Australian National University

ACQAO

LECTURE 3: Quantum Statistics, Coherence And Correlations

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Lecture 3 Outline

QUANTUM STATISTICS, COHERENCE AND CORRELATIONS:

- Coherence and correlations
- Hanbury Brown and Twiss experiments
- Correlation experiments elsewhere:
  - By quantum mechanics
  - By structure
  - By dissociation
  - By collisions

What correlates atoms?

- Quantum mechanics (with classical analogues)
  - The distinguishability or indistinguishability of particles e.g. the Hanbury-Brown and Twiss Effect
  - Applies to bosons and fermions
  - Applies to photons, electrons, neutrons, protons etc.

- Interactions
  - Structure e.g. optical lattices - Bloch (Mainz): Rb atoms
  - Molecular dissocation (the analogue of degenerate parametric down-conversion for photons)
    - Jin (JILA) - K molecules
  - Collisions (the analogue of optical four-wave-mixing)
    - Spontaneous four-wave-mixing : Institut d'Optique (He*)
    - Stimulated four-wave-mixing : ANU (He*)

Theory – Glauber (1963)

PHYSICAL REVIEW VOLUME 130, NUMBER 6 15 JUNE 1963

The Quantum Theory of Optical Coherence*

Roy J. Glauber
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 11 February 1963)

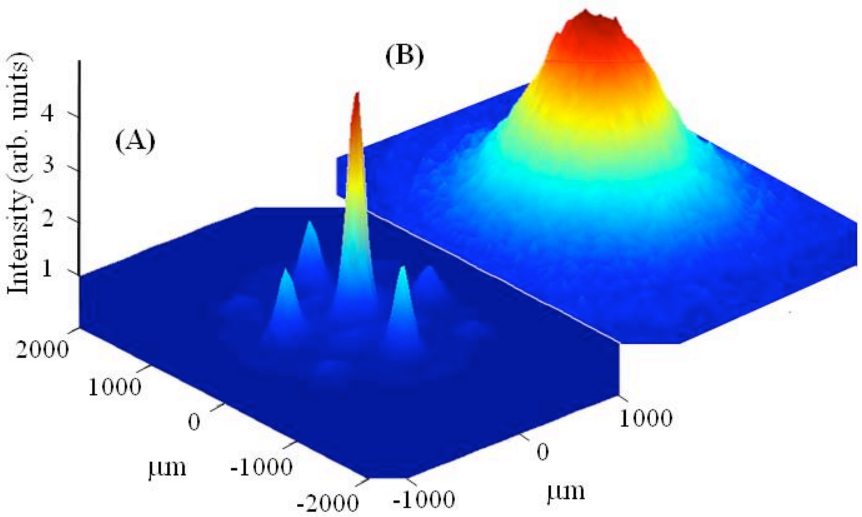
The concept of coherence which has conventionally been used in optics is found to be inadequate to the needs of recently opened areas of experiment. To provide a fuller discussion of coherence, a succession of correlation functions for the complex field strengths is defined. The nth order function expresses the correlation of values of the fields at $2n$ different points of space and time. Certain values of these functions are measurable by means of n-fold delayed coincidence detection of photons. A fully coherent field is defined as one whose correlation functions satisfy an infinite succession of stated conditions. Various orders of incomplete coherence are distinguished, according to the number of coherence conditions actually satisfied. It is noted that the fields historically described as coherent in optics have only first-order coherence. On the other hand, the existence, in principle, of fields coherent to all orders is shown both in quantum theory and classical theory. The methods used in these discussions apply to fields of arbitrary time dependence. It is shown, as a result, that coherence does not require monochromaticity. Coherent fields can be generated with arbitrary spectra.

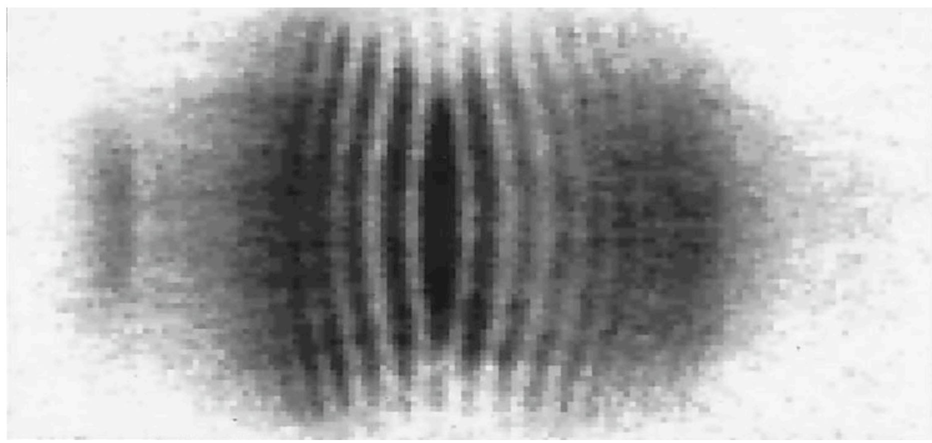
First Order Correlation Function $g^{(1)}$

- Measures single particles
- => Amplitude fluctuations

$$g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau)\rangle}{\langle|E(t)|\rangle\langle|E(t+\tau)|\rangle}$$

- Gives fringe visibility in interference





2nd Order Correlations $g^{(2)}$

- Measures coincidence of particle *pairs*
  => Intensity fluctuations

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle\langle I(t+\tau)\rangle}$$

- Second order coherence
  – speckle and the Hanbury Brown-Twiss Effect

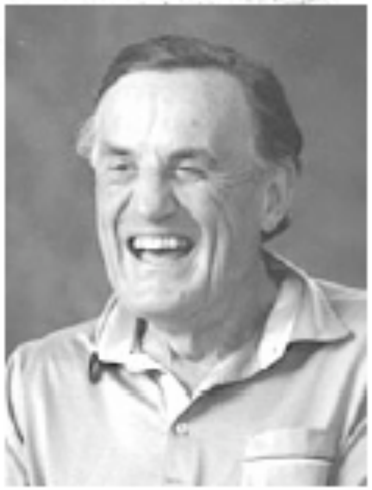
NATURE January 7, 1956 VOL. 177

CORRELATION BETWEEN PHOTONS IN TWO COHERENT BEAMS OF LIGHT

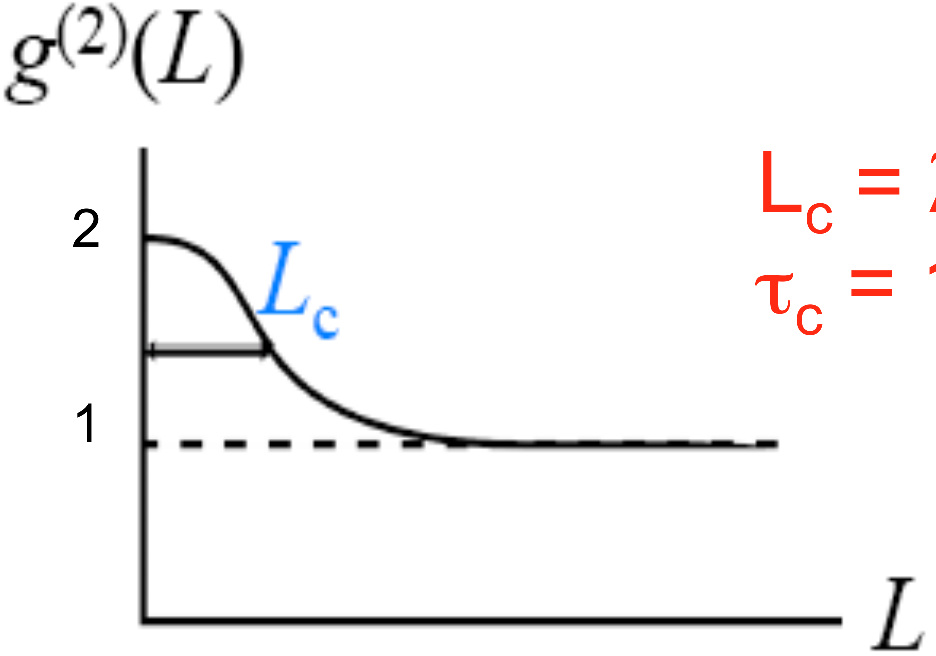
By R. HANBURY BROWN
University of Manchester, Jodrell Bank Experimental Station

AND

R. Q. TWISS
Services Electronics Research Laboratory, Baldock

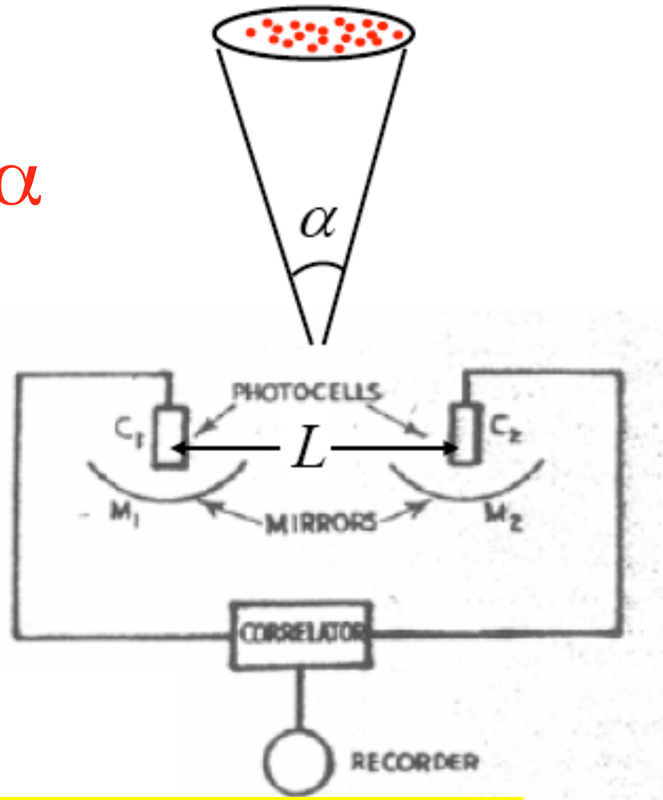


Hanbury Brown/Twiss effect



$L_c = \lambda / 2\pi\alpha$

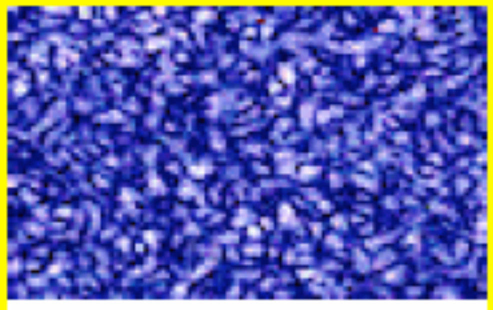
$\tau_c = 1 / \Delta\omega$



Varied the detector separation L to measure angular diameter (coherence area) of a star

Speckle in the observation plane:

- Correlation radius $L_c \approx \lambda / \alpha$
- Changes after $\tau_c \approx 1 / \Delta\omega$

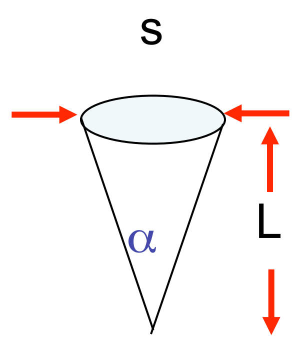


Correlation length

- For light, correlation length is

$$L_c = \lambda / 2\pi\, \alpha$$
$$\sim L\, \lambda / 2\pi\, s$$

which is identified as the speckle size

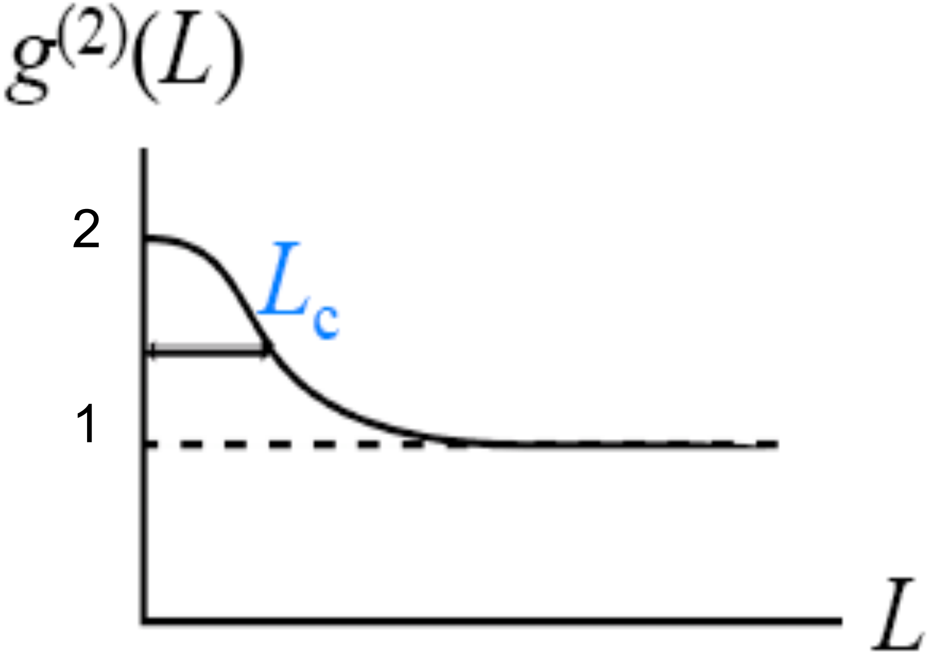


- For de Broglie waves $\lambda_{dB} = h/mv$ move from source for time t

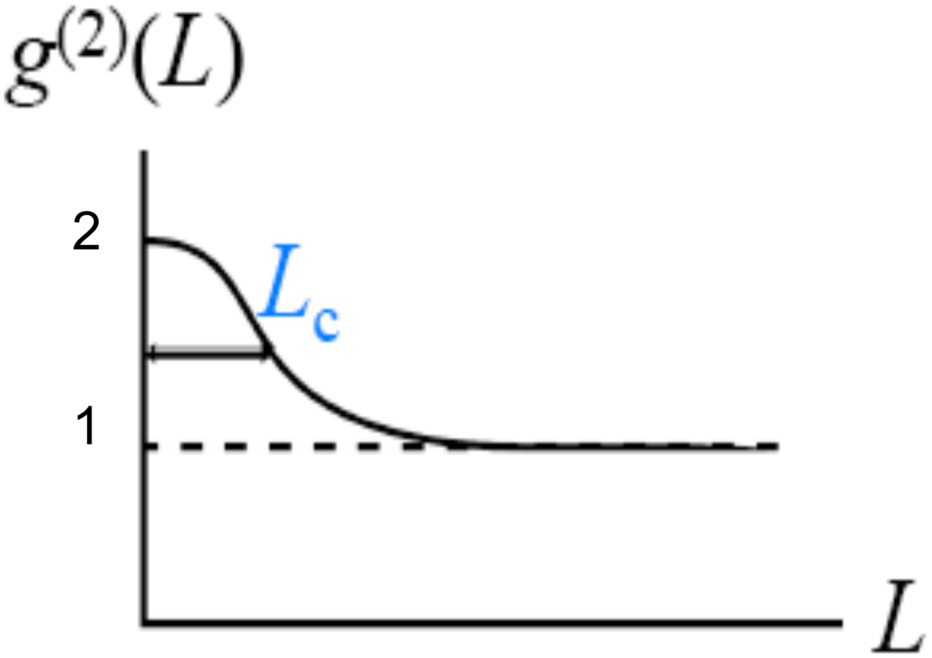
$$L_c = \lambda_{dB} / 2$$
$$\sim \hbar\, t / ms$$

If detector resolution
$d \ll L_c \Rightarrow G^{(2)}(0) = 2$

If detector resolution
$d \gg L_c \Rightarrow G^{(2)}(0) = 1$



HBT interpretation



joint detection probability

$$r_2;\tau) = \frac{\langle \pi(r_1, r_2, t)\rangle}{\langle \pi(r_1, t)\rangle \langle \pi(r_2, t)\rangle}$$

single detection probabilities

$g^{(2)} > 1$ => probability of finding two photons in the same place greater than the product of the two single photon probabilities

=> photon bunching

$g^{(2)} = 1$ => independent particles

Bunching and Antibunching

$g^{(2)}(\tau)$: the probability of measuring a particle at a time τ after the previous particle

Chaotic bosons are bunched

Coherent sources uncorrelated

Chaotic fermions are antibunched

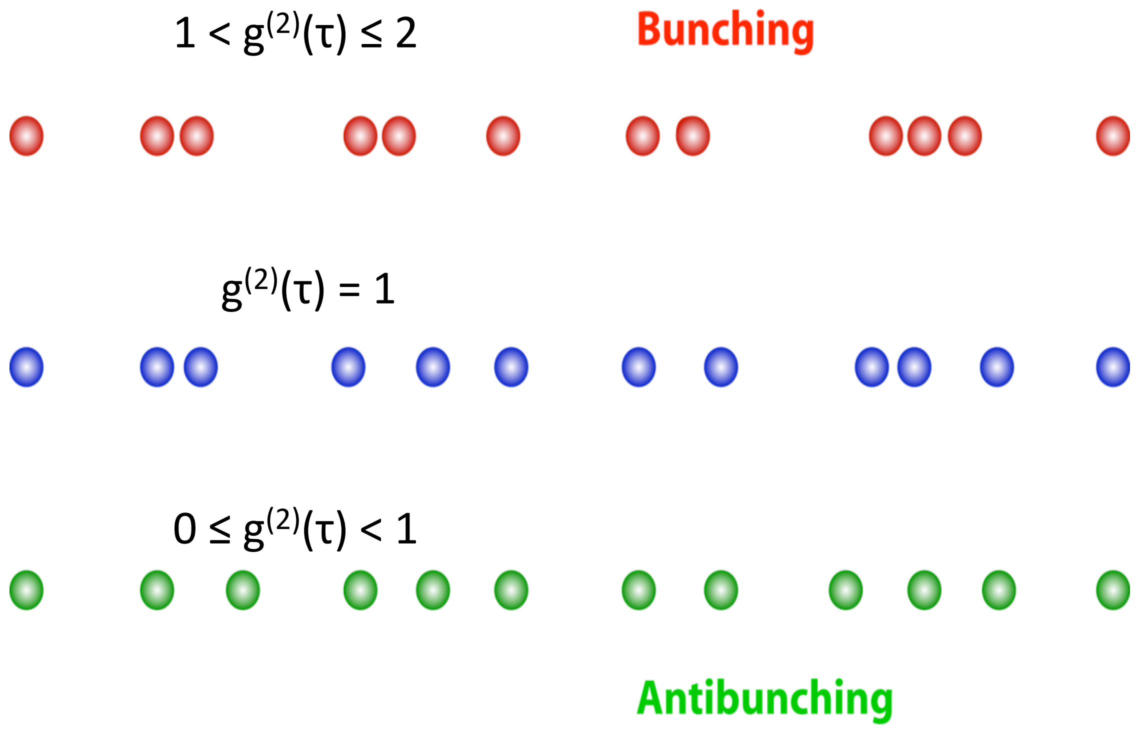
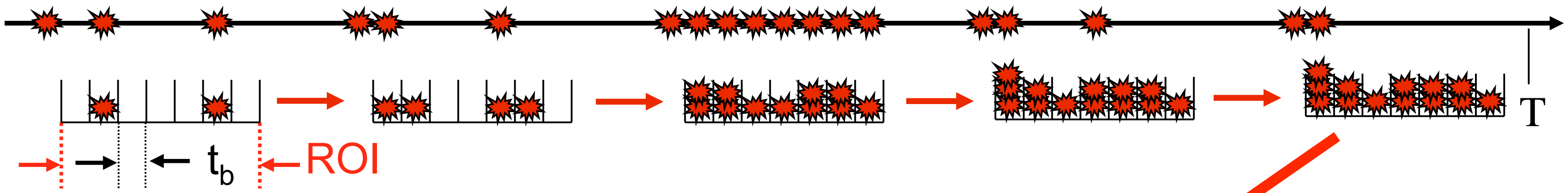


Image from http://atomoptic.iota.u-psud.fr/research/helium/pictures/bunch-antib-en.pdf

Temporal HBT



sliding bin vernier

ROI = region of interest = N x t_b

t_b = bin time

T = total data acquisition time

t_d = detector deadtime

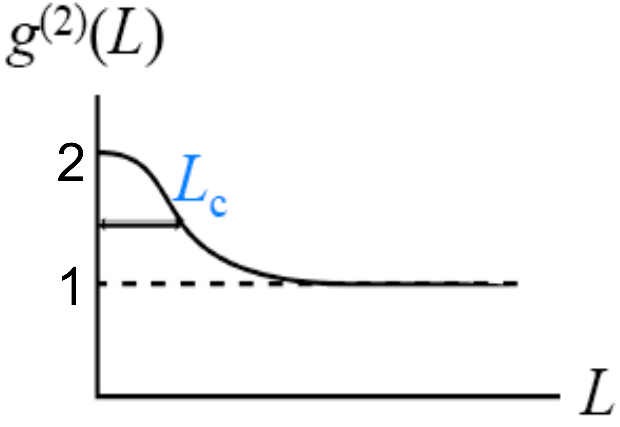
Δt = 1/mean count rate

τ_c = correlation time

Want: $t_d \ll \Delta t \leq t_b < \tau_c < \mathrm{ROI} \ll T$

Normalise

Classical Explanation



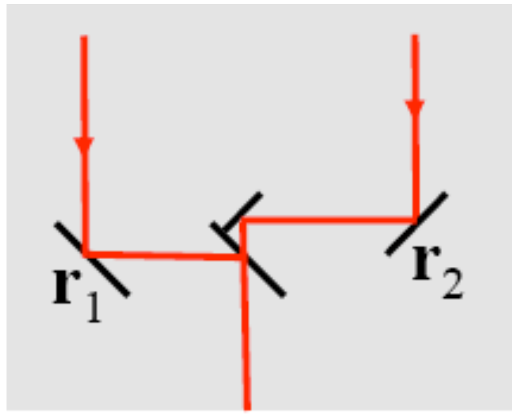
$$g^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = 1 + |g^{(1)}(\mathbf{r}_1, \mathbf{r}_2)|^2$$

Uncorrelated events from independent particles (shot noise)

Correlated events due to beat notes between spectral components of emitters within coherence volume
$\Rightarrow$ effective beat note wavelength
$\lambda_b = c / \omega' - \omega''$

This is why the HBT measurement is not affected by atmospheric fluctuations whose path lengths $L \ll \lambda_b$

In contrast, the Michelson interferometer measures $g^{(1)}$ and derives L_c from the fringe visibility, which is affected by atmospheric fluctuations $L \sim \lambda$.

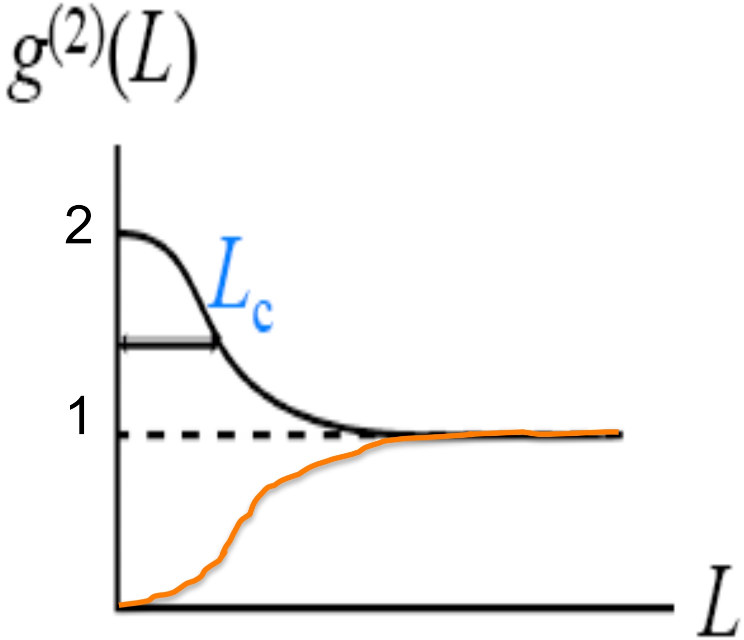


Quantum Interpretation

Fano and Glauber:

- Two path interference:
  - Path 1 amplitude <ab/cd>
  - Path 2 amplitude <ad/cb>
- Indistinguishable paths - combined amplitude square

  (<ab/cd> $\pm$ <ad/cb>) ^{2}



yields cross terms - 2! combinations

Bosons + (symmetric)

Fermions - (antisymmetric)

$$g^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = 1 \pm |g^{(1)}(\mathbf{r}_1, \mathbf{r}_2)|^2$$

Fermion $g^{(2)}(0) = 0$ is purely quantum - no classical analogue!

Atom correlation experiments

- Tokyo: Yasuda and Shimizu, *PRL* **77**, 3090 (1996)
  - first measurement of HBT effect in atoms (Ne*) showing coherent bunching
- Naraschewski and Glauber, *PRA* **59**, 4595 (1997) - THEORY
  - theory of 1st and 2nd order correlation functions for thermal and condensed gas mixtures
- Mainz: Folling, Bloch et al., *Nature* **434**, 481 (2005)
  - Correlations in the Mott insulator phase of a *bosonic* Rb optical lattice experiment
- JILA: Greiner, Jin et al., *PRL* **94**, 110401 (2005)
  - demonstrated correlations between the dissociating atom pairs from a K molecular BEC
- Orsay: Westbrook, Aspect - HBT in bosonic ^{4}He*, *Science* **310**, 648 (2005)
  - also measured correlations in *collisions* between ultracold atomic clouds (ICAP)
- ETH: Esslinger et al. *PRL* **95**, 090404 (2005)
  - $g^{(2)}$ for atom laser and pseudo-thermal beam
- Mainz: Bloch et al. *Nature* **444**, 733 (2006)
  - Correlations with *fermionic* Rb in Mott insulator phase
- Orsay & VU Amsterdam: Vassen et al., *Nature* **445**, 402 (2007)
  - used Orsay detector to measure HBT in fermions ^{3}He*

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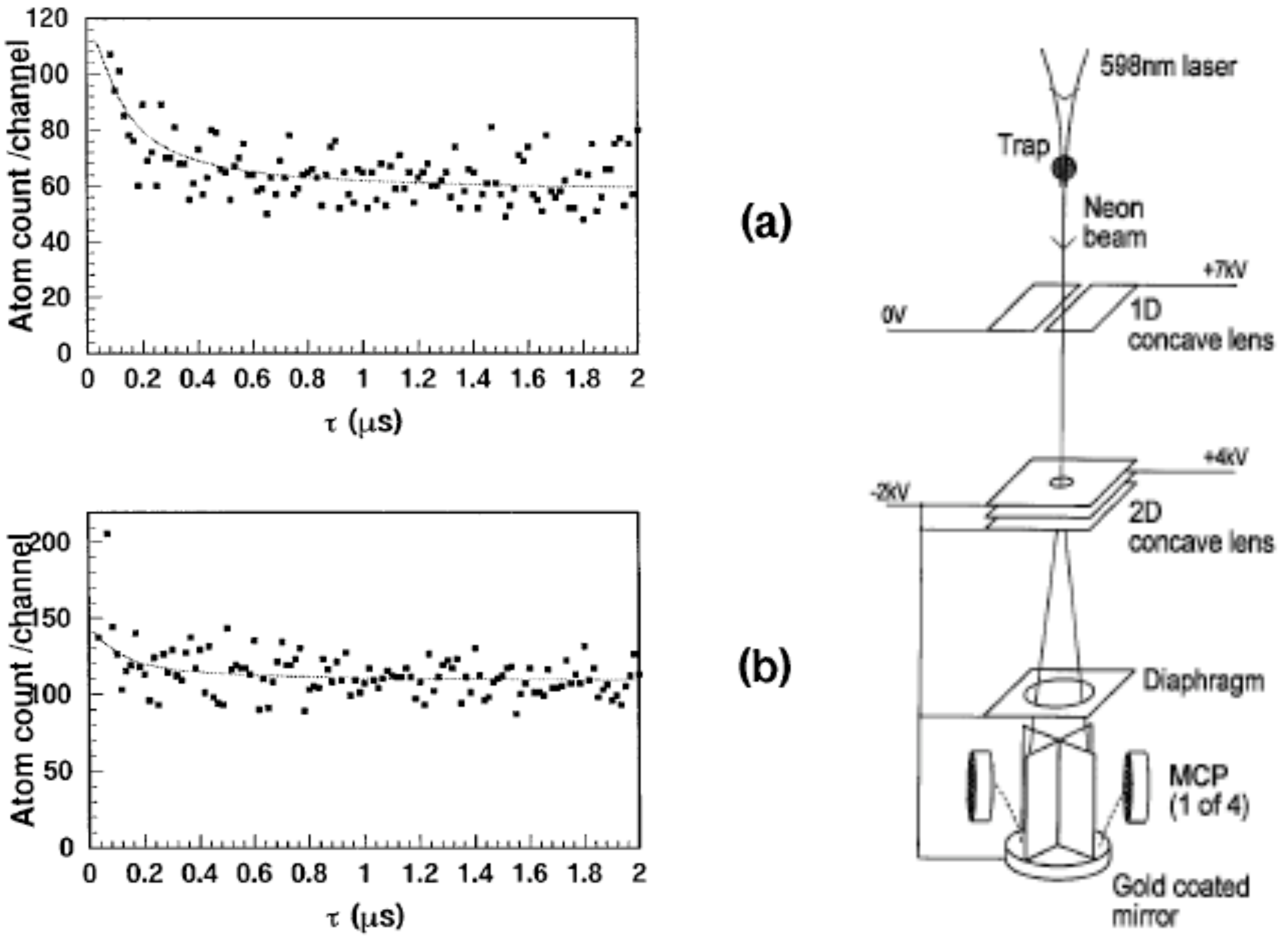
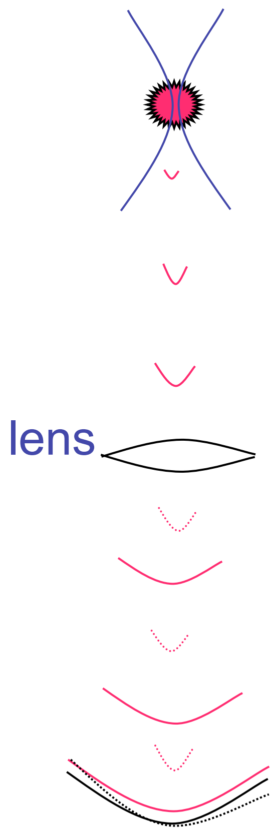
Yasuda and Shimizu

VOLUME 77, NUMBER 15 PHYSICAL REVIEW LETTERS 7 OCTOBER 1996

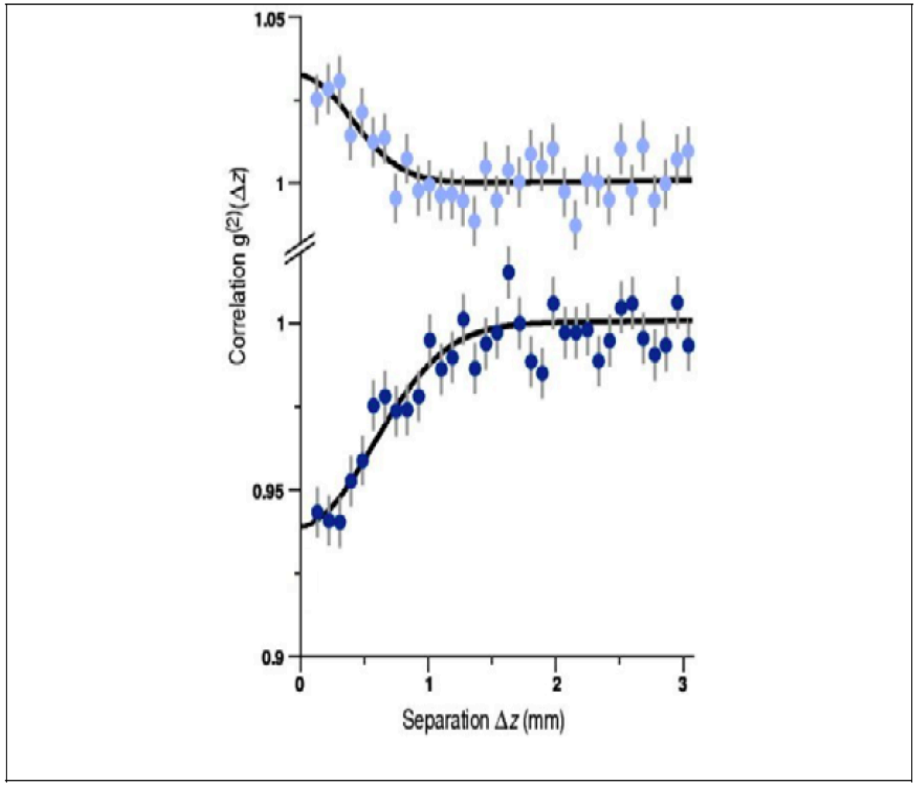
Observation of Two-Atom Correlation of an Ultracold Neon Atomic Beam

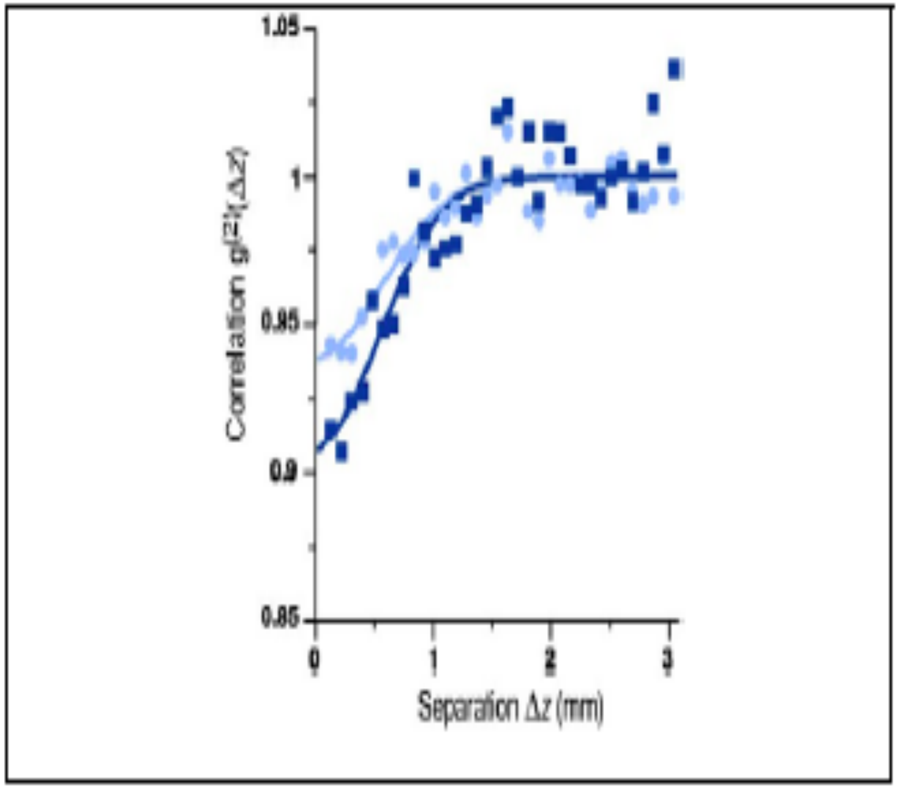




HBT - Fermions and Bosons

nature Vol 445|25 January 2007|doi:10.1038/nature05513





Top: Bosons
Bottom: Fermions

L_c ^{3}He = 4/3 (L_c ^{4}He)

Light blue: without lens
Dark blue: with demagnifying lens

Similar to Shimizu's experiment

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Correlation by structure

of TRACE data is acknowledged. This work was supported by the Japan–UK Cooperation Science Programme of the JSPS (Principal investigators K.S. and N.O. Weiss) and a Grant-in-Aid for the 21st Century COE 'Centre for Diversity and Universality in Physics' from MEXT, Japan. The numerical computation was performed on the Earth Simulator.

Competing interests statement The authors declare that they have no competing financial interests.

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Spatial quantum noise interferometry in expanding ultracold atom clouds

Simon Fölling, Fabrice Gerbier, Artur Widera, Olaf Mandel, Tatjana Gericke & Immanuel Bloch

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In a pioneering experiment[1], Hanbury Brown and Twiss (HBT) demonstrated that noise correlations could be used to probe the properties of a (bosonic) particle source through quantum statistics; the effect relies on quantum interference between possible detection paths for two indistinguishable particles. HBT correlations—together with their fermionic counterparts[2–4]—find numerous applications, ranging from quantum optics[5] to nuclear and elementary particle physics[6]. Spatial HBT interferometry has been suggested[7] as a means to probe hidden order in strongly correlated phases of ultracold atoms. Here we report such a measurement on the Mott insulator[8–10] phase of a rubidium Bose gas as it is released from an optical lattice trap. We show that strong periodic quantum correlations exist between density fluctuations in the expanding atom cloud. These spatial correlations reflect the underlying ordering in the lattice, and find a natural interpretation in terms of a multiple-wave HBT interference effect. The method should provide a useful tool for identifying complex quantum phases of ultracold bosonic and fermionic atoms[11–15].

Although quantum noise correlation analysis is now a basic tool in various areas of physics, applications to the field of cold atoms have been scarce. Most of these concentrate on photon correlation techniques from quantum optics[5,16]. It was not until 1996 that bunching of cold (but non-degenerate) bosonic atom clouds could be directly measured[17], followed by the observation of reduced inelastic losses due to a modification of local few-body correlations by quantum degeneracy[18–20].

In our experiment, we directly measure the spatial correlation function of the density fluctuations in a freely expanding atomic

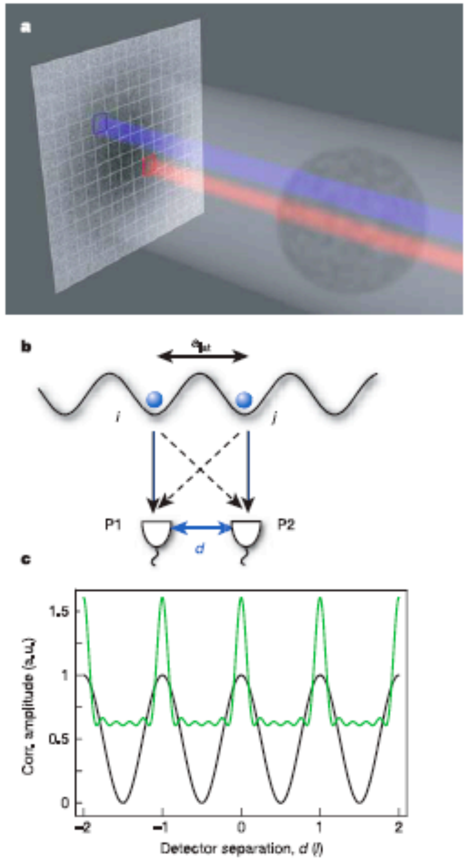
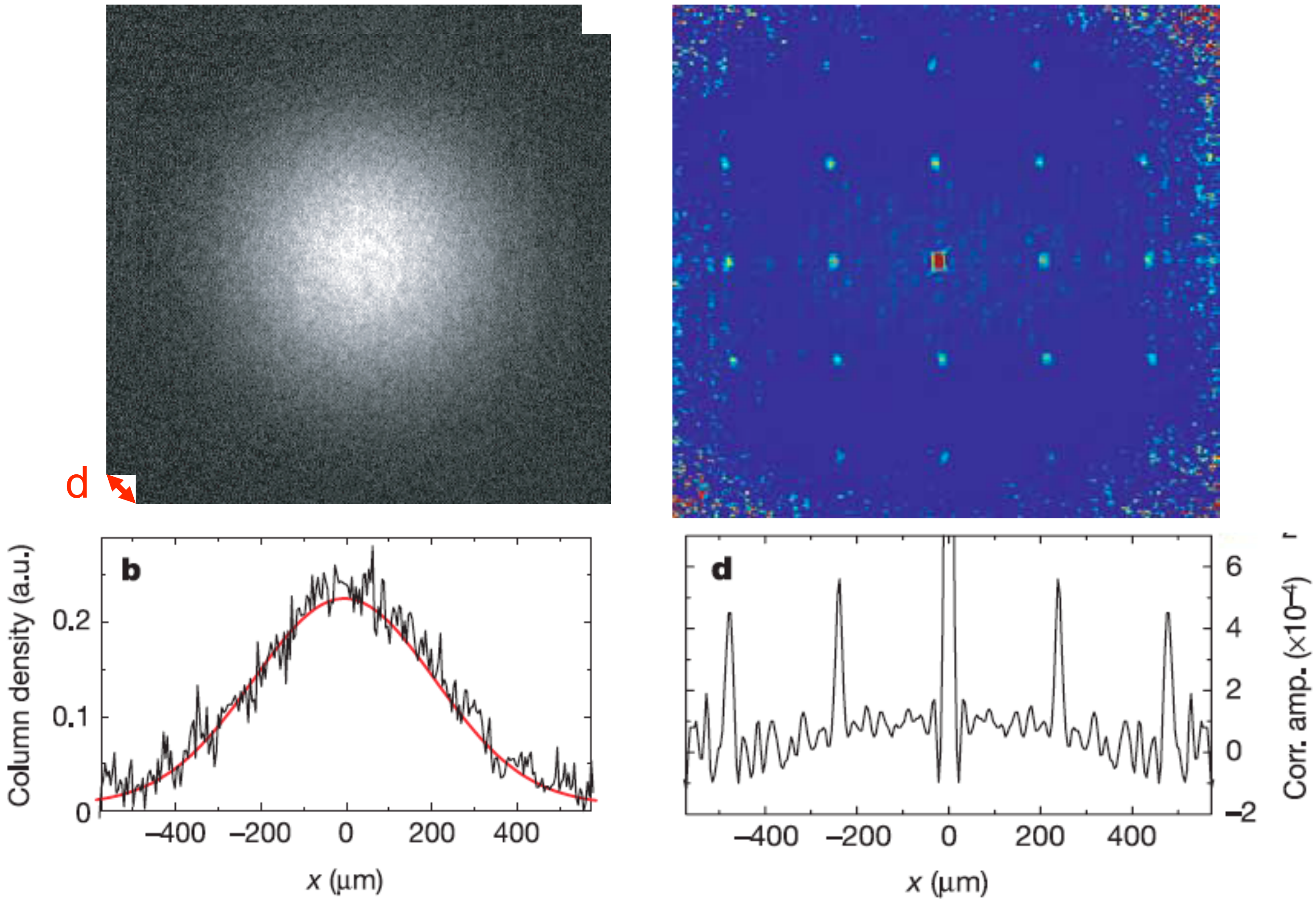


Figure 1 Illustration of the atom detection scheme and the origin of quantum correlations. **a**, The cloud of atoms is imaged to a detector plane and sampled by the pixels of a CCD camera. Two pixels P1 and P2 are highlighted, each of which registers the atoms in a column along its line of sight. Depending on their spatial separation *d*, their signals show correlated quantum fluctuations, as illustrated in **b**. **b**, When two atoms initially trapped at lattice sites *i* and *j* (separated by the lattice spacing a_{lat}) are released and detected independently at P1 and P2, the two indistinguishable quantum mechanical paths, illustrated as solid and dashed lines, interfere constructively for bosons (or destructively for fermions). **c**, The resulting joint detection probability (correlation amplitude) of simultaneously finding an atom at each detector is modulated sinusoidally as a function of *d* (black curve). The multiple wave generalization to a regular array of six sources with the same spacing is shown in green. a.u., arbitrary units.

- Initially prepare Rb atoms in the Mott insulator regime - with exactly 1 atom in each lattice site
- Use absorption imaging to detect atoms at pixel 1 and pixel 2
- HBT says you can't distinguish the paths
- Thus bosons interfere constructively

Correlation structure (cont'd)

- Displacement d gives conditional probability of finding two atoms in the same position
- Shows clear correlations at reciprocal lattice site positions in 2 D



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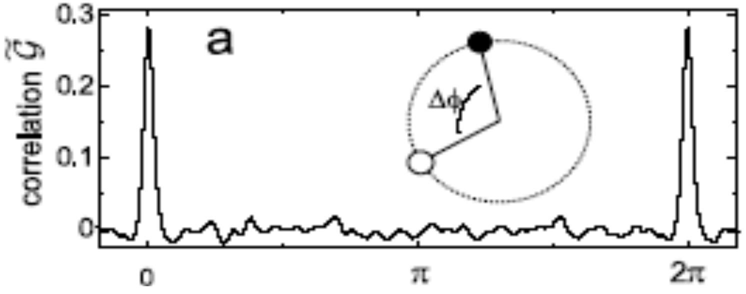
Correlation by dissociation

PRL 94, 110401 (2005) PHYSICAL REVIEW LETTERS week ending 25 MARCH 2005

Probing Pair-Correlated Fermionic Aton

M. Greiner,* C. A. Regal
JILA,‡ National Institute of Standards and Technology and
Department of Physics, University of Co
(Received 24 January 200



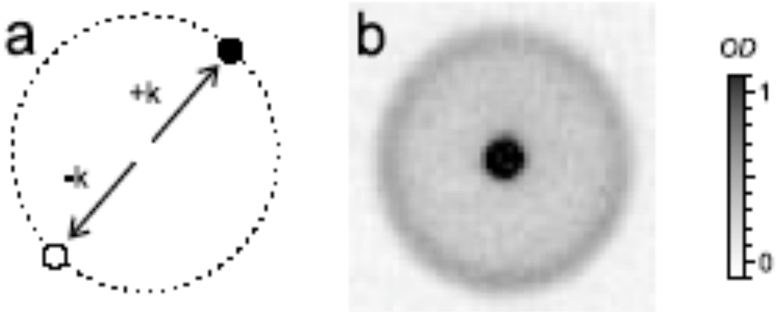
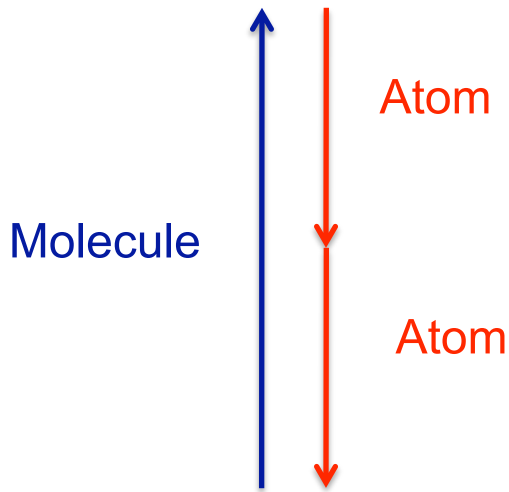


FIG. 3. (a) Atoms with equal but opposite momenta are found on opposite sides of the atom cloud in TOF expansion. (b) This atom absorption image was taken after rf photodissociation of weakly bound molecules using an rf detuning of $\Delta\nu_{rf} =$ 1.3 MHz. The pair-correlated atoms compose an expanding spherical shell, containing approximately 1.3×10^5 atoms per spin state, which appears as a ring around a small cloud of residual atoms and molecules in the 2D absorption image.

Parametric
down conversion



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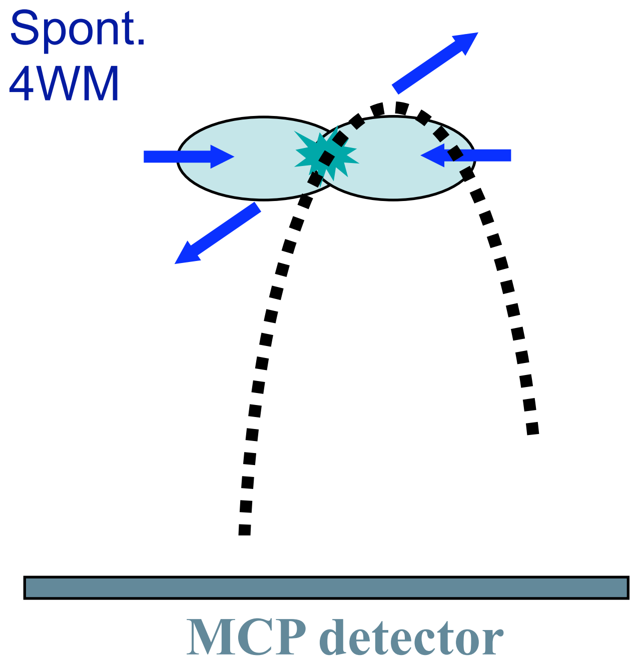
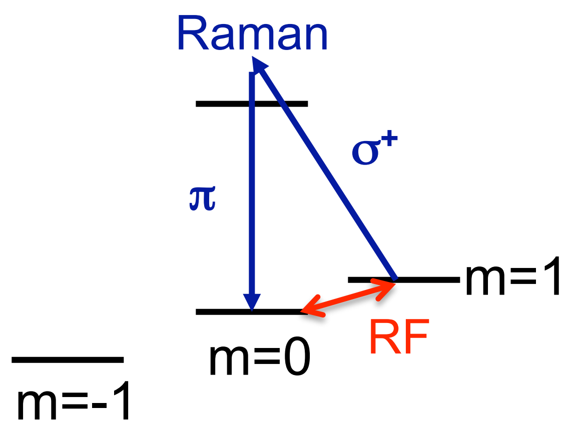
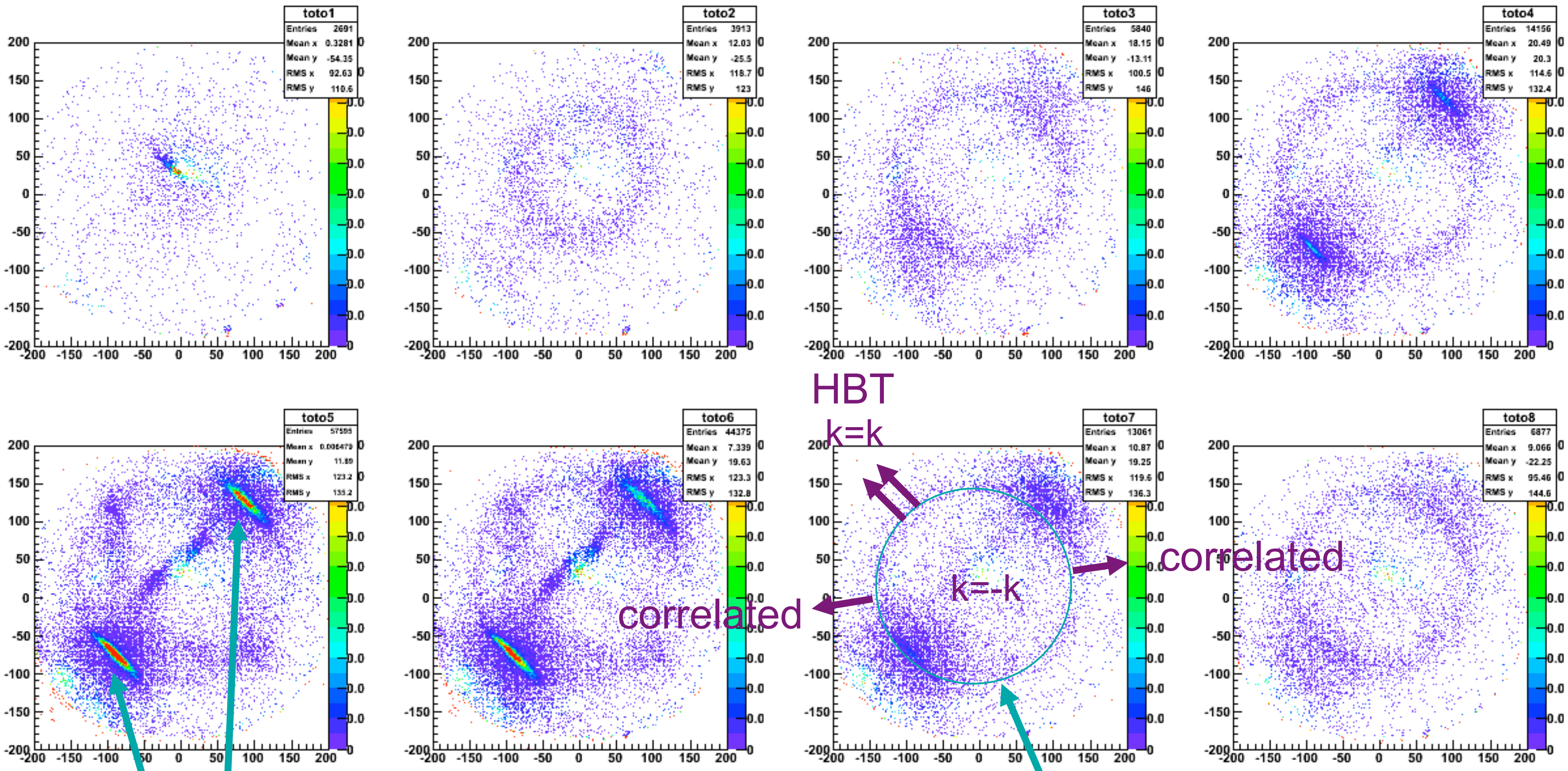
Correlation by collisions

- He* atoms magnetically trapped in $m_f = 1$ state
- Make transition to untrapped $m_f = 0$ state - either Raman (Orsay) or RF (ANU)
- This causes collisions - equivalent to spontaneous 4 wave mixing
- If the coupling is sufficiently strong or prolonged, *stimulated* 4WM occurs





Orsay 4WM correlations



BECs

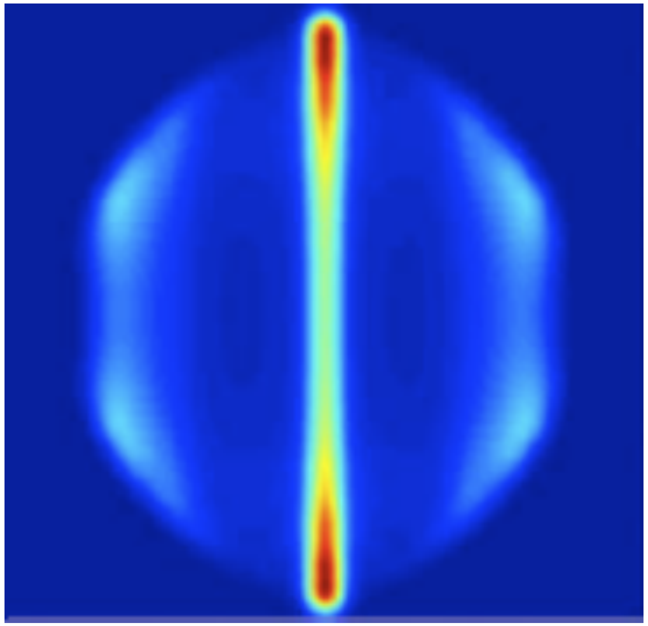
successive slices of atom distribution at the detector

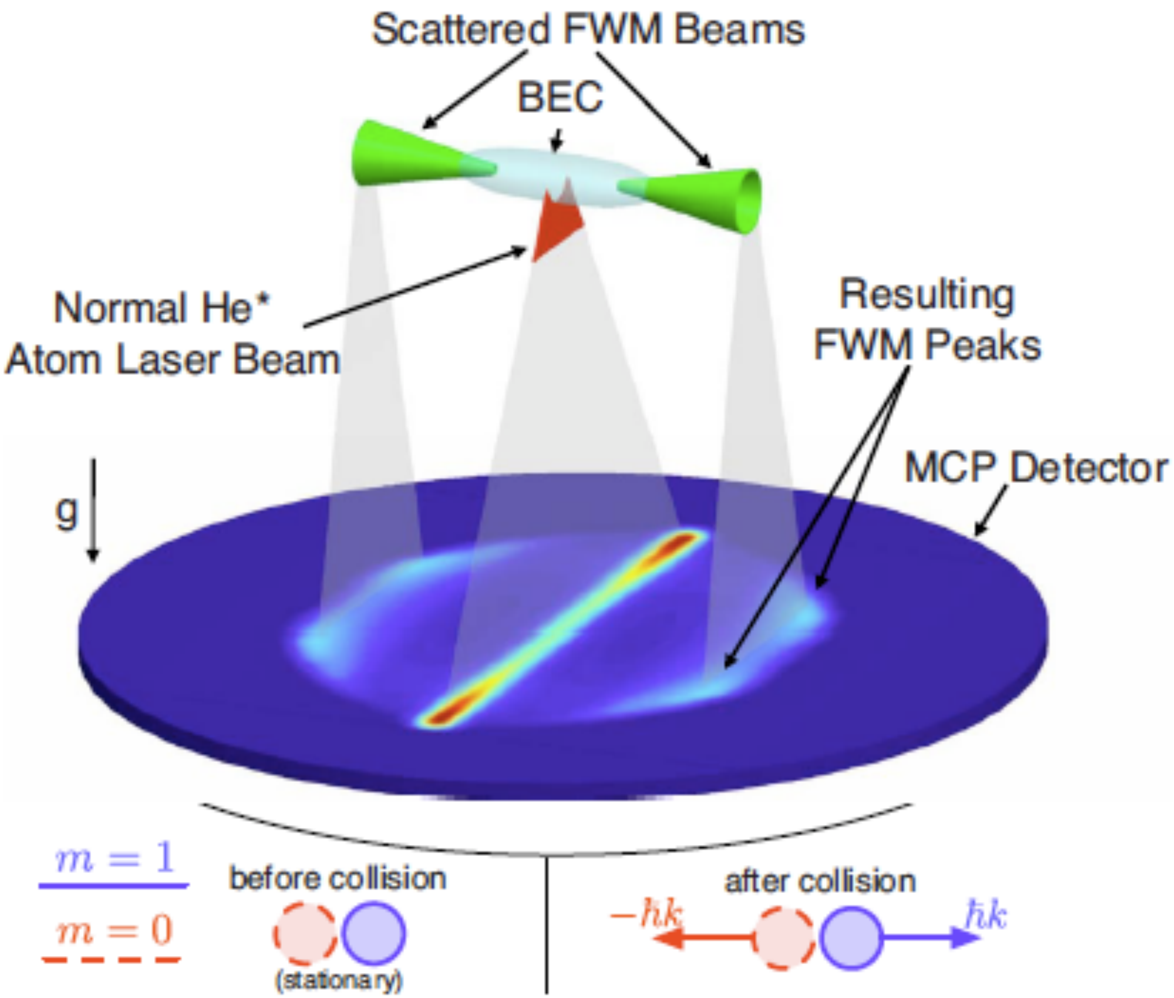
Scattered atoms

Jaskula et al., *PRL* **105**, 109402 (2010)

ANU 4WM correlations

- Scattering of He* in a pairwise process occurs when RF outcoupling
- Prove that these atom pairs are correlated – and perhaps entangled





R.G. Dall et al., *Phys. Rev. A* **79**, 011601 R (2009)

The Future

- Correlations can be used as a diagnostic of the coherence of matter wave devices
- Applications might include squeezed atom interferometry, atom holography
- Correlations can lead to entanglement
- Studies of entanglement enable investigation of fundamental questions in quantum mechanics, such as the Einstein-Podolsky-Rosen paradox

Australian National University

Coherence and correlation experiments at ANU

Next lecture

Baruntse 7129m - Nepal, 1988