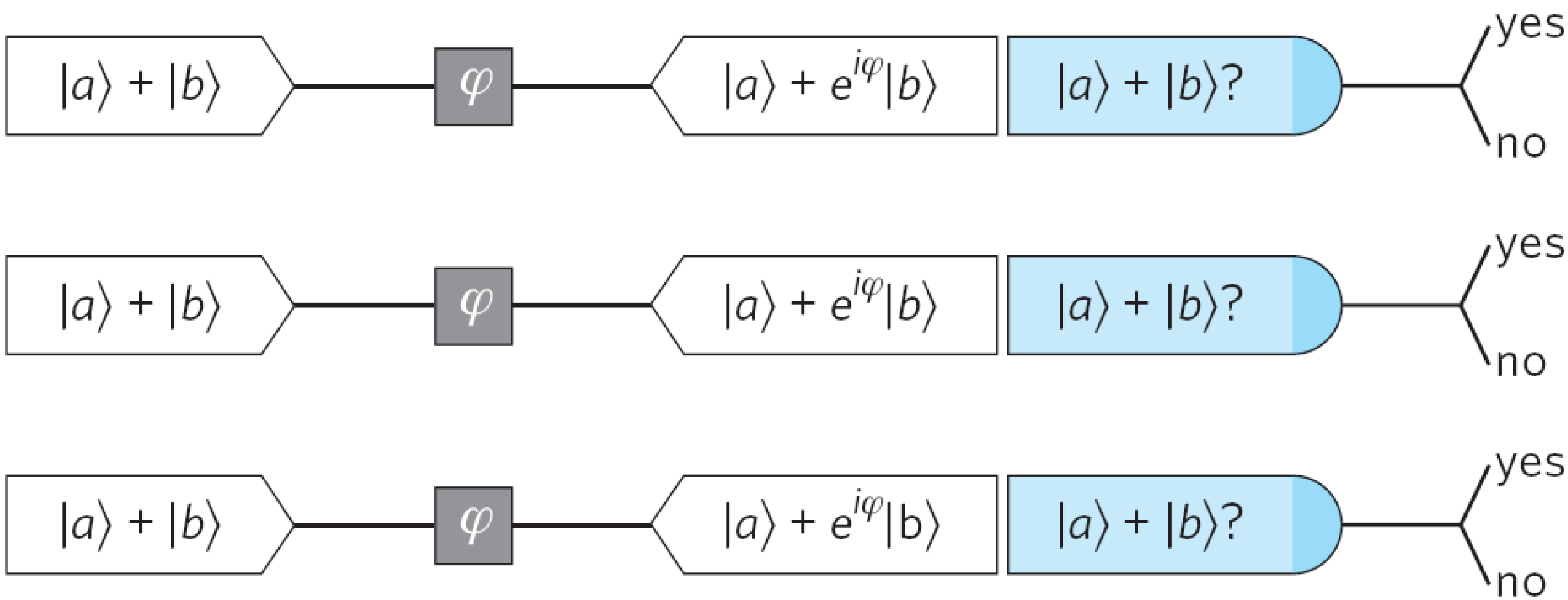
(central limit theorem)

Cramer-Rao bound

$$\Delta X \geqslant \frac{\hbar}{\sqrt{\nu}\,2\Delta p} \equiv \frac{1}{\sqrt{\nu}\,\Delta H}$$

Remember, we're looking for the **AVERAGE** position (not *the* position)

Quantum metrology with independent particles



$$p = \frac{\#\text{yes (in } n \text{ repetitions)}}{n} \longrightarrow \frac{1 - \cos\varphi}{2}$$

Frequency measurement

***N* independent "particles"**

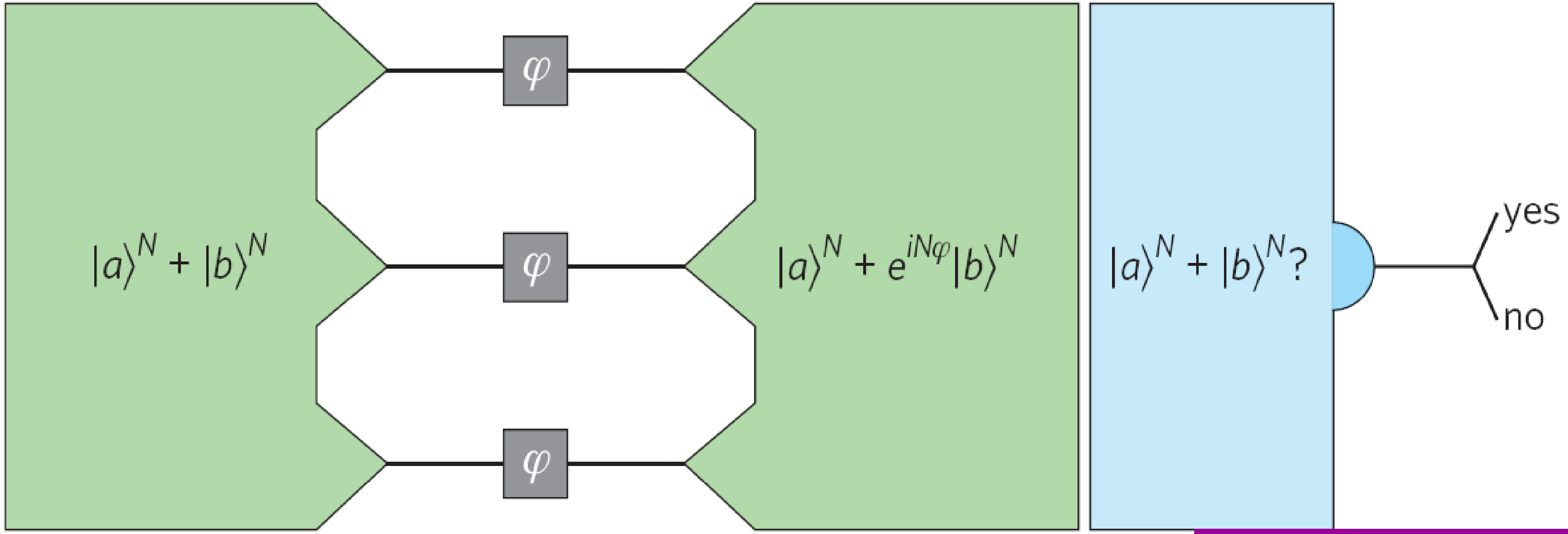
$$(\text{signal}) = \langle \sigma_z \rangle = -\cos\omega T$$

$$(\text{noise}) = \Delta\sigma_z = \sqrt{1 - \cos^2\omega T} = |\sin\omega T|$$

$$\Delta(\omega T) = \frac{1}{\sqrt{N}} \frac{(\text{noise})}{|d(\text{signal})/d(\omega T)|} = \frac{1}{\sqrt{N}}$$

standard quantum limit
(shot noise limit)

Quantum metrology with entangled particles



$$p_{\text{ent}} = \frac{\text{\#yes (in } v \text{ repetitions)}}{v} \longrightarrow \frac{1 - \cos N\varphi}{2}$$

Fringe pattern with period 2π/*N*

Frequency measurement

$$(\text{signal}) = \langle \sigma_z \rangle = -\cos N\omega T$$

$$(\text{noise}) = \Delta\sigma_z = \sqrt{1 - \cos^2 N\omega T} = |\sin N\omega T|$$

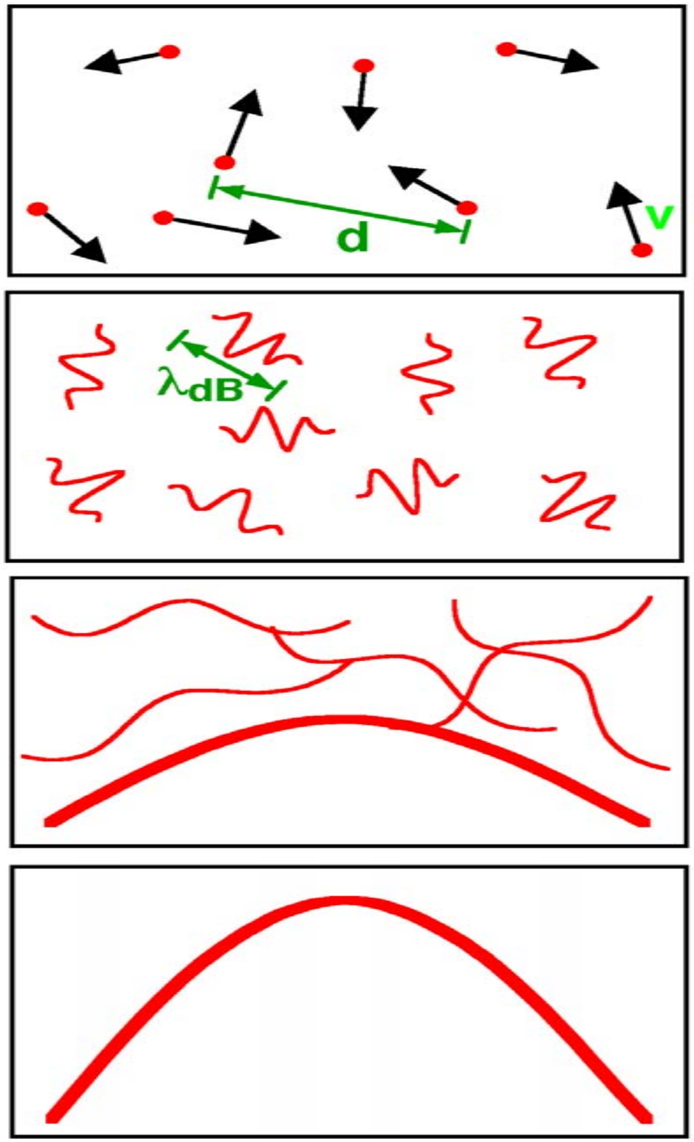
$$\Delta(\omega T) = \frac{1}{\sqrt{\nu}} \frac{(\text{noise})}{|d(\text{signal})/d(\omega T)|} = \frac{1}{\sqrt{\nu}} \frac{1}{N}$$

Heisenberg limit

$\nu = (\text{number of trials})$

***N* cat-state atoms**

1.2. Interferometry with Bose condensed atoms



High Temperature T:

thermal velocity v

density d^{-3}

"Billiard balls"

Low Temperature T:

De Broglie wavelength

$\lambda_{dB} = h/mv \propto T^{-1/2}$

"Wave packets"

$T = T_{crit}$:

Bose-Einstein Condensation

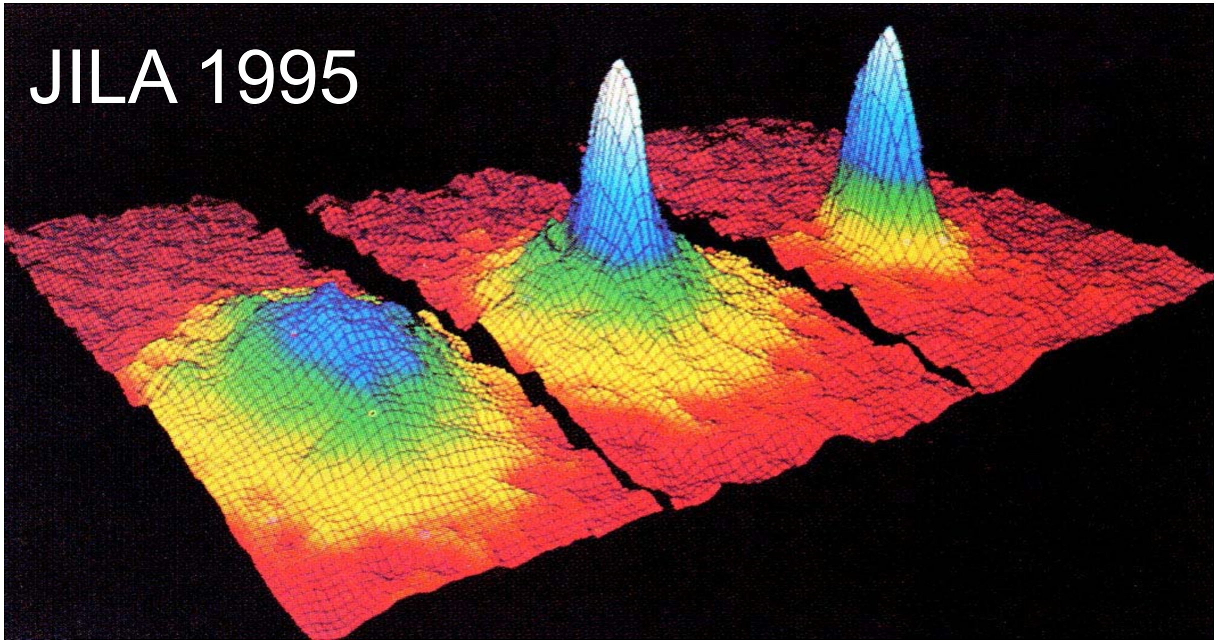
$\lambda_{dB} \approx d$

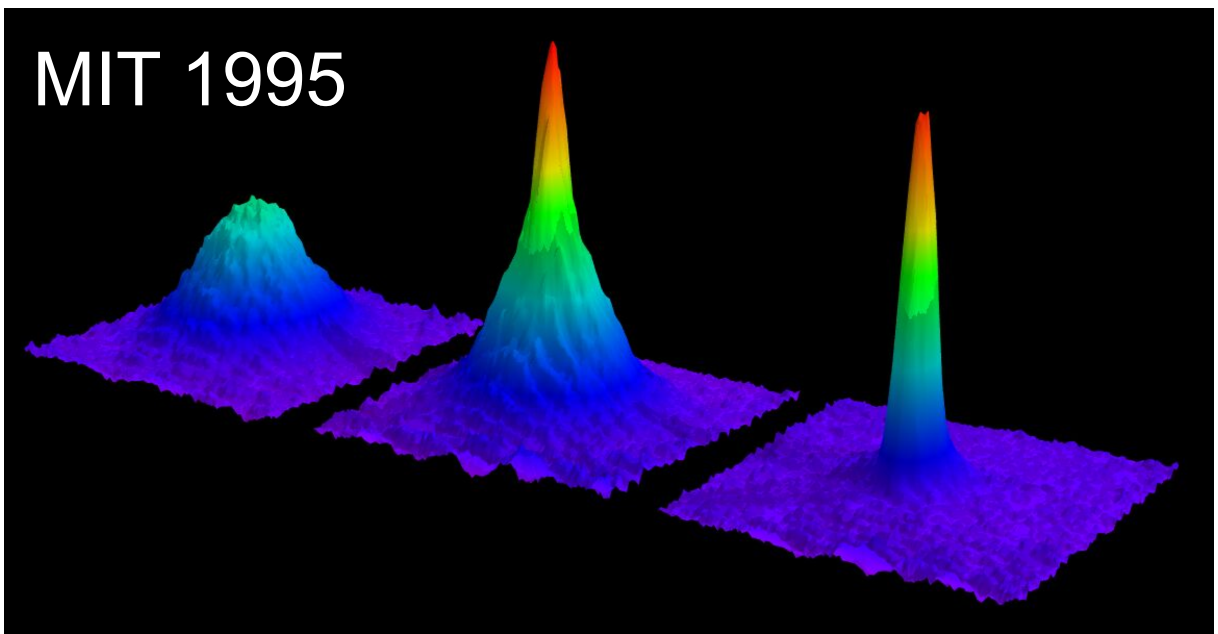
"Matter wave overlap"

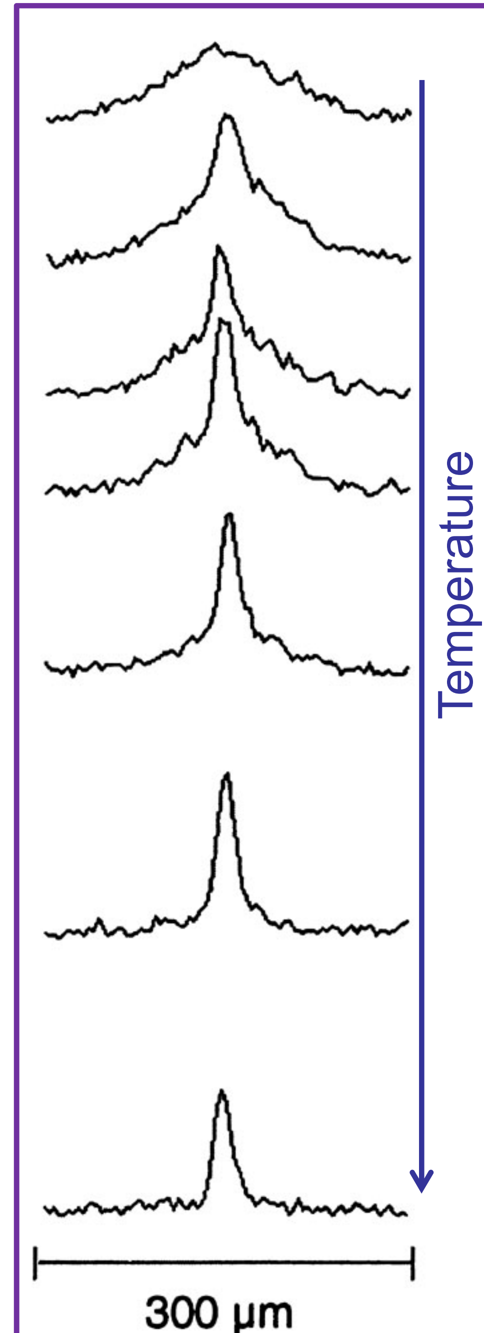
T=0:

Pure Bose condensate

"Giant matter wave"


JILA 1995


MIT 1995

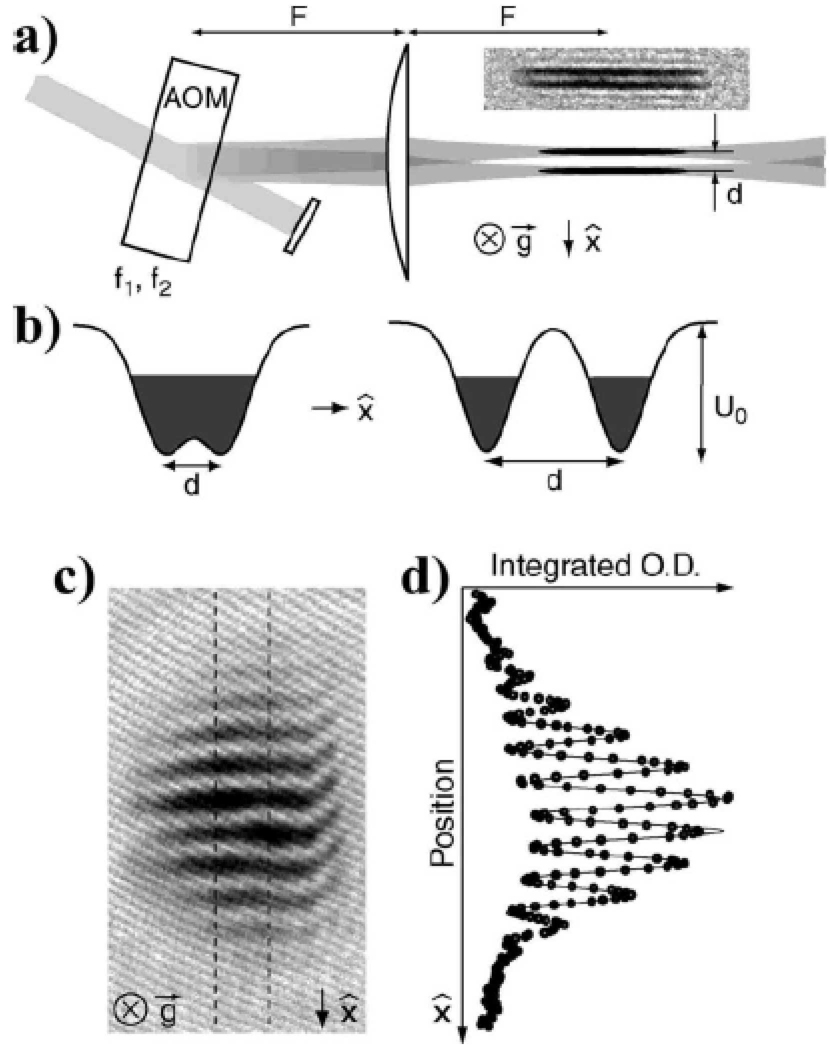

Temperature
300 μm

Double-well interferometers

(1) Coherent splitting of the wave function by slowly deforming a single trap into a double well is the generic trapped atom beam splitter, achieving physical separation of two wave-function components that start with the same phase.

(2) When the two wells are well-separated, an interaction may be applied to either.

(3) Finally the split atoms in the two wells are recombined to observe the interference.

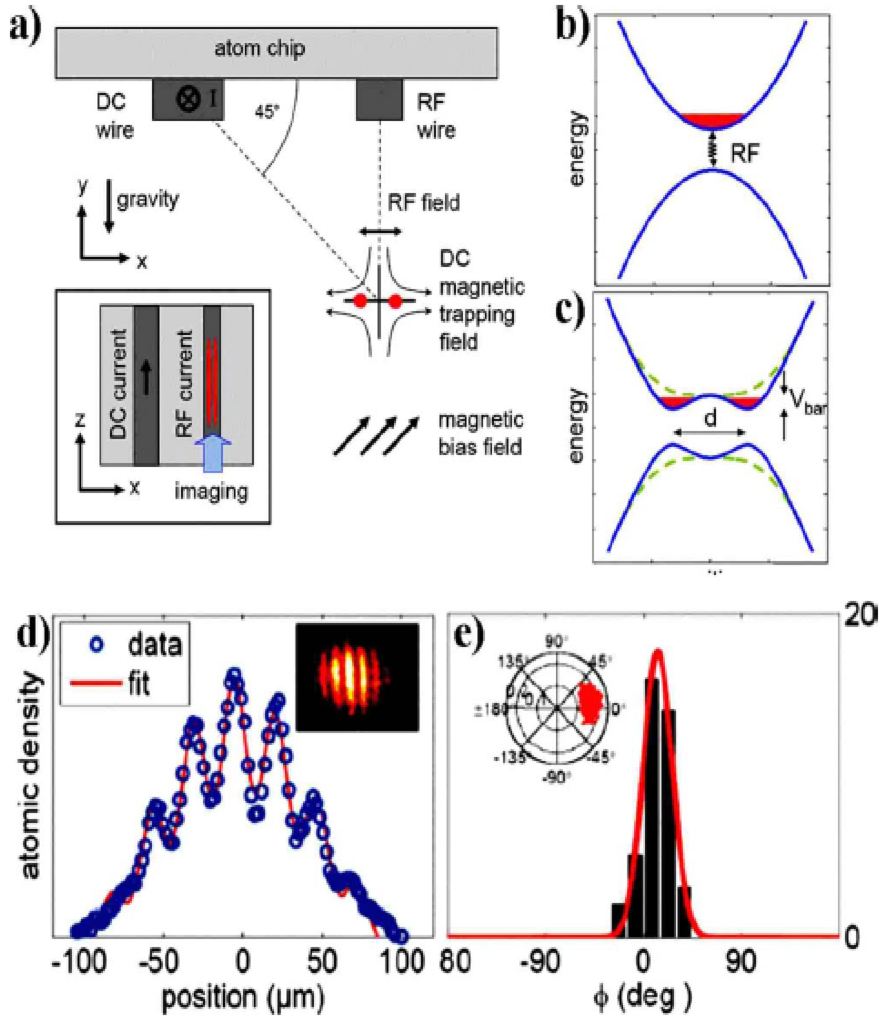


Shin et al., 2004, "Atom interferometry with Bose-Einstein condensates in a double-well potential," Phys. Rev. Lett. **92, 050405**.

Atom-chip interferometers

Atoms are manipulated by electric, magnetic, and optical fields created by microfabricated structures containing conductors designed to produce the desired potentials such as harmonic potential and double-well potential.

Atom chips have been demonstrated to be capable of quickly creating BECs and also of complex manipulation of ultracold atoms on a microscale, such as splitting and recombination.



Schumm et al, 2005, "Matter-wave interferometry in a double well on an atom chip," Nature Phys. **1, 57.**

Ramsey interferometers

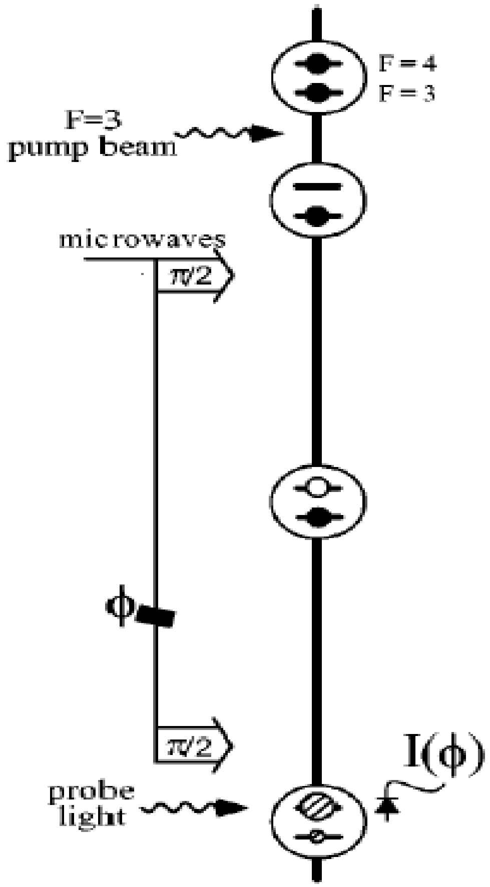
(1) prepare an initial state |1>;

(2) apply the first half-Pi pulse to create an equal superposition of |1> and |2>;

(3) accumulate a relative phase between |1> and |2> in the free evolution;

(4) recombine |1> and |2> via the second half-Pi pulse;

(5) detect the final state.



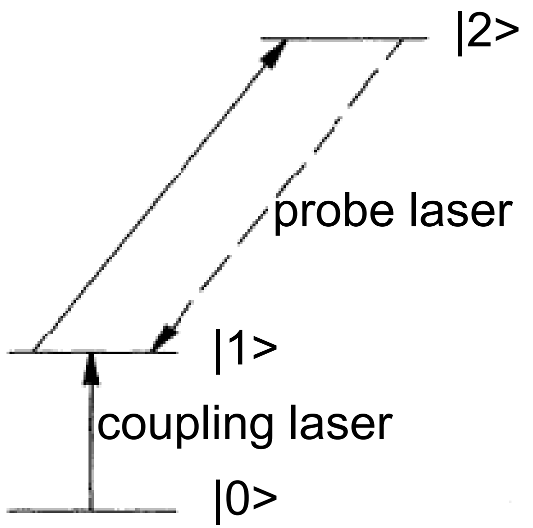
Atom Interferometry,
edited by P. Berman (Academic Press, San Diego, 1997)

Cronin, Schmiedmayer, Pritchard, Rev. Mod. Phys. 81, 1051 (2009)

Applications in precision measurement

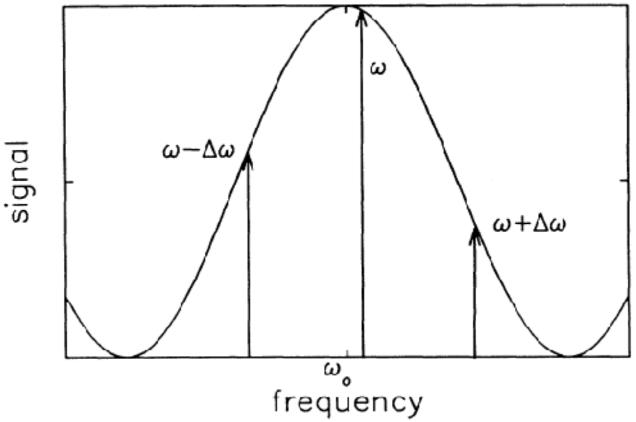
Quantum frequency standard

Atomic transitions are very useful to measure time or frequency with very high accuracy that the definition of a second is based on them.



Starting with a system of N non-interacting atoms in the ground state |0>, an electromagnetic pulse is applied to create equal superposition of |0> and of an excited state |1> for each atom.

A subsequent free evolution of the atoms for a time t introduces a phase factor between the two states, *wt*, where *w* is the frequency of the transition between |0> and |1>.



At the end of the free evolution, a second electromagnetic pulse is applied and then the probability for the final state in |0> (Ramsey interferometry) is measured.

Other applications

Gravimeters (gravity), gryroscopes (rotation), and gradiometers

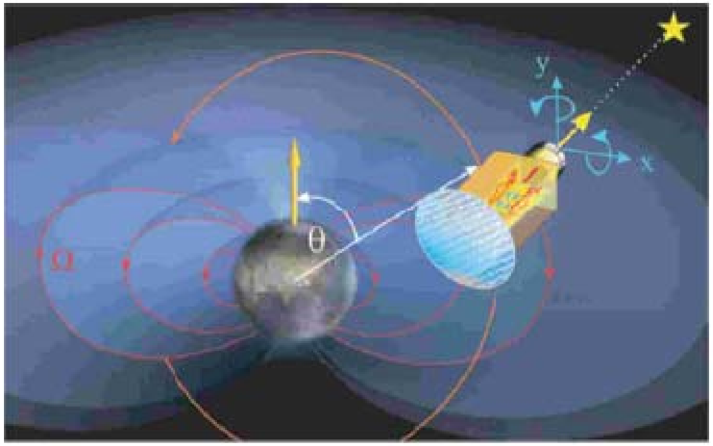
$$\phi = (\vec{G}\cdot\vec{g})\tau^2 + 2\vec{G}\cdot(\vec{\Omega}\times\vec{v})\tau^2,$$

Newton's constant G

$$\frac{\phi_{\text{atom}}}{\phi_{\text{light}}} = \frac{mc^2}{\hbar\omega} = \frac{\lambda_{\text{ph}}}{\lambda_{\text{dB}}}\frac{c}{v} \approx 10^{10}.$$

Tests of relativity

Interferometers in orbit (GPS)



Fine structure constant and ħ/M

$$\alpha^2 = (e^2/\hbar c)^2 = (2R_\infty/c)h/m_e.$$

Cronin, Schmiedmayer, Pritchard, Rev. Mod. Phys. 81, 1051 (2009)

2. Matter-wave interferometry

2.1. Macroscopic quantum coherence of atomic BECs

Hamiltonian in quantum field theory

$$\hat{H} = \int d\mathbf{r}\hat{\Psi}^{\dagger}(\mathbf{r})\left[-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\mathbf{r})\right]\hat{\Psi}(\mathbf{r}) + \frac{1}{2}\int d\mathbf{r}d\mathbf{r}'\hat{\Psi}^{\dagger}(\mathbf{r})\hat{\Psi}^{\dagger}(\mathbf{r}')V(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}')\hat{\Psi}(\mathbf{r}),$$

$$V(\mathbf{r}'-\mathbf{r}) = g\delta(\mathbf{r}'-\mathbf{r}),$$

contact interaction at ultralow temperature

$$\hat{H} = \int d\mathbf{r}\hat{\Psi}^{\dagger}(\mathbf{r})\left[-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\mathbf{r})\right]\hat{\Psi}(\mathbf{r}) + \frac{g}{2}\int d\mathbf{r}\hat{\Psi}^{\dagger}(\mathbf{r})\hat{\Psi}^{\dagger}(\mathbf{r})\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}).$$

$$\hat{H} = \int d\mathbf{r}\hat{\Psi}^{\dagger}(\mathbf{r})\left[-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\mathbf{r})\right]\hat{\Psi}(\mathbf{r})$$
$$+ \frac{g}{2}\int d\mathbf{r}\hat{\Psi}^{\dagger}(\mathbf{r})\hat{\Psi}^{\dagger}(\mathbf{r})\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}).$$

$$\hat{\Psi}(\mathbf{r},t) = \Phi(\mathbf{r},t) + \hat{\Psi}'(\mathbf{r},t),$$

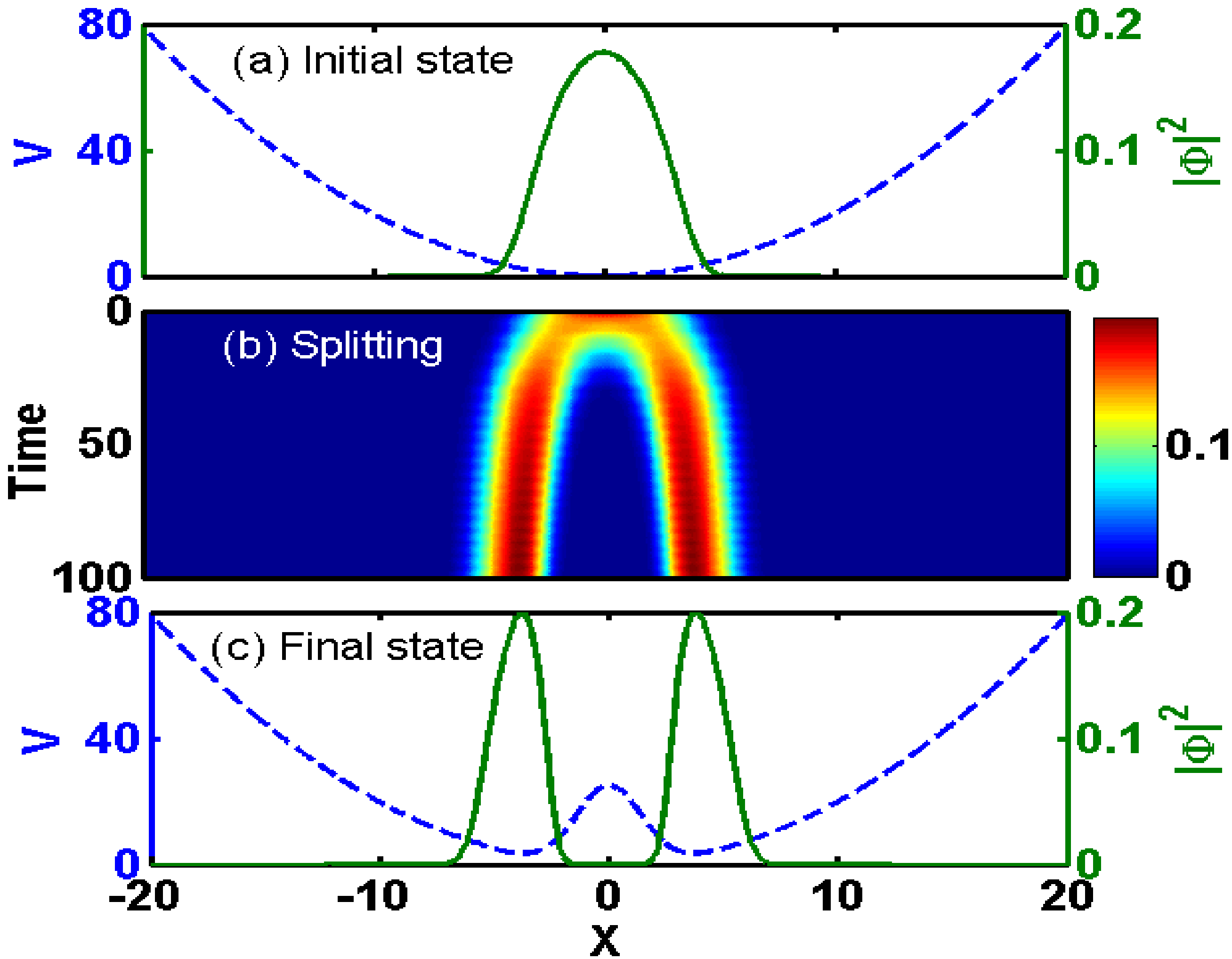
$$\Phi(\mathbf{r},t) \equiv \langle\hat{\Psi}(\mathbf{r},t)\rangle$$

mean-field approximation

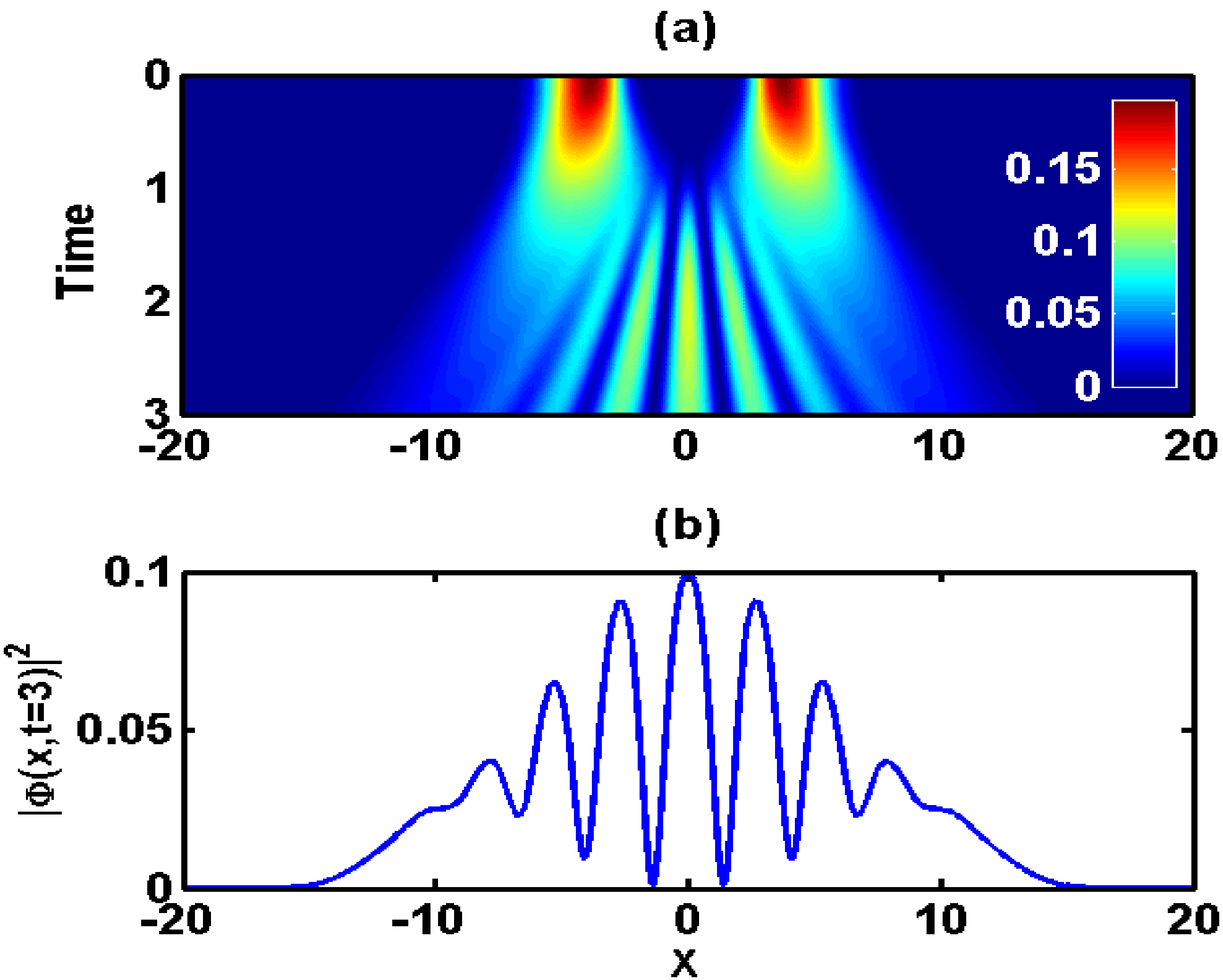
$$H_{MF} = \int d\mathbf{r}\Phi^{*}(\mathbf{r})\left[-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\mathbf{r})\right]\Phi(\mathbf{r})$$
$$+ \frac{g}{2}\int d\mathbf{r}\Phi^{*}(\mathbf{r})\Phi^{*}(\mathbf{r})\Phi(\mathbf{r})\Phi(\mathbf{r}.$$

$$i\hbar\frac{\partial}{\partial t}\Phi(\mathbf{r},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\mathbf{r}) + g|\Phi(\mathbf{r},t)|^2\right]\Phi(\mathbf{r},t)$$

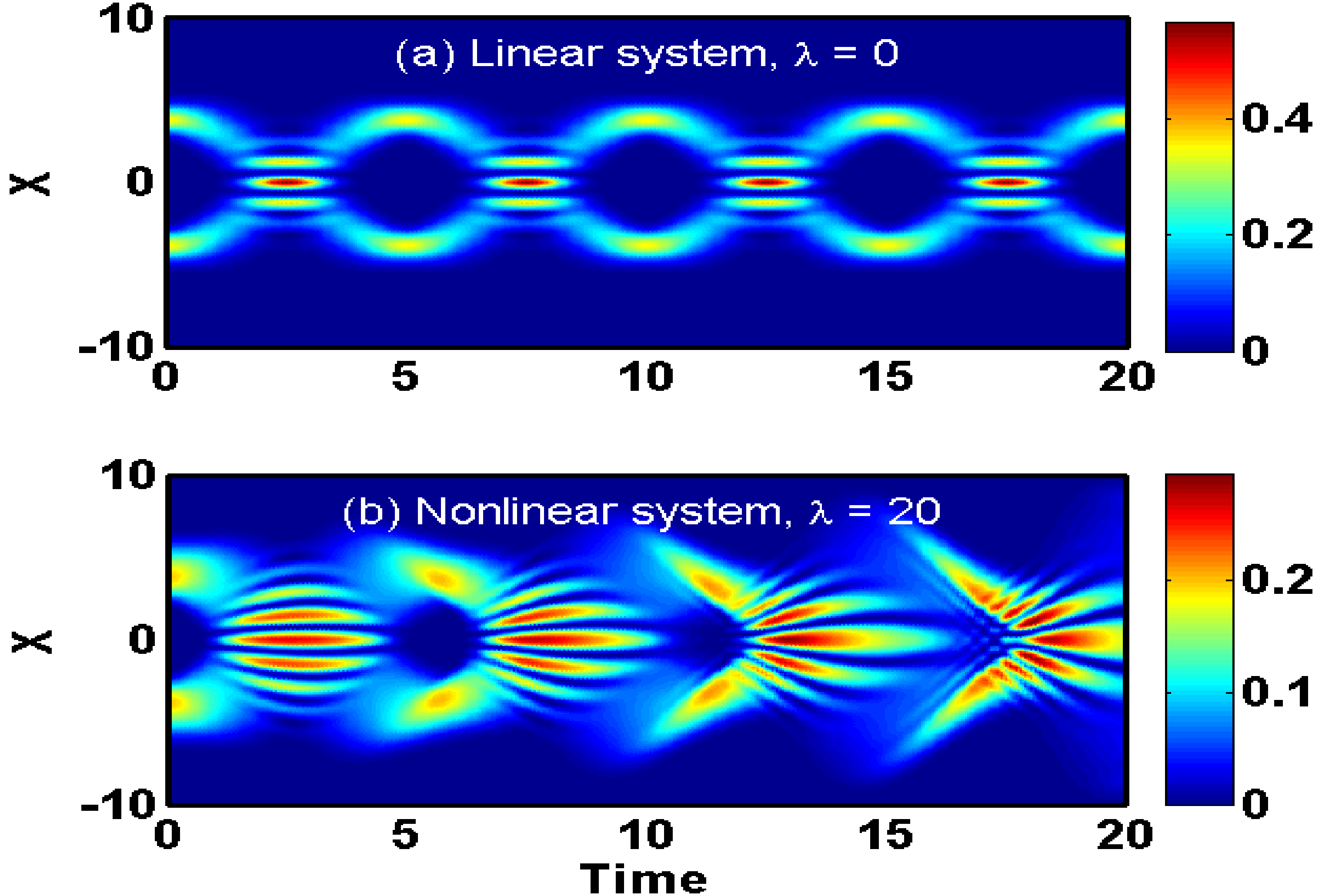
2.2. Atomic matter-wave interference and nonlinear excitations



Schematic diagram for 1D BEC interferometry

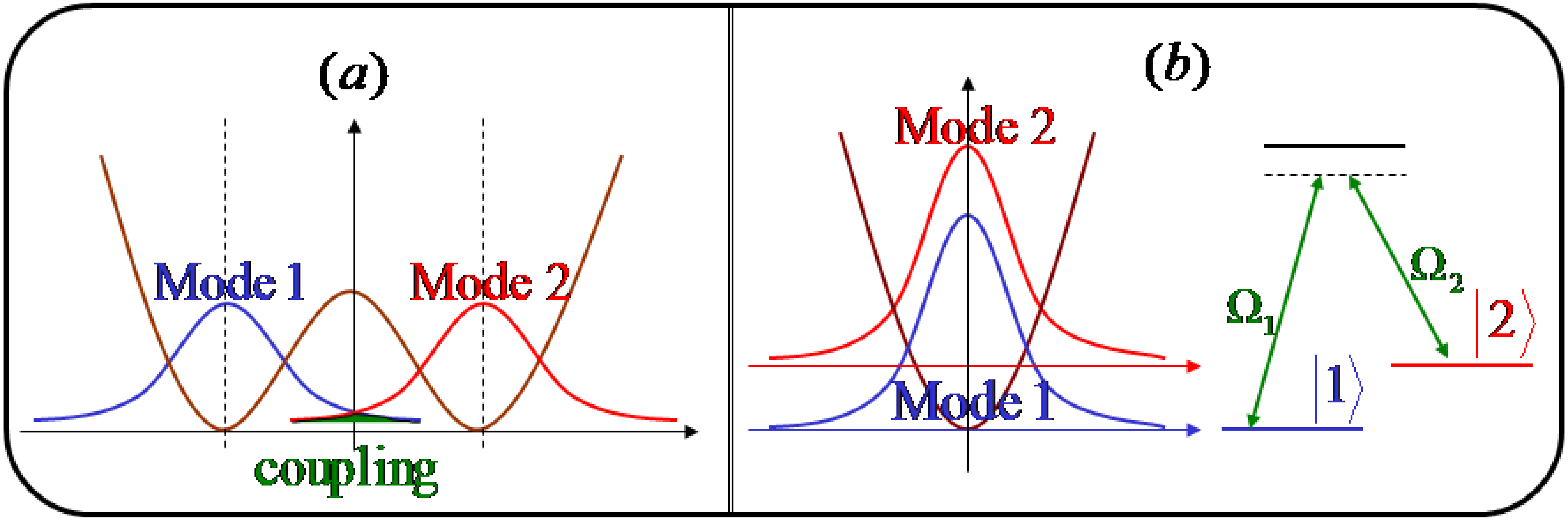


Interference of two freely expanding condensates



Nonlinear excitations in matter-wave interference

2.3. Bose-Josephson junction



Schematic diagrams for Bose-Josephson junctions:

(a) an external Bose-Josephson junction linked by quantum tunneling, and

(b) an internal Bose-Josephson junction via a two-component BEC linked by Raman fields.

External Bose-Josephson junction under two-mode approximation

$$\Phi(\mathbf{r},t) = \psi_1(t)\phi_1(\mathbf{r}) + \psi_2(t)\phi_2(\mathbf{r})$$

$$\begin{aligned} H_{MF} =& - J\left(\psi_1^*\psi_2 + \psi_2^*\psi_1\right) + \varepsilon_1 \left|\psi_1\right|^2 + \varepsilon_2 \left|\psi_2\right|^2 \\ & + \frac{1}{2}U_{11}\left|\psi_1\right|^4 + \frac{1}{2}U_{22}\left|\psi_2\right|^4 . \end{aligned}$$

Internal Bose-Josephson junction under single-mode approximation

$$\Phi_j(\mathbf{r},t) = \psi_j(t)\phi(\mathbf{r})$$

$$\begin{aligned} H_{MF} =& - J\left(\psi_1^*\psi_2 + \psi_2^*\psi_1\right) + \varepsilon_1 \left|\psi_1\right|^2 + \varepsilon_2 \left|\psi_2\right|^2 \\ & + \frac{1}{2}U_{11}\left|\psi_1\right|^4 + \frac{1}{2}U_{22}\left|\psi_2\right|^4 \\ & + U_{12}\left|\psi_1\right|^2\left|\psi_2\right|^2 , \end{aligned}$$

Unified form for both external and internal BJJs

$$H = \frac{\delta}{2}\left(n_2 - n_1\right) + \frac{E_c}{8}\left(n_2 - n_1\right)^2 - J\left(\psi_1^* \psi_2 + \psi_2^* \psi_1\right),$$

with $n_j = \psi_j^* \psi_j = |\psi_j|^2$,

$$\delta = \varepsilon_2 - \varepsilon_1 + N\left(U_{22} - U_{11}\right)/4,$$

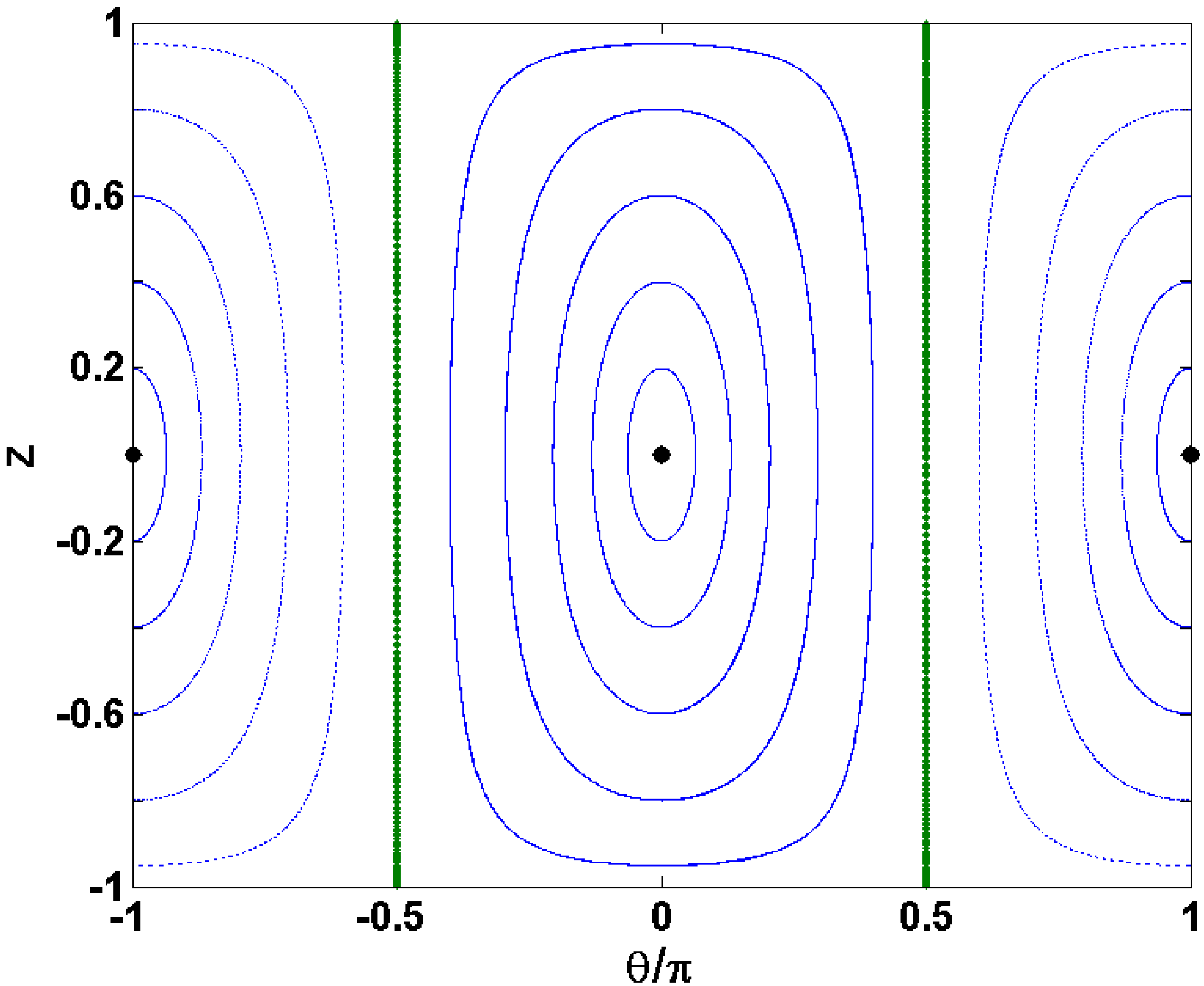
$$E_c = U_{11} + U_{22} \text{ for external BJJs}$$

$$E_c = U_{11} + U_{22} - 2U_{12} \text{ for internal ones.}$$

$$i\hbar \frac{d\psi_1}{dt} = -\frac{\delta}{2}\psi_1 + \frac{E_c}{4}\left(|\psi_1|^2 - |\psi_2|^2\right)\psi_1 - J\psi_2,$$

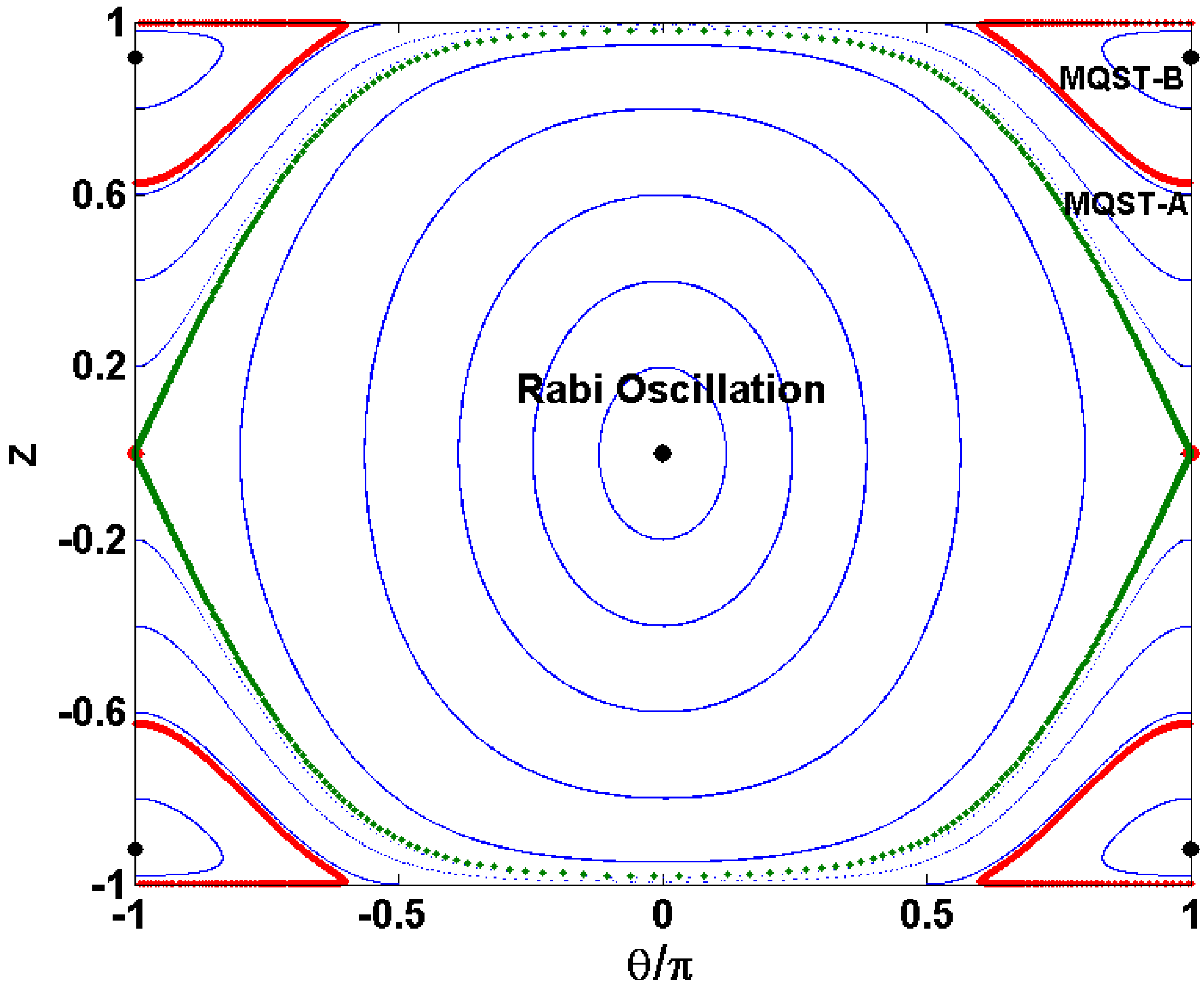
$$i\hbar \frac{d\psi_2}{dt} = +\frac{\delta}{2}\psi_2 + \frac{E_c}{4}\left(|\psi_2|^2 - |\psi_1|^2\right)\psi_2 - J\psi_1.$$

Rabi oscillation in a linear system (Ec = 0)



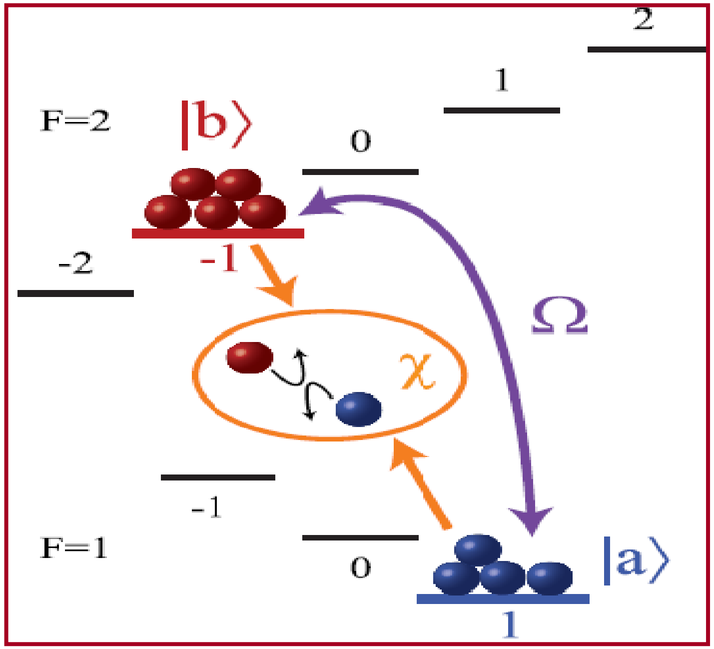
Trajectories in the phase-space (z, ɵ) of a linear BJJ
z - the fractional population imbalance, ɵ - the relative phase

Rabi oscillation and macroscopic quantum self-trapping



Trajectories in the phase-space (z, ɵ) of a nonlinear BJJ

Experimental observation of MQST

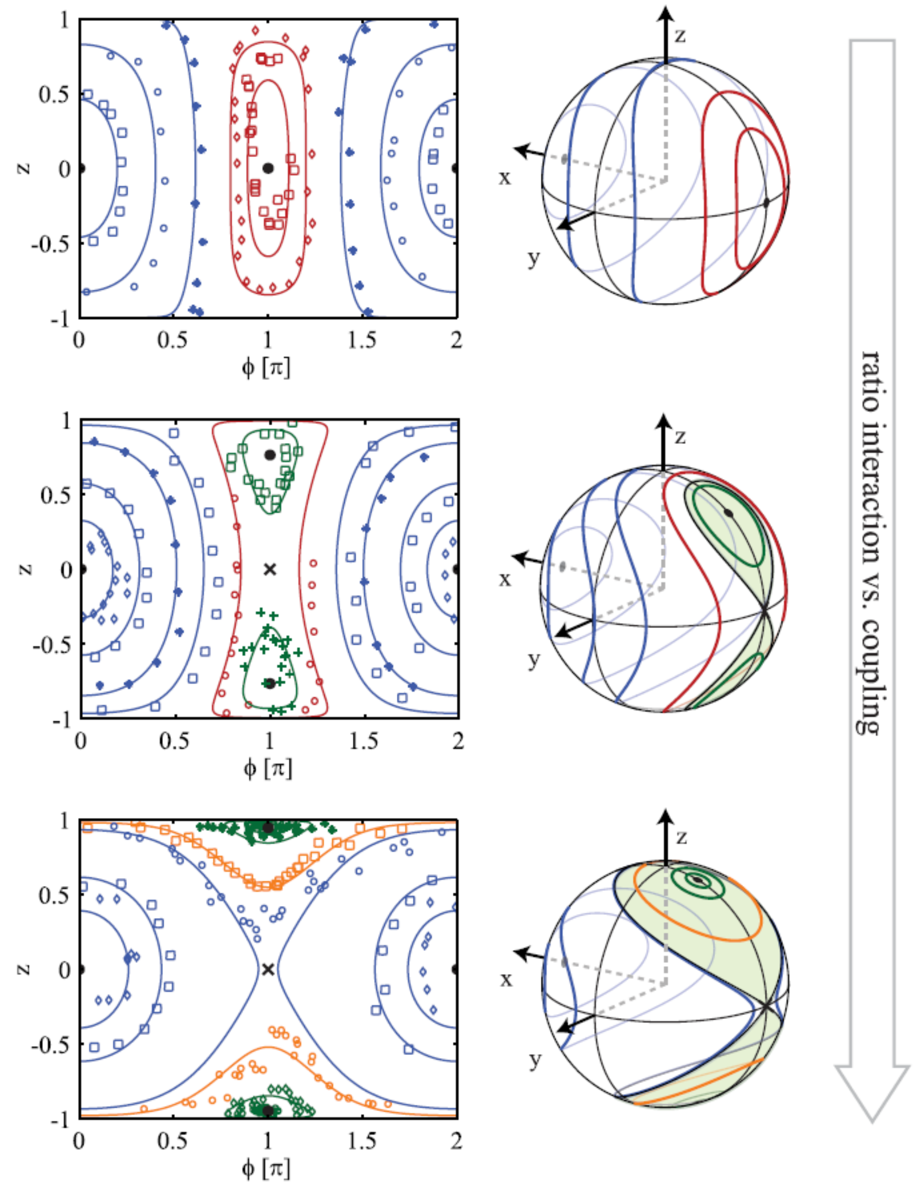


$$\hat{H} = \chi \hat{J}_z^2 - \Omega \hat{J}_x, \qquad z = \frac{n_1 - n_2}{n_1 + n_2}$$

classical non - rigid pendulum

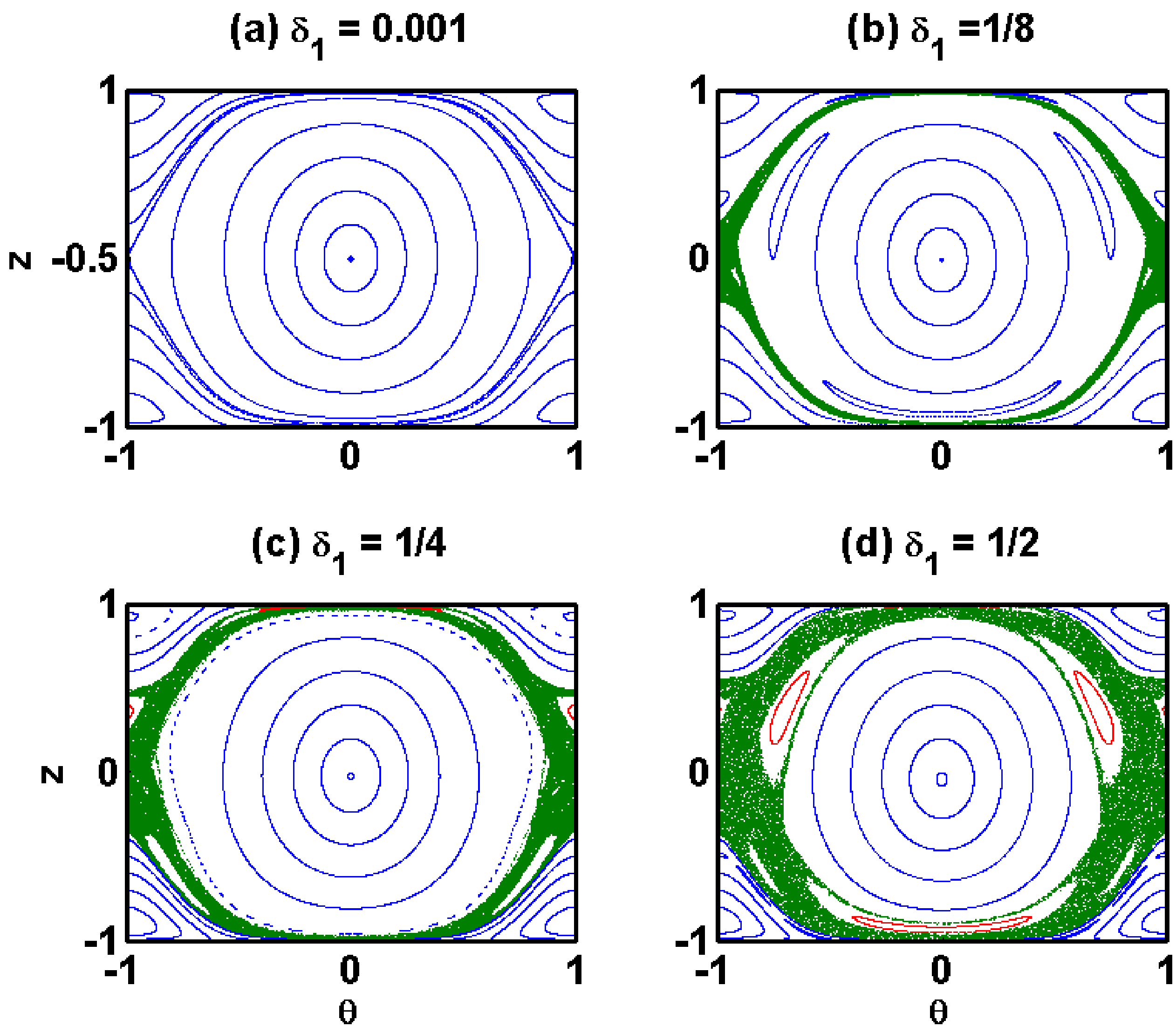
$$H = \chi m^2 - \Omega \sqrt{\left(\frac{N}{2}\right)^2 - m^2} \cos(\phi)$$

Theory:
Smerzi et al, PRL 79, 4950 (1997)



Experiment: Oberthaler et al., PRL 95,010402 (2005); PRL 105, 204101 (2010)

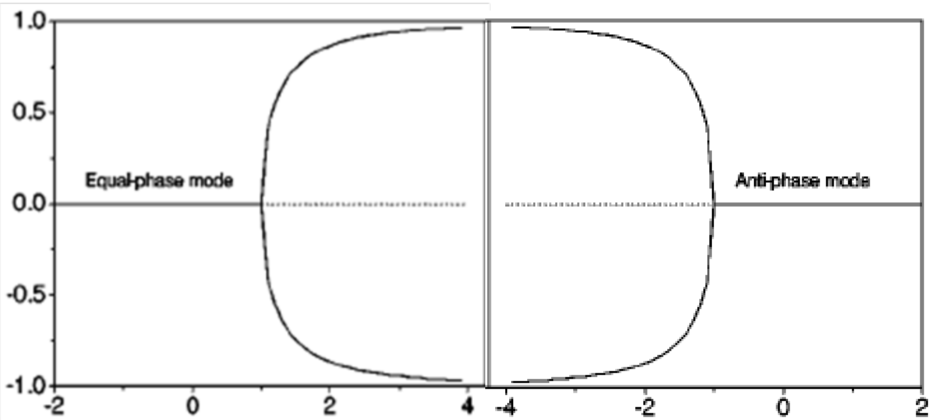
Shapiro resonance and chaos

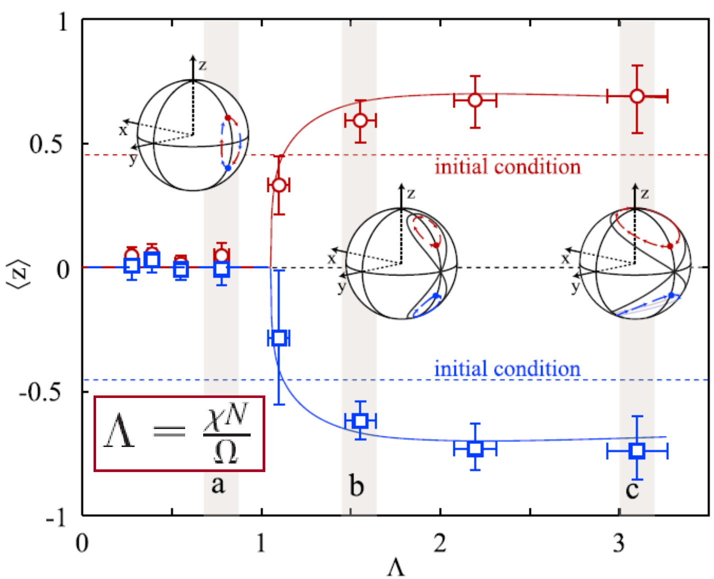


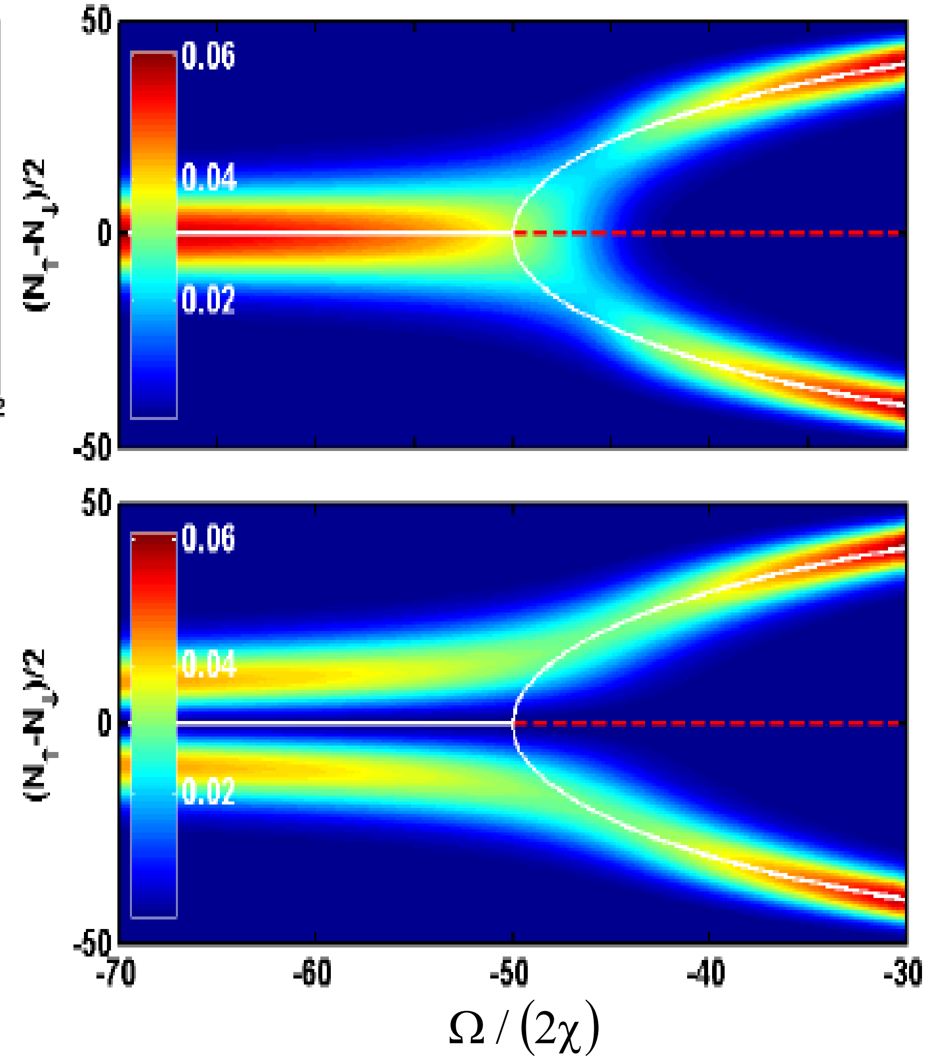
Poincare sections for a BJJ with a driving

$$\delta(t) = \delta_1 \cos(2\pi t).$$

Symmetry-breaking transition







Theory: Lee et al., PRA 69, 033611 (2004); Lee, PRL 102, 070401 (2009); etc.
Experiment: Oberthaler et al., PRL 105, 204101 (2010).

Universal dynamics near critical point

Two characteristic time scales for slow dynsmics across the crtical point,

(1) reaction time (how fast the system follows its ground state),

$$\tau_{\mathrm{r}} = \hbar / \Delta_g(t)$$

(2) transition time (how fast the system is driven),

$$\tau_{\mathrm{t}} = \Delta_g(t) / \left| \frac{d\Delta_g(t)}{dt} \right|$$

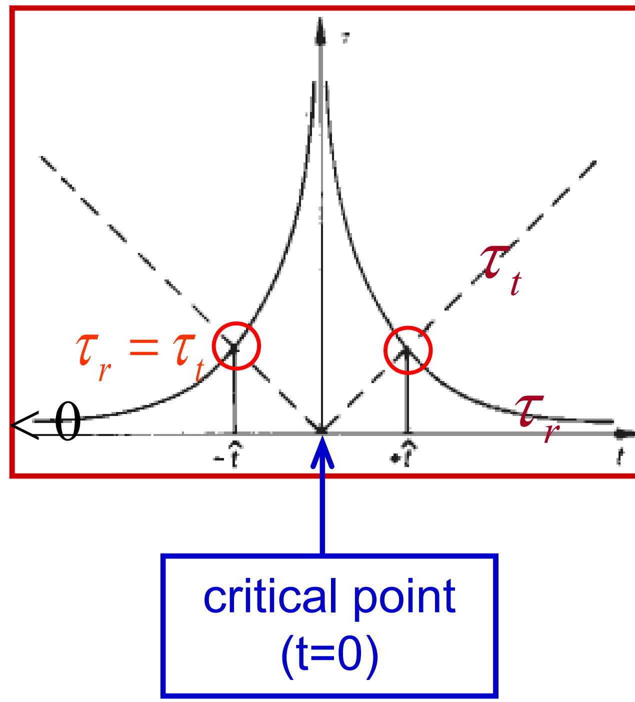
The excitation gap over the ground state

$$\Delta_g(t) = \begin{cases} \sqrt{\hbar\Omega(\hbar\Omega + E_C L)} & \text{for } \left|\hbar\Omega / E_C\right| \geq \mathrm{L} \\ \sqrt{(E_C L)^2 - (\hbar\Omega)^2} & \text{for } \left|\hbar\Omega / E_C\right| \leq \mathrm{L} \end{cases}$$

where, $\mathrm{L} = \mathrm{N}/2$, $E_C = 2\chi \propto \left(g_{11} + g_{22} - 2g_{12}\right) < 0$

$\tau_{\mathrm{r}} < \tau_{\mathrm{t}}$, adiabatic evolution

$\tau_{\mathrm{r}} > \tau_{\mathrm{t}}$, non-adiabatic evolution



Kibble-Zurek scalings near critical point

$$\tau_r(\hat{t}) = \tau_t(\hat{t})$$

slow transitions, $\tau_q \gg 1$

$$|\hat{t}| \sim \tau_0^{2/3}\tau_q^{1/3}$$

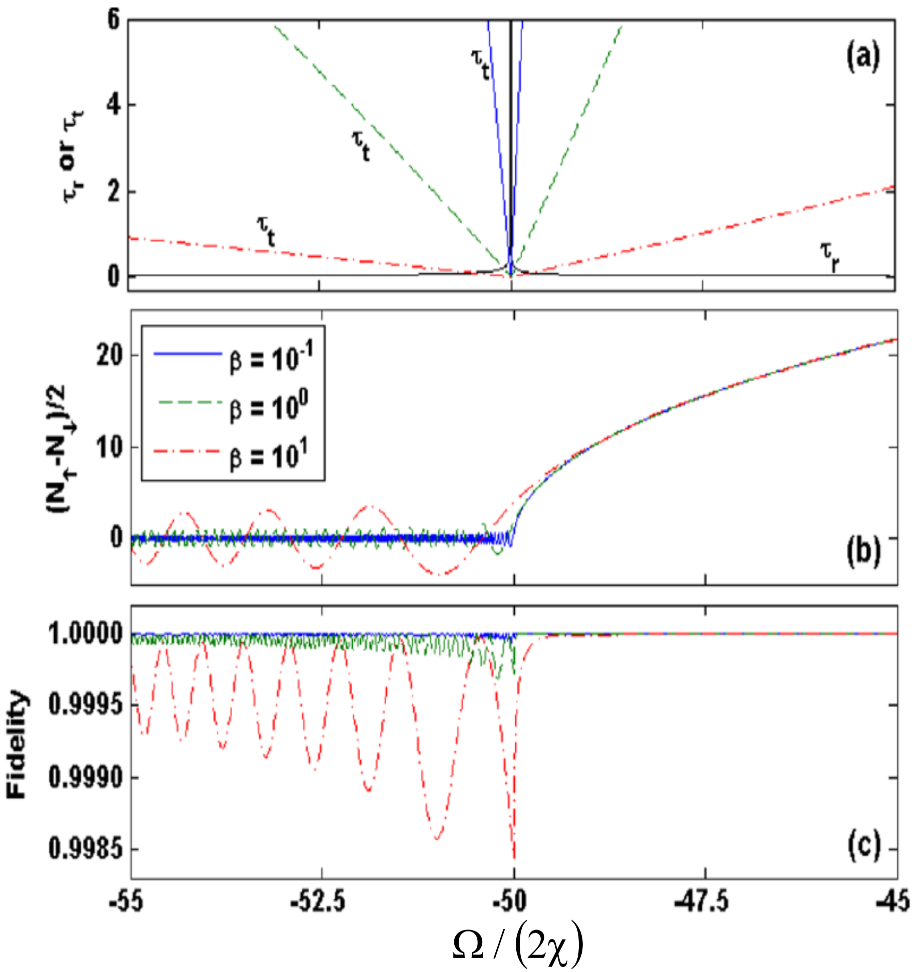
$$\varepsilon \sim \tau_0^{-2/3}\tau_q^{-2/3}$$

Kibble-Zurek scalings

$$|\hat{t}| \sim \tau_0^{1/(1+z\nu)}\tau_q^{z\nu/(1+z\nu)}$$

$$\varepsilon \sim \tau_0^{-1/(1+z\nu)}\tau_q^{-1/(1+z\nu)}$$

with $z = 1$ and $\nu = 1/2$



$$\Omega(t) = \Omega_c(1 \pm t/\tau_q) = \Omega_c \pm \beta t$$

$$\varepsilon = |[\Omega(t) - \Omega_c]/\Omega_c| = |t|/\tau_q$$

$$\tau_0 = 1/\Omega_c$$

Lee, PRL 102, 070401 (2009)

3. Many-body quantum interferometry

Hamiltonian in second quantization

$$H = -\frac{\hbar\Omega}{2}\left(e^{+i\varphi}\hat{b}_1^+\hat{b}_2 + e^{-i\varphi}\hat{b}_2^+\hat{b}_1\right) + G_{12}\hat{b}_1^+\hat{b}_2^+\hat{b}_2\hat{b}_1 + \sum_{j=1,2}\left(E_{0j}\hat{b}_j^+\hat{b}_j + \frac{1}{2}G_{jj}\hat{b}_j^+\hat{b}_j^+\hat{b}_j\hat{b}_j\right)$$

define the collective spin operators

$$\vec{J} = \left(\hat{b}_1^+, \hat{b}_2^+\right)\cdot\frac{\vec{\sigma}}{2}\cdot\begin{pmatrix}\hat{b}_1\\ \hat{b}_2\end{pmatrix}$$ with Pauli matrices $\vec{\sigma}$

$H = -\vec{B}\cdot\vec{J} + \chi J_z^2 + O(\hat{N}) + O(\hat{N}^2)$ with $\vec{B} = \left(B_x, B_y, B_z\right)$, $\hat{N} = \hat{b}_1^+\hat{b}_1 + \hat{b}_2^+\hat{b}_2$

if total number of atoms $\hat{N}$ is conserved, $O(\hat{N})$ and $O(\hat{N}^2)$ can be eliminated.

That is,

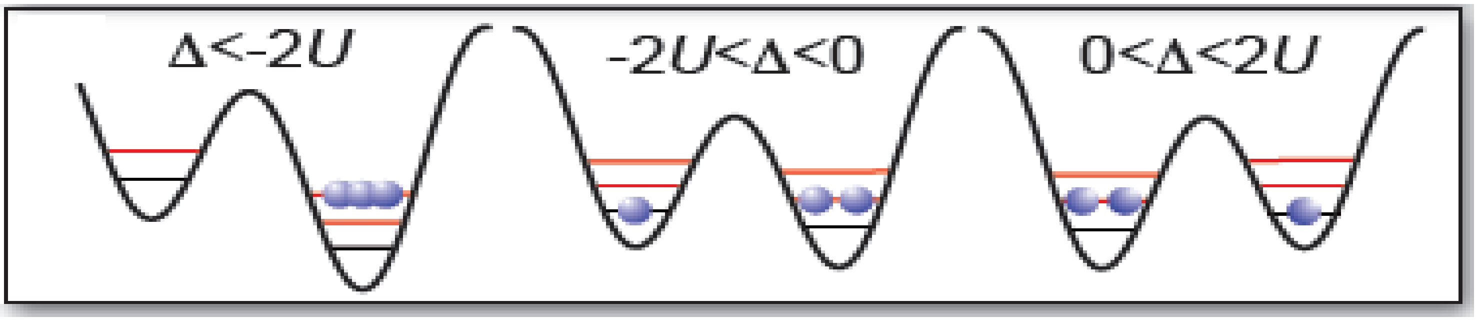
$$H = \delta J_z - \hbar\Omega\left(\cos\varphi\cdot J_x + \sin\varphi\cdot J_y\right) + \chi J_z^2$$

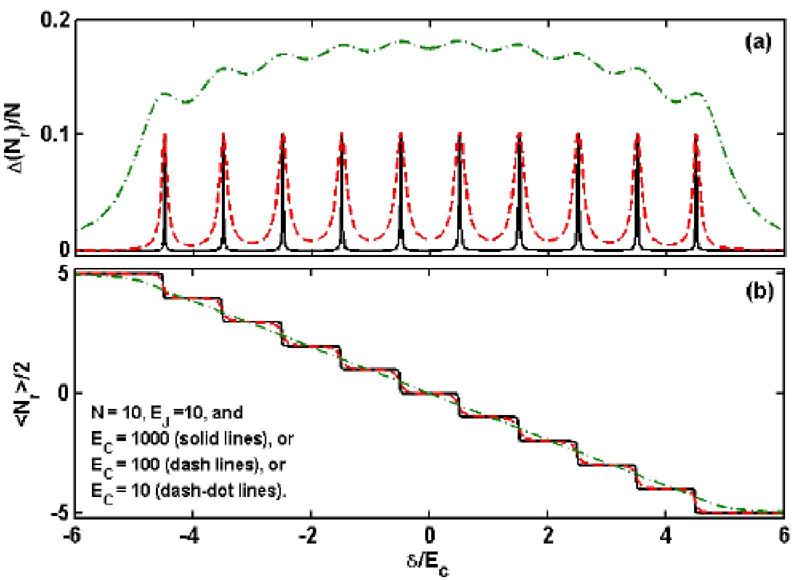
Ground states for symmetric Bose-Josephson junction

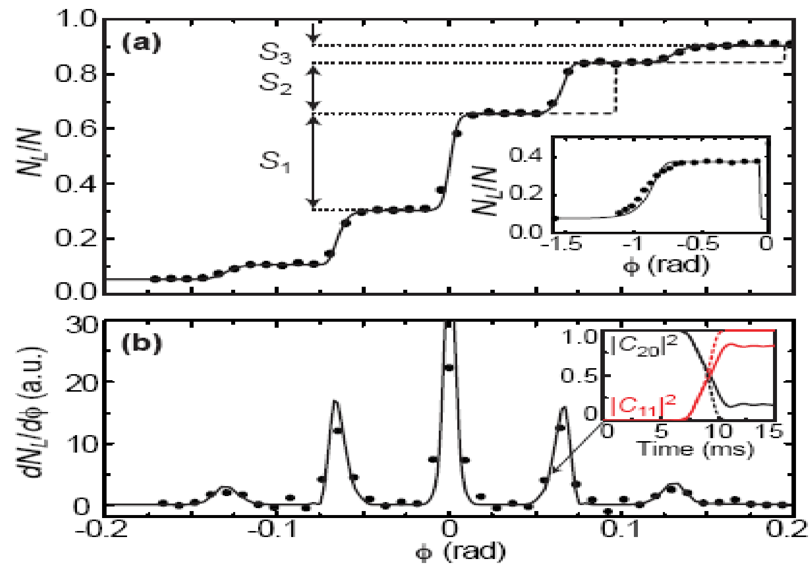
$$H/\hbar = -\frac{\Omega}{2}(a_2^+ a_1 + a_1^+ a_2) + \frac{E_C}{8}(n_2 - n_1)^2 = -\Omega J_x + \chi J_z^2$$

Regime	$\lvert\chi/\Omega\rvert >> 1$ $\chi > 0$	$\lvert\chi/\Omega\rvert \approx 0$	$\lvert\chi/\Omega\rvert >> 1$ $\chi < 0$
State form	$\frac{(a_1^+)^{N/2}(a_2^+)^{N/2}\lvert 0\rangle}{(N/2)!}$	$\frac{(a_1^+ + a_2^+)^N \lvert 0\rangle}{2^{N/2}\sqrt{N!}}$	$\frac{((a_1^+)^N + (a_2^+)^N)\lvert 0\rangle}{2^{1/2}\sqrt{N!}}$
Coherent matrix $\langle a_i^+ a_j \rangle$	$\frac{N}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\frac{N}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\frac{N}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
Fluctuations	$\Delta N_i \sim 0$	$\Delta N_i \sim \sqrt{N}$	$\Delta N_i \sim N$

Resonant tunneling and interaction blockade in asymmetric systems







Theory

C. Lee, L.-B. Fu, and Yu. S. Kivshar, EPL 81, 60006 (2008); Carr et al., …

Experiment

P. Cheinet, I. Bloch, et al., Phys. Rev. Lett. 101, 090404 (2008)

3.1. Quantum spin squeezing and many-particle entanglement

Quantum spin squeezing

Squeezing parameter based on the Heisenberg uncertainty relation

$[J_\alpha, J_\beta] = i\varepsilon_{\alpha\beta\gamma} J_\gamma$, $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Civita symbol.

The uncertainty relation is $(\Delta J_\alpha)^2 (\Delta J_\beta)^2 \geq |\langle J_\gamma \rangle|^2 / 4$.

$$\xi_H^2 = \frac{2(\Delta J_\alpha)^2}{|\langle J_\gamma \rangle|}, \; \alpha \neq \gamma \in (x, y, z), \text{ squeezing parameter}$$

if $\xi_H^2 < 1$, the state is squeezed.

Squeezing parameter ξ_S^2 *given by Kitagawa and Ueda*

$$\xi_S^2 = \frac{\min\left(\Delta J_{\vec{n}_\perp}^2\right)}{j/2} = \frac{4 \min\left(\Delta J_{\vec{n}_\perp}^2\right)}{N},$$

$\vec{n}_\perp$ refers to an axis perpendicular to the MSD

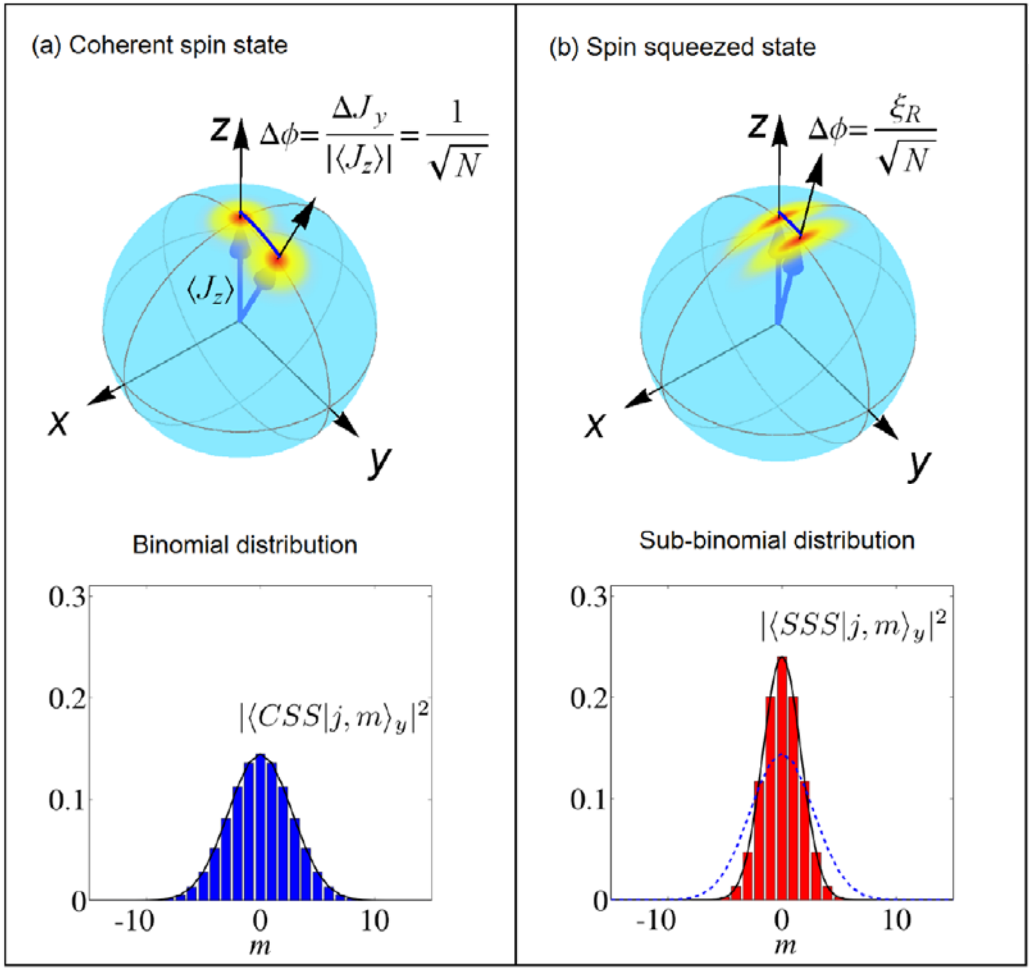
the mean-spin direction (MSD) $\vec{n}_0 = \dfrac{\langle \vec{J} \rangle}{|\langle \vec{J} \rangle|}$

the minimization is over all directions

J. Ma, X.G. Wang, C. P. Sun, and F. Nori, arXiv:1011.2978 (Phys. Rep.)

Squeezing parameter ξ_R^2 given by Wineland et al.

$$\xi_R^2 = \left(\frac{\Delta\phi}{(\Delta\phi)_{\mathrm{CSS}}}\right)^2 = \frac{N\left(\Delta J_{\vec{n}_\perp}\right)^2}{\left|\langle\vec{J}\rangle\right|^2}$$



rotate the state around the x-axis.

$$J_y^{\mathrm{out}} = \exp(i\phi J_x) J_y \exp(-i\phi J_x)$$

$$= \cos\phi J_y - \sin\phi J_z$$

the phase sensitivity $\Delta\phi$

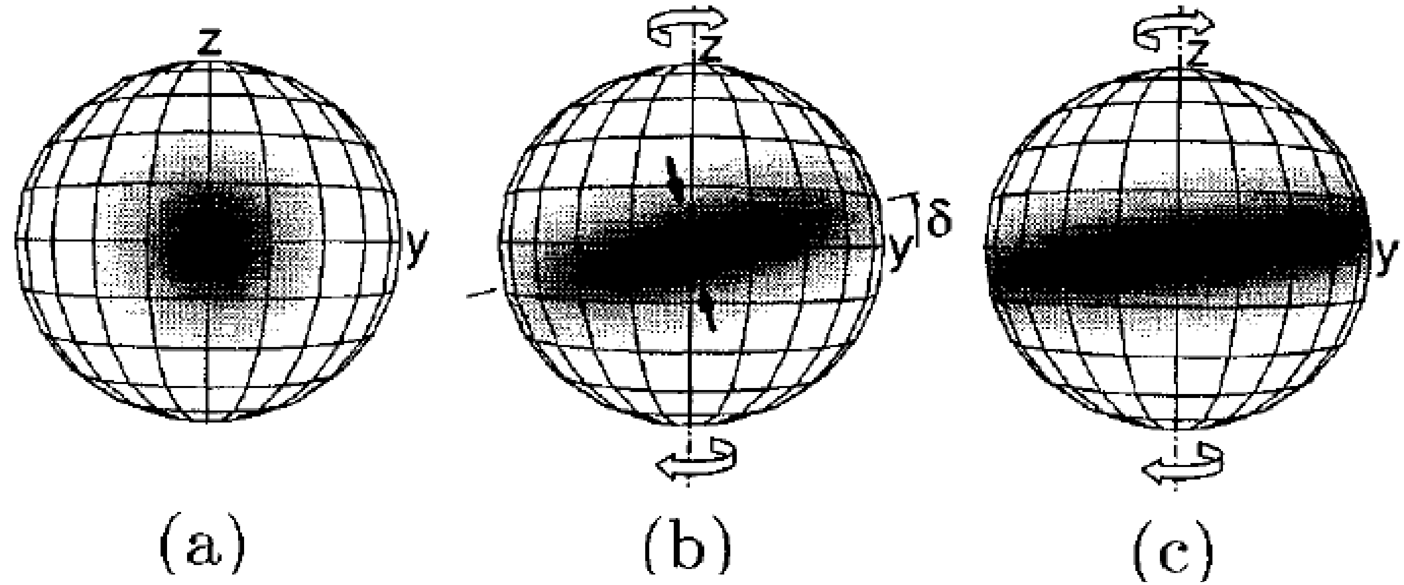
$$\Delta\phi = \frac{\Delta J_y^{\mathrm{out}}}{\left|\frac{\partial\langle J_y^{\mathrm{out}}\rangle}{\partial\phi}\right|} = \frac{\Delta J_y^{\mathrm{out}}}{|\cos\phi\langle J_z\rangle|}$$

standard quantum limit (SQL)

$$(\Delta\phi)_{\mathrm{CSS}} = \frac{\sqrt{j/2}}{j} = \frac{1}{\sqrt{2j}} = \frac{1}{\sqrt{N}},$$

$$\xi_R^2 = \left(\frac{j}{|\langle\vec{J}\rangle|}\right)^2 \xi_S^2$$

Spin squeezing by nonlinear interactions (Kitagawa and Ueda)



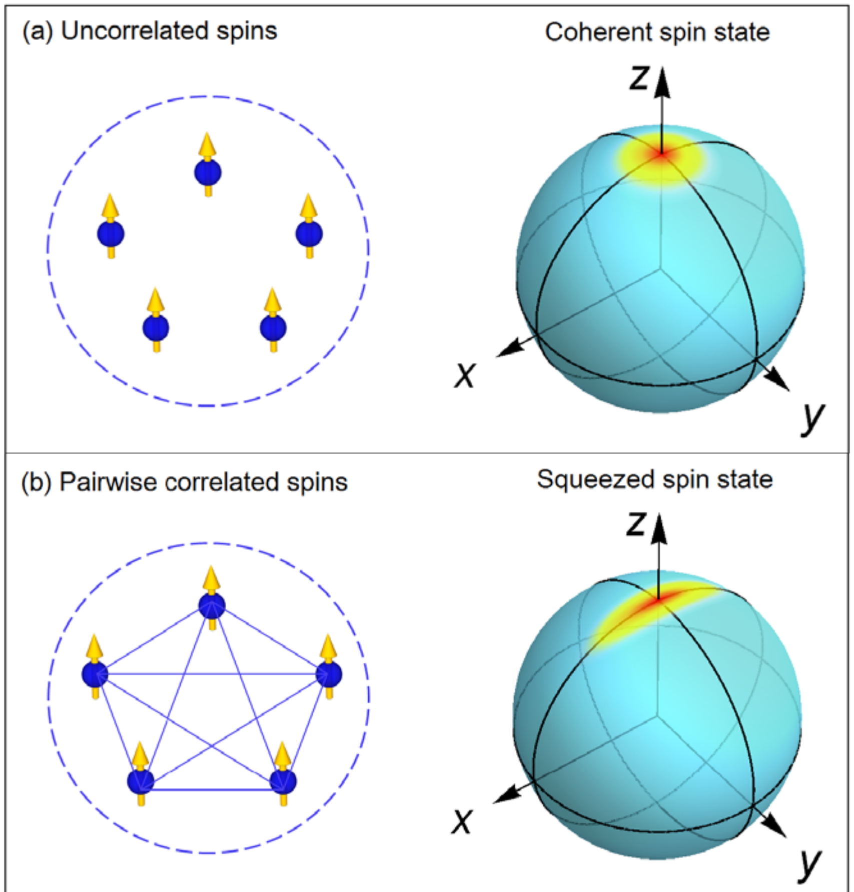
One-axis twisting can reduce the noise down to the order of $S^{1/3}$

FIG. 2. State evolutions by one-axis twisting in terms of the quasiprobability distribution (QPD) on the sphere for $S = 20$. The densities of the figures are normalized by the maximum value $Q_{\max}$ of $Q(\theta, \phi)$. (a) shows the initial coherent spin state $|\theta=\frac{\pi}{2}, \phi=0\rangle$ ($Q_{\max} = 1$). (b) and (c) show one-axis twisted states generated by the unitary transformation $U = \exp[-i\mu S_z^2/2]$; (b) optimally squeezed at $\mu = 0.199$ ($Q_{\max} = 0.445$) and (c) excessively twisted at $\mu = 0.399$ ($Q_{\max} = 0.241$). Although not clear from the figure, the QPD of (c) deviates from a geodesic (swirliness).

Spin squeezing and entanglement

A symmetric state is entangled if and only if it violates the inequality,

$$1 - \frac{4\langle J_{\vec{n}}\rangle^2}{N^2} \geq \frac{4(\Delta J_{\vec{n}})^2}{N} \Leftrightarrow \xi_D^2 = \frac{N(\Delta J_{\vec{n}})^2}{N^2/4 - \langle J_{\vec{n}}\rangle^2} = \frac{N(\Delta J_{n_1})^2}{\langle J_{n_2}\rangle^2 + \langle J_{n_3}\rangle^2} \geq 1$$



.............**Dynamical Evolution**........

Many-particle entanglement with Bose–Einstein condensates

A. Sørensen*, L.-M. Duan†, J. I. Cirac† & P. Zoller†

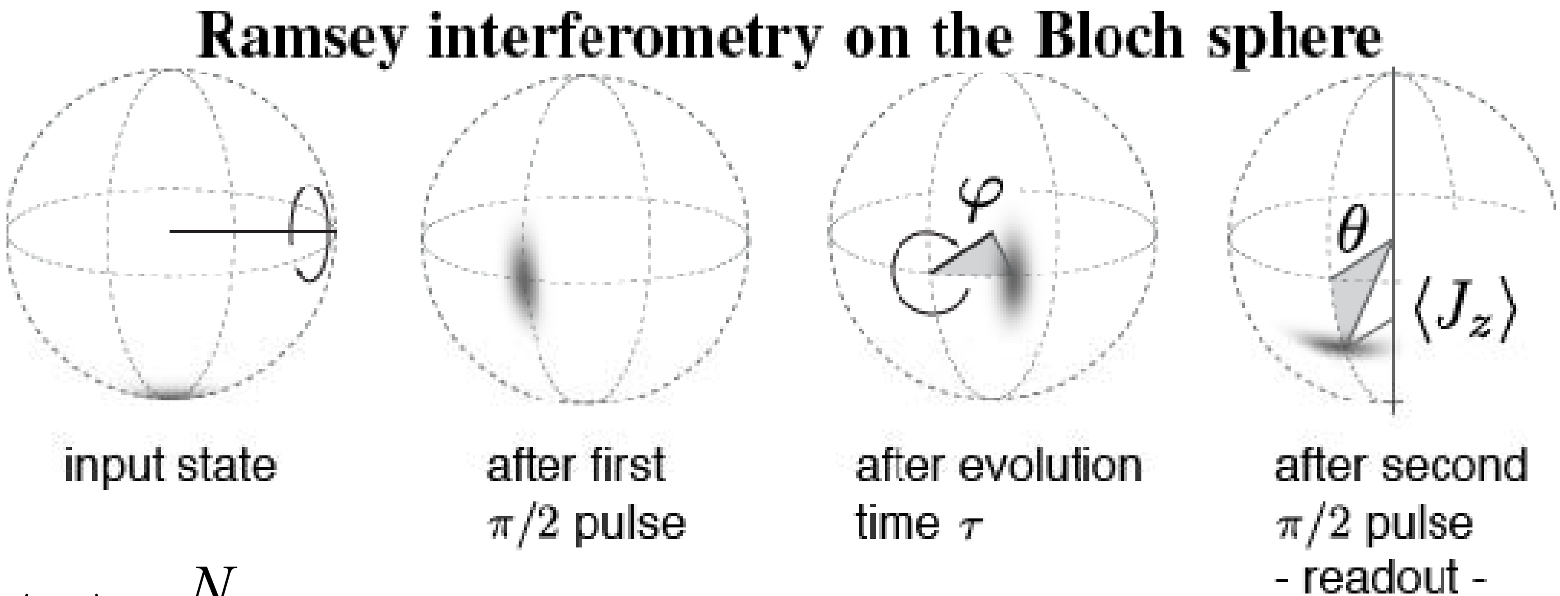
NATURE | VOL 409 | 4 JANUARY 2001 | www.nature.com

S. Raghavan, H. Pu, P. Meystre, and N. Bigelow, Generation of arbitrary Dicke states in spinor Bose-Einstein condensates, Opt. Commun. **188**, 149 (2001)

M. Zhang, K. Helmerson, and L. You, Entanglement and spin squeezing of Bose-Einstein-condensed atoms, Phys. Rev. A 68, 043622 (2003)

X. X. Yang and Y. Wu, Effective Two-State Model and NOON States for Double-Well Bose-Einstein Condensates in Strong-Interaction Regime, Commun. Theor. Phys. 52, 244 (2009)

3.2. High-precision interferometry via spin squeezed states



$$\langle J_z \rangle = \frac{N}{2}\cos\phi,$$

$$\left(\partial\langle J_z\rangle / \partial\phi\right)_{\max} = \frac{N}{2},$$

$$\Delta(J_z) = \frac{\sqrt{N}}{2}\xi_R,$$

$$\Delta(\phi) = \frac{\Delta(J_z)}{\partial\langle J_z\rangle / \partial\phi} = \frac{\xi_R}{\sqrt{N}} \rightarrow \text{phase sensitivity}$$

$\xi_R = 1$, spin coherent state

$\xi_R < 1$, spin squeezed state

Dependent on ξ_R, $\Delta(\phi)$ achieves from standard quantum limit, Heisenber g limit, to super - Heisenberg limit.

Fast diabatic spin squeezing by one axis twisting evolution

$H/\hbar = \chi J_z^2 + \Omega J_\gamma + \Delta\omega_0 J_z$, where $J_\gamma = J_x \cos\gamma + J_y \sin\gamma$ (Kitagawa, Ueda)

nature

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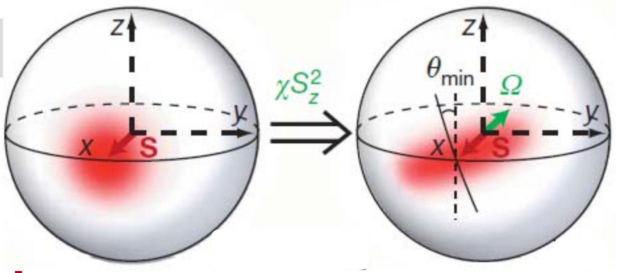


strong nonlinearity via controlling spatial overlap

Atom-chip-based generation of entanglement for quantum metrology

Max F. Riedel[1,2], Pascal Böhi[1,2], Yun Li[3,4], Theodor W. Hänsch[1,2], Alice Sinatra[3] & Philipp Treutlein[1,2,5]

Vol 464 | 22 April 2010 | doi:10.1038/nature08919 nature

LETTERS

strong nonlinearity via using Feshbach resonance

Nonlinear atom interferometer surpasses classical precision limit

C. Gross[1], T. Zibold[1], E. Nicklas[1], J. Estève[1]† & M. K. Oberthaler[1]

Twin Matter Waves for Interferometry Beyond the Classical Limit

B. Lücke, *et al.*

Science **334**, 773 (2011);
DOI: 10.1126/science.1208798

pair-correlated states from spin dynamics

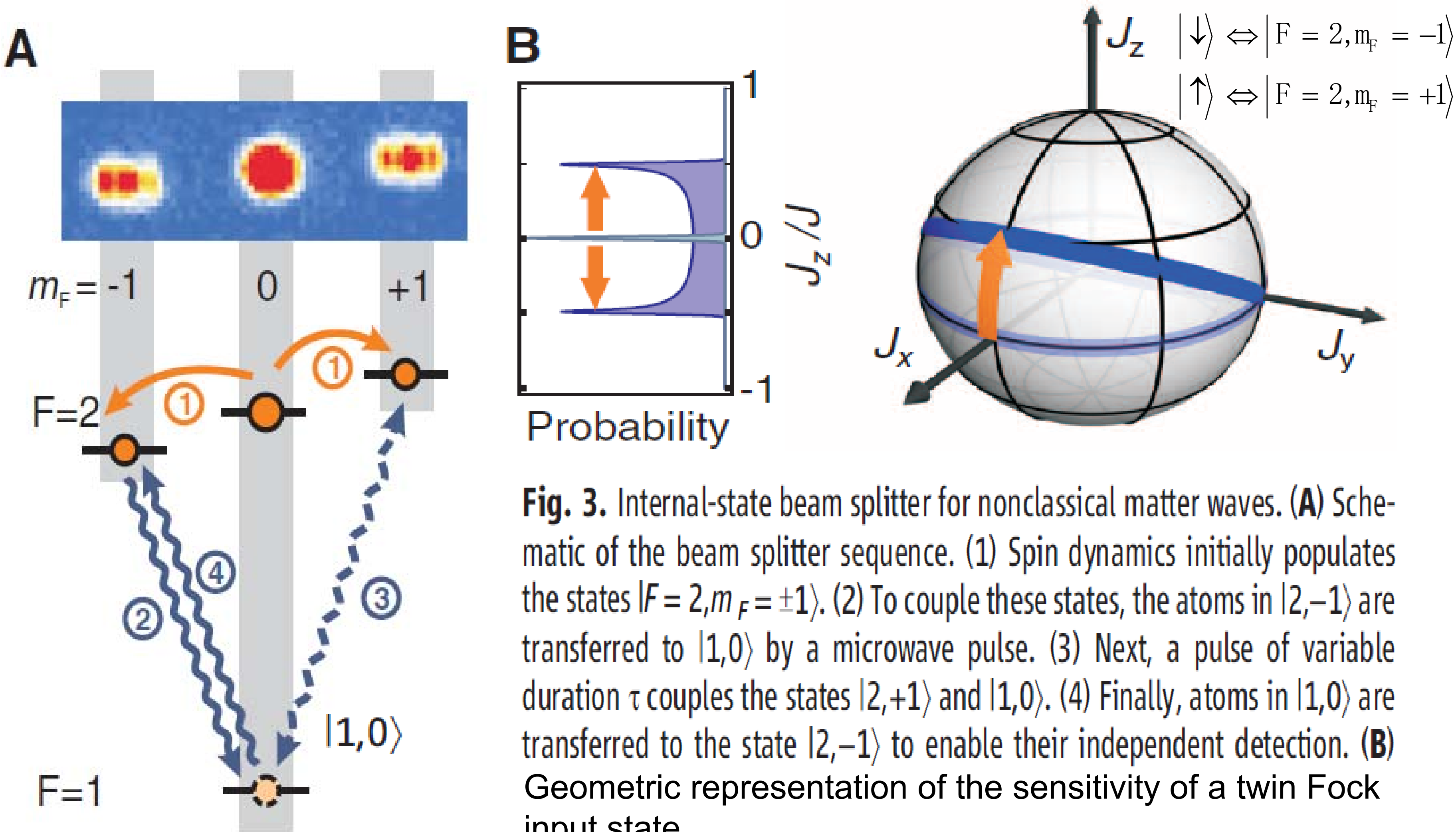


Fig. 3. Internal-state beam splitter for nonclassical matter waves. (**A**) Schematic of the beam splitter sequence. (1) Spin dynamics initially populates the states $|F = 2, m_F = \pm1\rangle$. (2) To couple these states, the atoms in $|2,-1\rangle$ are transferred to $|1,0\rangle$ by a microwave pulse. (3) Next, a pulse of variable duration τ couples the states $|2,+1\rangle$ and $|1,0\rangle$. (4) Finally, atoms in $|1,0\rangle$ are transferred to the state $|2,-1\rangle$ to enable their independent detection. (**B**) Geometric representation of the sensitivity of a twin Fock input state.

nature physics

LETTERS

PUBLISHED ONLINE: 26 FEBRUARY 2012 | DOI: 10.1038/NPHYS2245

Spin-nematic squeezed vacuum in a quantum gas

C. D. Hamley, C. S. Gerving, T. M. Hoang, E. M. Bookjans and M. S. Chapman*

Collisions in ultracold atomic gases have been used to **induce quadrature spin squeezing in two-component Bose condensates**…
Here, we **generalize** this finding to a higher-dimensional spin space by measuring **squeezing in a spin-1 Bose condensate**. Following **a quench through a quantum phase transition (between FM and AFM states)**, we demonstrate that **spin-nematic quadrature squeezing improves on the standard quantum limit** by up to 8-10 dB…
The observation has implications for **continuous variable quantum information** and **quantum-enhanced magnetometry**.

$$\mathcal{H} = \lambda \hat{S}^2 + \frac{1}{2} q \hat{Q}_{zz}$$ SU(3) Cartesian dipole–quadrupole basis

$$\hat{S}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2 \qquad \hat{Q}_{zz} = (2/3)\hat{a}_1^\dagger \hat{a}_1 - (4/3)\hat{a}_0^\dagger \hat{a}_0 + (2/3)\hat{a}_{-1}^\dagger \hat{a}_{-1}$$

3.3. High-precision interferometry via NOON states

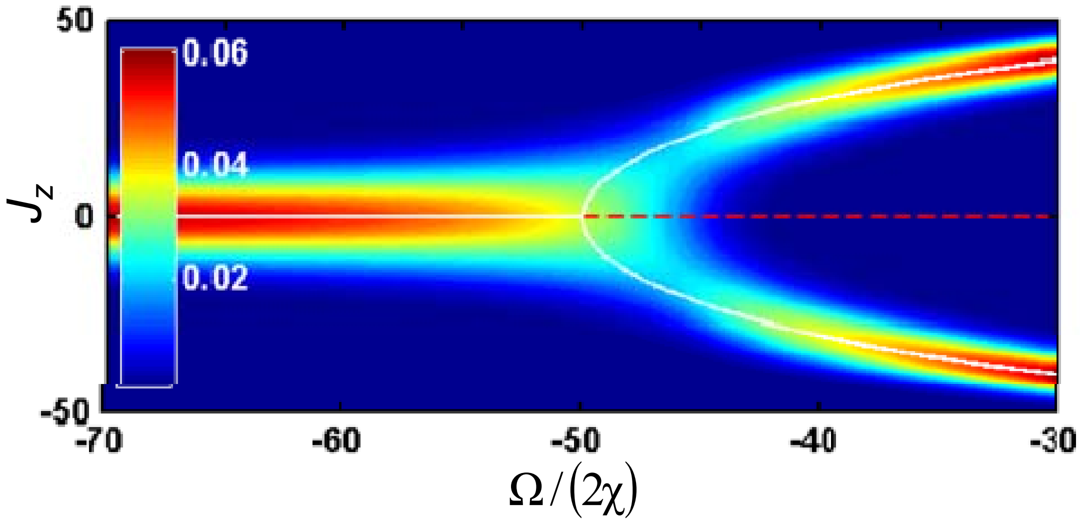
$$H/\hbar = \frac{\delta}{2}(n_1 - n_2) - \frac{\Omega}{2}(a_2^+ a_1 + a_1^+ a_2) + \frac{E_C}{8}(n_1 - n_2)^2 = \delta J_z - \Omega J_x + \chi J_z^2$$

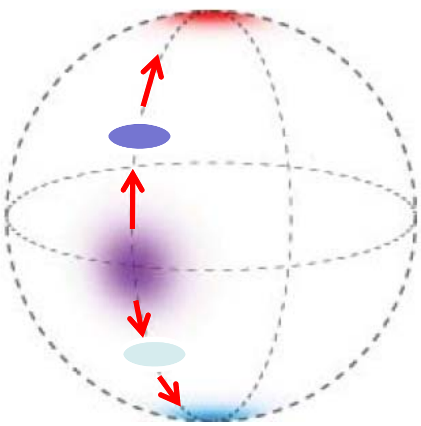
$$\text{Fock basis}: |\mathrm{NOON}\rangle = \frac{1}{\sqrt{2}}\left(|n_1 = N, n_2 = 0\rangle + |n_1 = 0, n_2 = N\rangle\right)$$

$$\text{spin basis}: |\mathrm{NOON}\rangle = \frac{1}{\sqrt{2}}\left(\left|J = \frac{N}{2}, J_z = -\frac{N}{2}\right\rangle + \left|J = \frac{N}{2}, J_z = +\frac{N}{2}\right\rangle\right)$$

The NOON state is a ground state for system of $\delta = 0$, $\chi < 0$ and $|\Omega/\chi| << 1$

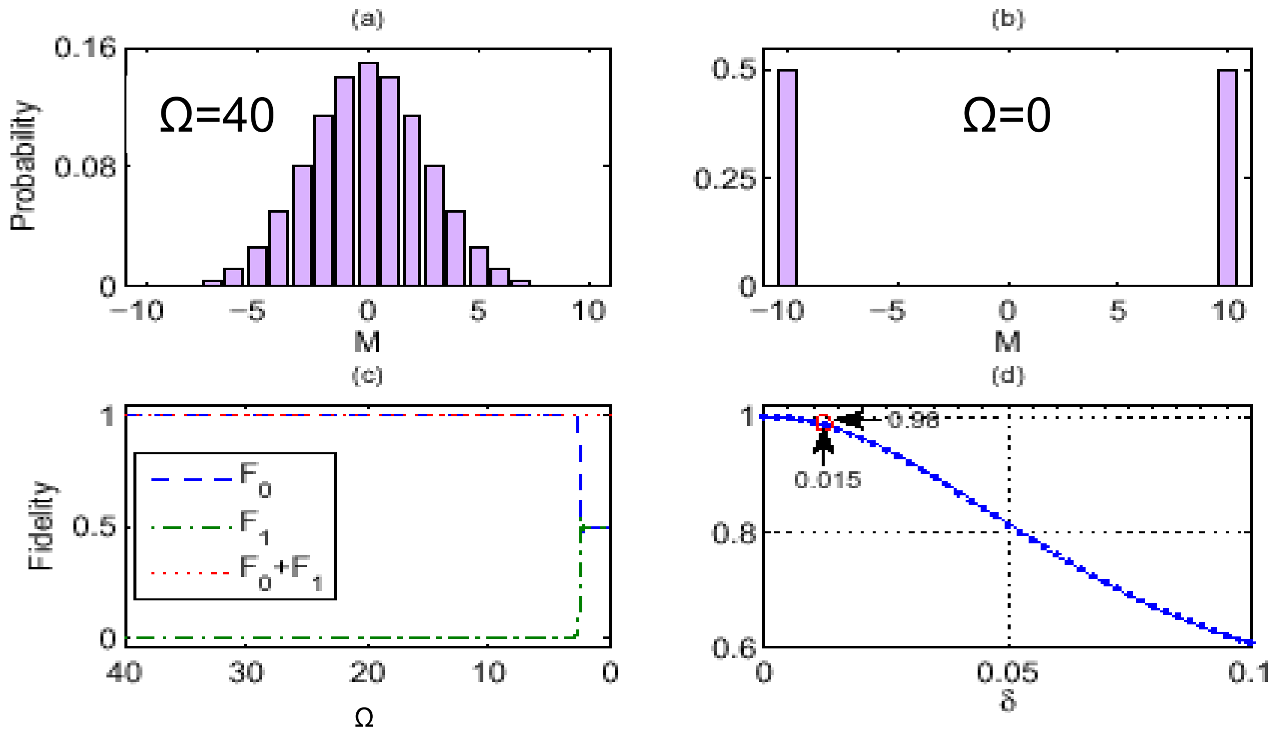
Adiabatic preparation of NOON state via dynamical bifurcation





C. Lee, PRL 97, 150402 (2006)

Beam splitting and recombination via dynamical bifurcation



For a system of $\delta = 0$ and $\chi < 0$, if $\Omega = 40 \rightarrow \Omega = 0$,

$$|GS\rangle = |CS\rangle_{SU(2)} \rightarrow |NOON\rangle = \left(|P1\rangle + |P2\rangle\right)/\sqrt{2}.$$

Here, $|P1\rangle = |J = N/2, M = -N/2\rangle$ and $|P2\rangle = |J = N/2, M = +N/2\rangle$ are the ground and first-excited states for the system of $\Omega = 0$ and $0 < \delta < |\chi|$, respectively. They can be used as two paths of a MZ interferometer.

Phase accumulation via the term of δJ_z

Switch on the term δJ_z for a period of time T,

$$|\mathrm{NOON}\rangle \to \frac{1}{\sqrt{2}}\left(e^{-i\delta T\cdot(N/2)}|P1\rangle + e^{+i\delta T\cdot(N/2)}|P2\rangle\right)$$

with $\varphi = \delta T$, which is the phase accumulated in a single - atom system.

Extract the relative phase from the population information via a dynamical bifurcation from $|\Omega/\chi<<1$ to $|\Omega/\chi|>>1$

Due to the indistinguishability, we can not use the proposals of Wineland et al. and Caves et al.

At the side of $|\Omega/\chi| << 1$, the ground [first excited] states will be $\left(|P1\rangle + |P2\rangle/\sqrt{2}\right)$ $[\left(|P1\rangle - |P2\rangle\right)/\sqrt{2}]$ even for a very small Ω.

Therefore, the state after the dynamical bifurcation becomes

$$\cos(N\varphi/2)|GS\rangle - i\cdot\sin(N\varphi/2)|FS\rangle,$$

whose populations are $P_{GS} = \cos^2(N\varphi/2) = (1+\cos(N\varphi))/2$

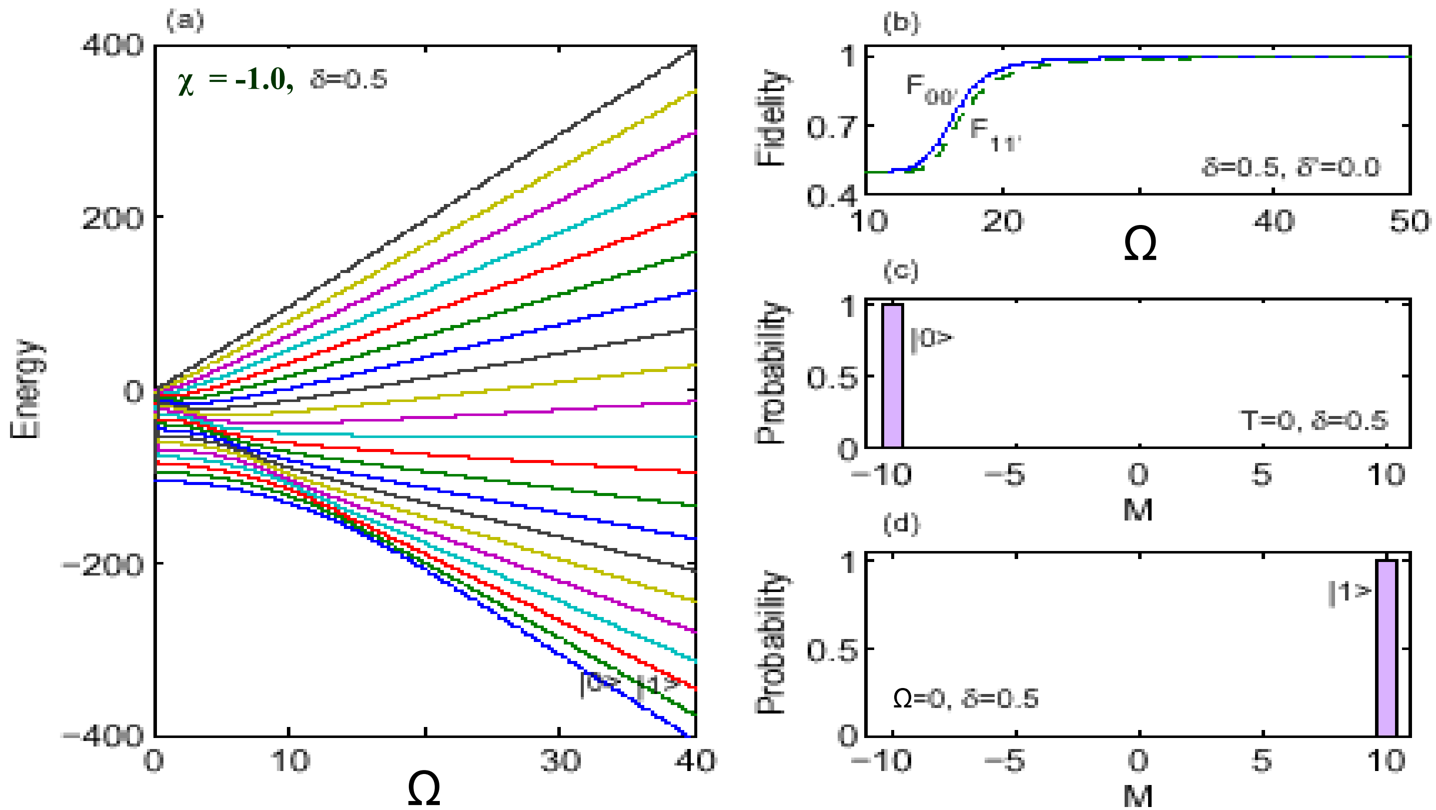
and $P_{FS} = \sin^2(N\varphi/2) = (1-\cos(N\varphi))/2$.

Detection

Usually, it is not easy to distinguish the |GS> and |FS> at the side of $|\Omega/\chi|>>1$.

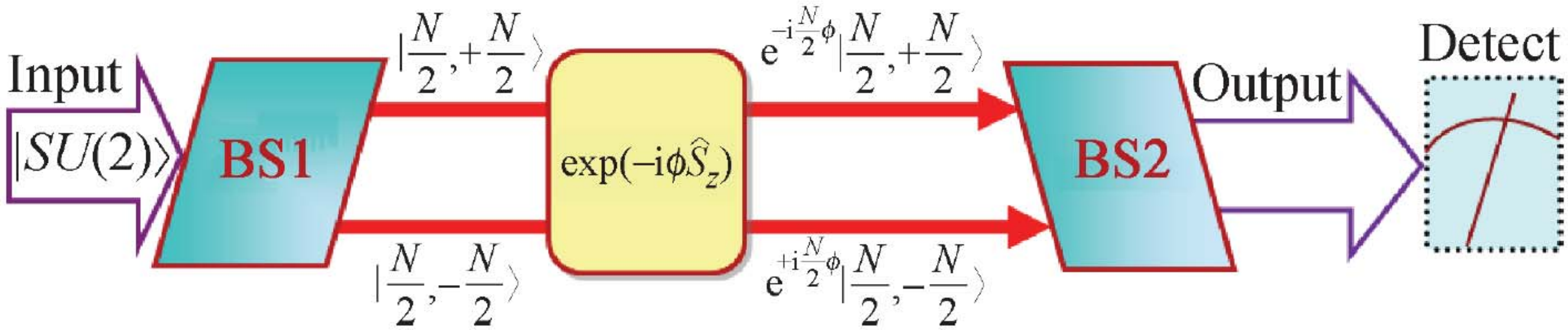
Note that the degeneracy of |P1> and |P2> can be broken by a suitable bias δ, we can suddenly switch on δJz with $\delta<<\Omega$ at the side of $|\Omega/\chi|>>1$.

Then keep δ unchanged and adiabatically switch off Ω, the |GS> and |FS> will adiabatically evolve into |P1> and |P2>, respectively.



Schematic diagram for MZ interferometry via NOON states of indistinguishable systems

Hamiltonian, $H/\hbar = \delta J_z - \Omega J_x + \chi J_z^2$



State Evolution

$$\left|\frac{N}{2},-\frac{N}{2}\right\rangle \Rightarrow |CS\rangle_{SU(2)} \Rightarrow \frac{|P1\rangle + |P2\rangle}{\sqrt{2}} \Rightarrow \frac{e^{-\frac{i}{2}N\varphi}|P1\rangle + e^{+\frac{i}{2}N\varphi}|P2\rangle}{\sqrt{2}}$$

$$\Rightarrow \cos\left(\frac{N\varphi}{2}\right)|GS\rangle - i\cdot\sin\left(\frac{N\varphi}{2}\right)|FS\rangle \Rightarrow \cos\left(\frac{N\varphi}{2}\right)|P1\rangle - i\cdot\sin\left(\frac{N\varphi}{2}\right)|P2\rangle$$

with

$$|P1\rangle = \left|\frac{N}{2},-\frac{N}{2}\right\rangle \text{ and } |P2\rangle = \left|\frac{N}{2},+\frac{N}{2}\right\rangle$$

Keynotes

- negative nonlinearity ($\chi < 0$) $\rightarrow$ Feshbach resonance
- coupling $\rightarrow$ tunnelling (double-well system), or
  Raman transition (two-component condensate)
- two paths $\rightarrow$ two degenerated ground states for the system of $\chi < 0$
- beam splitting/recombination $\rightarrow$ dynamical bifurcation
- path entangled state (NOON state) $\rightarrow$ dynamical bifurcation

Advantages

- large total number of particle (in order of 10^3, 10 for systems of photons and trapped ions)
- reduced influence of environment (adiabatic evolution and closed sub-Hilbert space)
- measurement precision of Heisenberg limit (path entangled states)
- experimental possibility (double-well or two-component systems)

Challenge

- adiabatic evolution requests long coherent time

C. Lee, PRL 97, 150402 (2006)

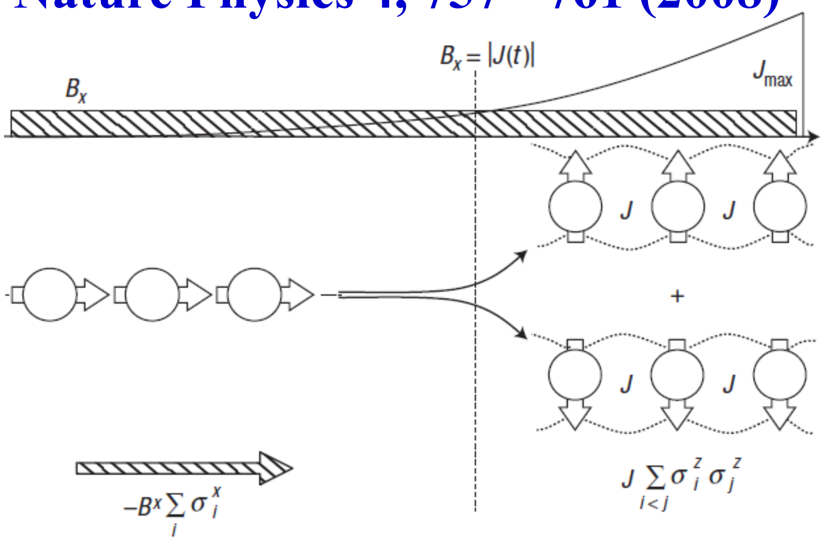
Adiabatic MZ interferometer with ultracold trapped ions

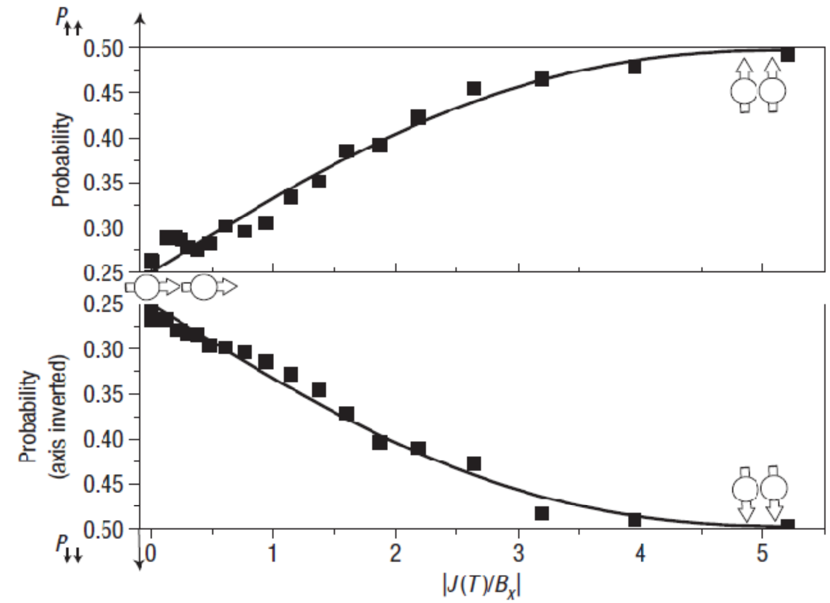
Simulating a quantum magnet with trapped ions

A. FRIEDENAUER*, H. SCHMITZ*, J. T. GLUECKERT, D. PORRAS AND T. SCHAETZ†

$$H_{\text{Ising}} = H_B + H_J = -B_x \sum_i \sigma_i^x + \sum_{i<j} J_{ij}\sigma_i^z \sigma_j^z$$

Nature Physics 4, 757 - 761 (2008)





Quantum Phase transition via increase the interaction strength J – imperfection from finite J (whose final state is not the exact NOON state)

nature COMMUNICATIONS

ARTICLE

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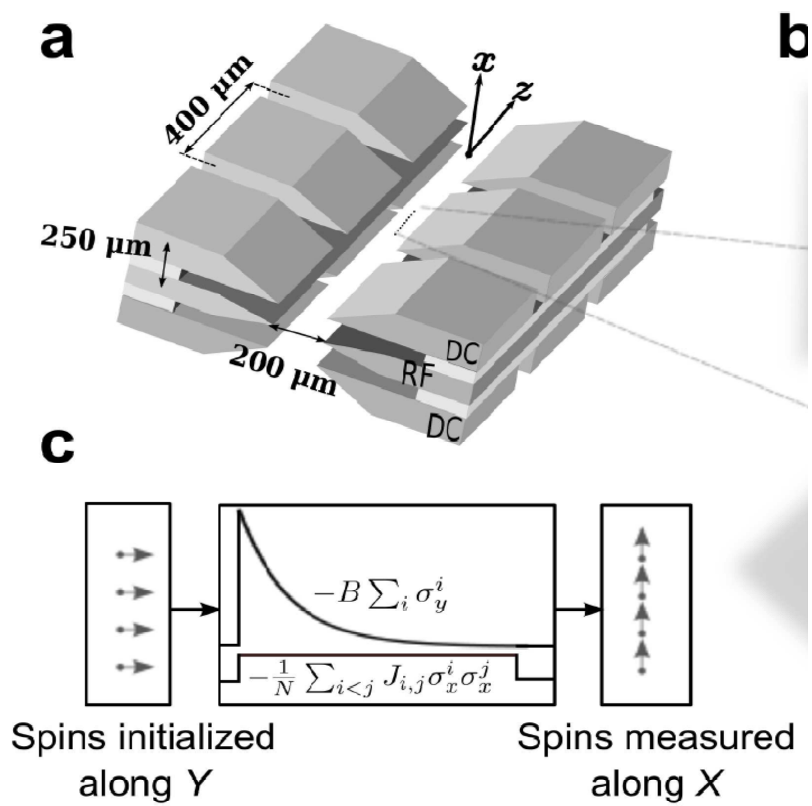
DOI: 10.1038/ncomms1374

Onset of a quantum phase transition with a trapped ion quantum simulator

R. Islam[1], E.E. Edwards[1], K. Kim[1], S. Korenblit[1], C. Noh[2], H. Carmichael[2], G.-D. Lin[3], L.-M. Duan[3], C.-C. Joseph Wang[4], J.K. Freericks[4] & C. Monroe[1]

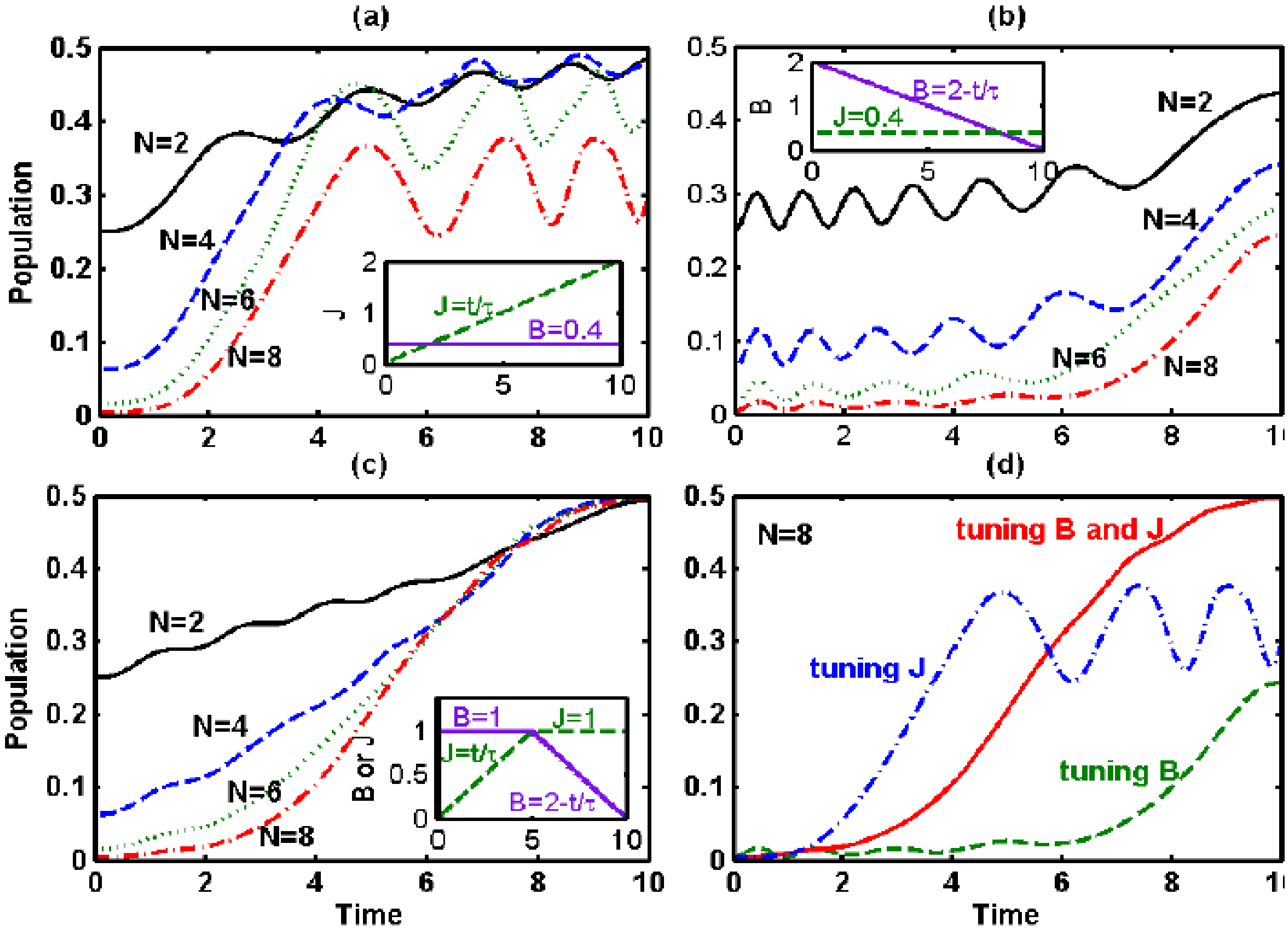


Up to nine 171Yb⁺ ions

$$H = -\frac{1}{N}\sum_{i<j} J_{i,j}\sigma_x^i\sigma_x^j - B\sum_i \sigma_y^i$$

Quantum phase transition via decreasing the transverse field strength B – imperfection from the initial state of a non-zero J [which is not the exact SU(2) coherent state]

Our proposal: Quantum phase transition (Beam Splitting) via two-step sweeping of increasing J and then decreasing B [no theoretical imperfection under adiabatic evolution]



YM Hu, M Feng, C Lee, Phys. Rev. A 85, 043604 (2012)

4. Summary and open problems

Summary

- In interferometers of Bose condensed atoms, the atom-atom interaction brings the nonlinearity to the system.
- Tuning the ratio of nonlinearity and coupling, symmetry-breaking transitions appear and the dynamics near the critical point obey the universal Kibble-Zurek mechanism.
- The spin squeezed states and NOON state can be prepared by controlling the nonlinearity and these states can used for high-precision interferometry beyond the standard quantum limit.

Open Problems

- noises (quantum fluctuations and technical noises)
- imperfect effects (atom loss and environment)
- coupling between internal and external degrees of freedom
- finite-temperature effects

Our related works on this topic

[1] **C. Lee***, J. Huang, H. Deng, H. Dai, and J. Xu, Nonlinear quantum interferometry with Bose condensed atoms, Front. Phys. 7, 109-130 (2012) (review article)

[2] Y.-M. Hu, M. Feng, and **C. Lee***, Adiabatic Mach-Zehnder interferometer via an array of trapped ions, Phys. Rev. A 85, 043604 (2012)

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[9] W. Hai, **C. Lee**, G. Chong, and L. Shi, Chaotic probability density in two periodically driven and weakly coupled Bose-Einstein condensates, Phys. Rev. E 66, 026202 (2002)

[10] **C. Lee***, W. Hai, L. Shi, X. Zhu, and K. Gao, Chaotic and frequency-locked atomic population oscillations between two coupled Bose-Einstein condensates, Phys. Rev. A 64, 053604 (2001)

Current interests of my group:
Cold Atomic Physics and Quantum Technologies

- **Quantum interferometry** (Bose-condensed atoms, ultracold trapped ions)
- Cavity-QED with Bose condensed atoms
- Quantum dynamics of ultracold atomic systems (nonlinear dynamics of BECs, quenching dynamics, dynamics of quantum phase transition, dynamics of open quantum systems, etc)
- Potential applications of ultracold atoms in quantum technologies (high-precision measurements, quantum simulation, atom clocks, etc) (joint research projects with WIPM@CAS-China and ANU/SUT-Australia)

Group Members and Collaborators

- Group leader: Chaohong Lee
- Postdoc: Honghua Zhong
- PhD students: Haiming Deng, Jun Xu, Qinzhou Ye (from September 2012)
- Master Students: Jiahao Huang, Hui Dai, Zhilin Lei
- Undergraduate students: Shuyuan Wu, Haifen Qiu
- Visiting PhD students: Yanmin Hu (WIPM), Wanju Fan, Xizhou Qin
- Visiting professors: Xiwen Guan (ANU), Xiaobing Luo, Xiaolong Zhang, Qiongtao Xie
- Other national collaborators: Kelin Gao (WIPM), Mang Feng (WIPM), Wenhua Hai (Changsha)…
- Other international collabrators: Joachim Brand (Germany-NZ), Yuri Kivshar (ANU), Elena Ostrovskaya (ANU), Murray Batchelor (ANU)…

Thanks for your attention!

International postdoctoral positions available!