Dynamic systems

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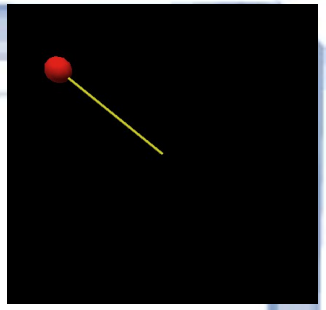
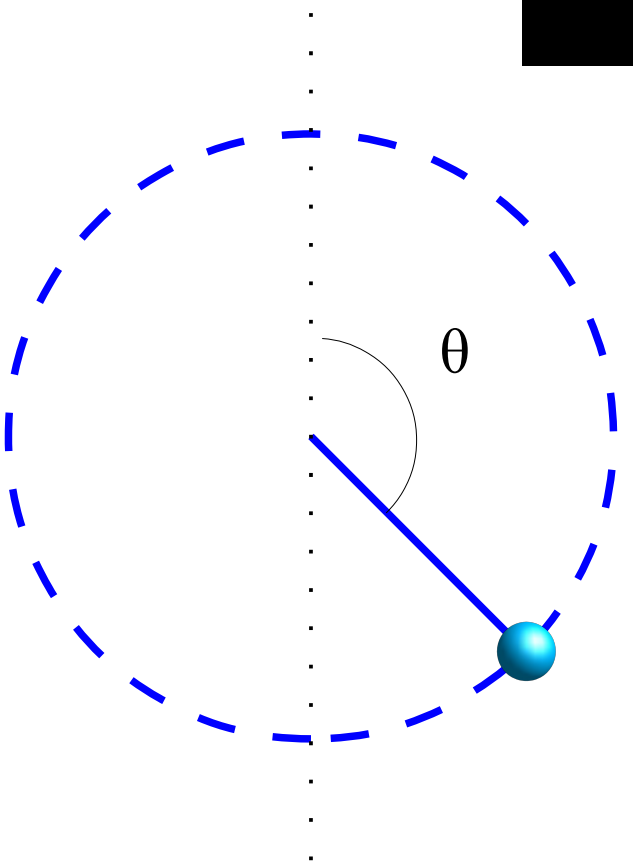
Today Outline

- Delta kicked rotors
- Classical rotors
- Atom optics implementation
- Quantum resonances

Delta kicked rotor



- Ball with mass *m* at the end of a massless stick
- Potential switched
- Non-linear dynamics



$$H = \frac{p_\theta^2}{2I} + V_0 \cos\theta \sum_{n=1}^{N} \delta(t - nT)$$

Theorist units

$$H = \frac{p_\theta^2}{2\mathrm{I}} + V_0 \cos\theta \sum_{n=1}^{N} \delta(t - nT)$$

$$H' = \frac{\rho^2}{2} + \kappa \cos\theta \sum_{n=1}^{N} \delta(\tau - n)$$

$$\rho = p_\theta T / I$$

$$\kappa = V_0 T^2 / I$$

$$\tau = t / T$$

$$H' = H T^2 / I$$

$$\dot{\theta} = \frac{dH'}{d\rho} = \rho$$

$$\dot{\rho} = \frac{-dH'}{d\theta} = -\kappa \sin\theta \sum_{n=1}^{N} \delta(\tau - n)$$

Standard map

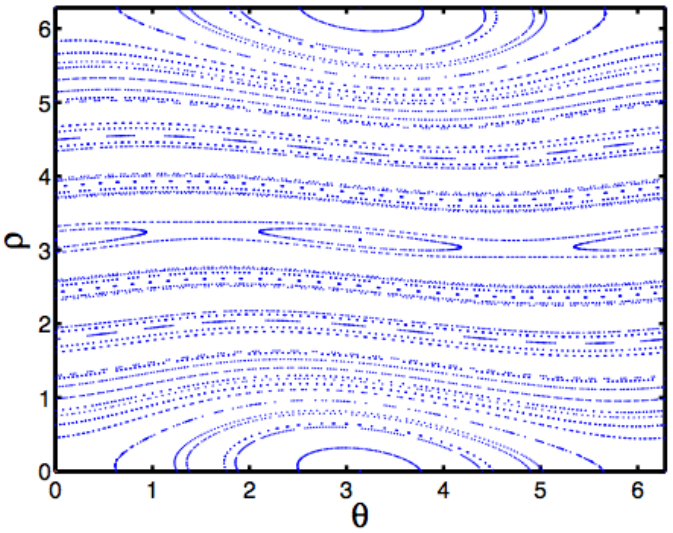
$$\dot{\theta} = \frac{dH'}{d\rho} = \rho$$

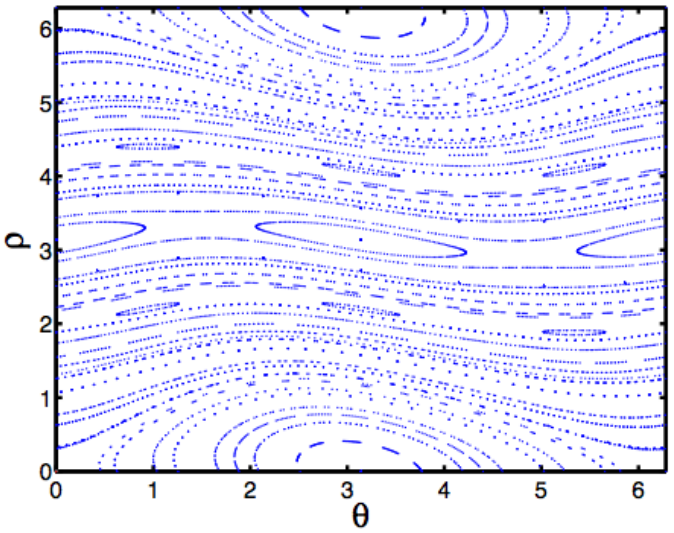
$$\dot{\rho} = \frac{-dH'}{d\theta} = -\kappa \sin\theta \sum_{n=1}^{N} \delta(\tau - n)$$

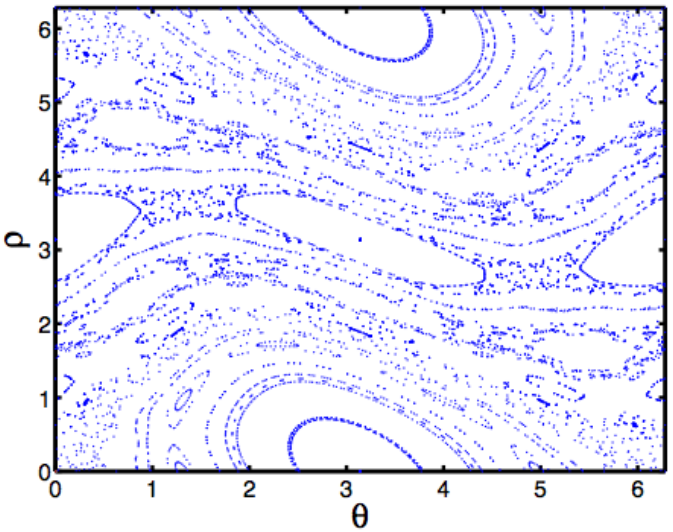
- Each kick the momentum ρ changes by κ sin θ
- Between kicks the angle changes by ρ

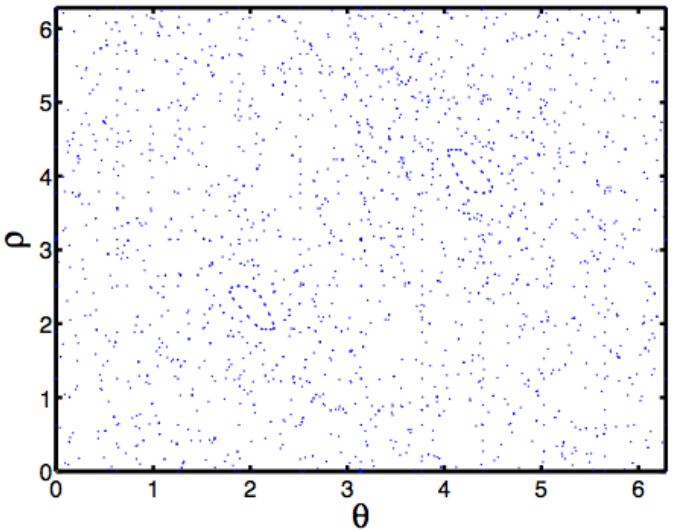
Define the Energy as $\langle \rho^2/2 \rangle$

Phase space









Quantum kicked rotor

- Replace observables by operators

$$H' = \frac{\hat{\rho}^2}{2} + \kappa \cos\hat{\theta} \sum_{n=1}^{N} \delta(\tau - n)$$

- Commutation relations are

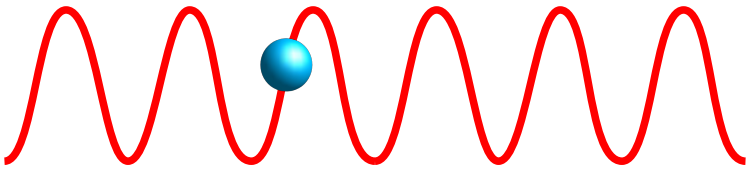
$$[\hat{\theta}, \hat{\rho}] = i\hbar$$

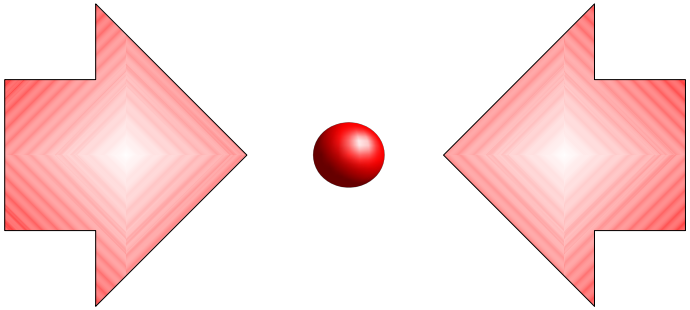
- with

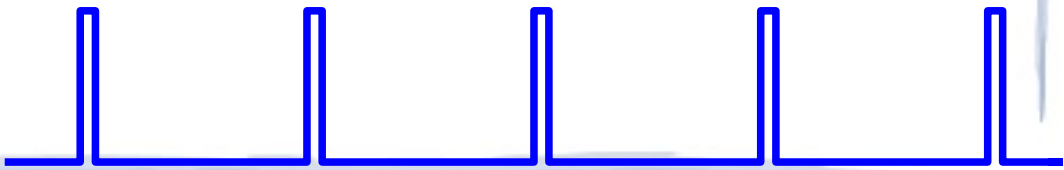
$$\hbar \propto T$$

Atom optics kicked rotor

- Atoms in standing wave
  - Potential varies as cos 2kx
- MOT:
  - Hot, integrate over initial momentum states
  - Large, integrate over initial positions
- BEC:







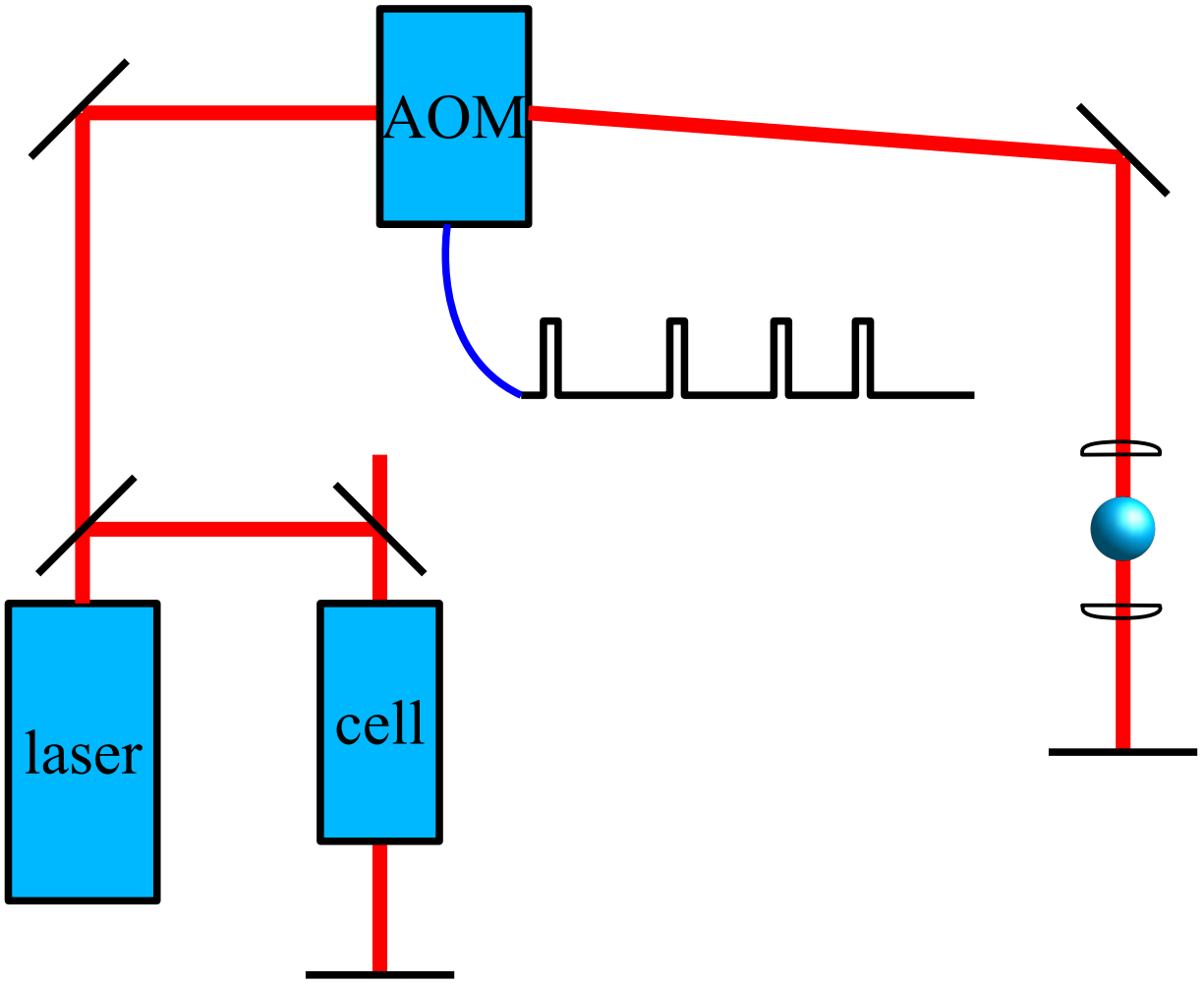
Quantum Kicked Rotor

- In position space:
  - Kick introduces phase shift $\Delta\varphi=\Omega\sin 2kx$
- In momentum space:
  - Delta kick transforms delta function into comb
  - Free evolution accumulates linear phase shift $\varphi=ct$
  - Talbot effect:
    - After free propagation $\varphi=2n\pi$
    - Wave function the same
    - Constructive interference

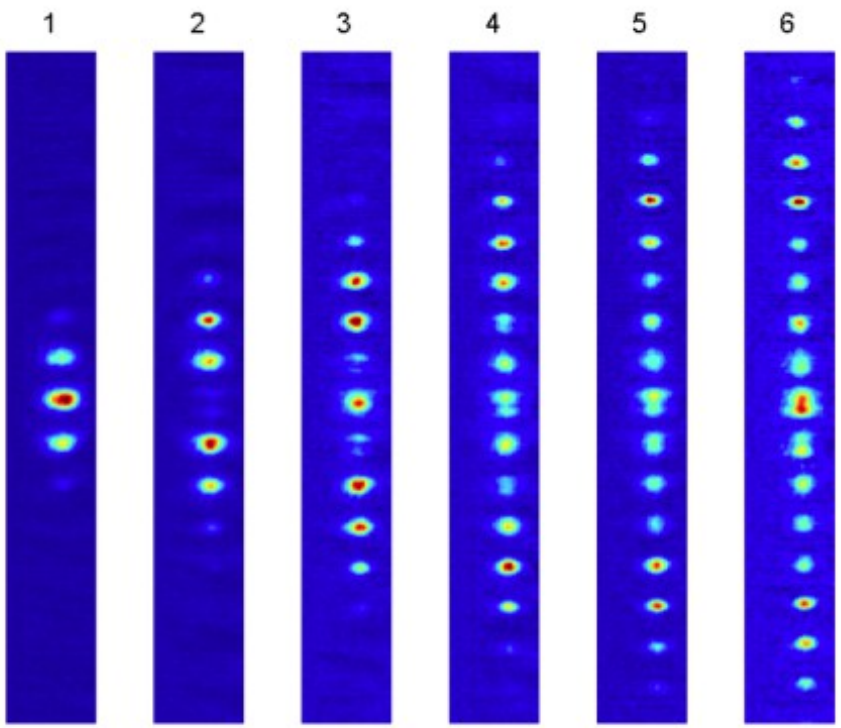
$$\hbar k = 8\,\omega_R T$$

*Duffy et al, Physical Review E **70**, 056206 (2004)*

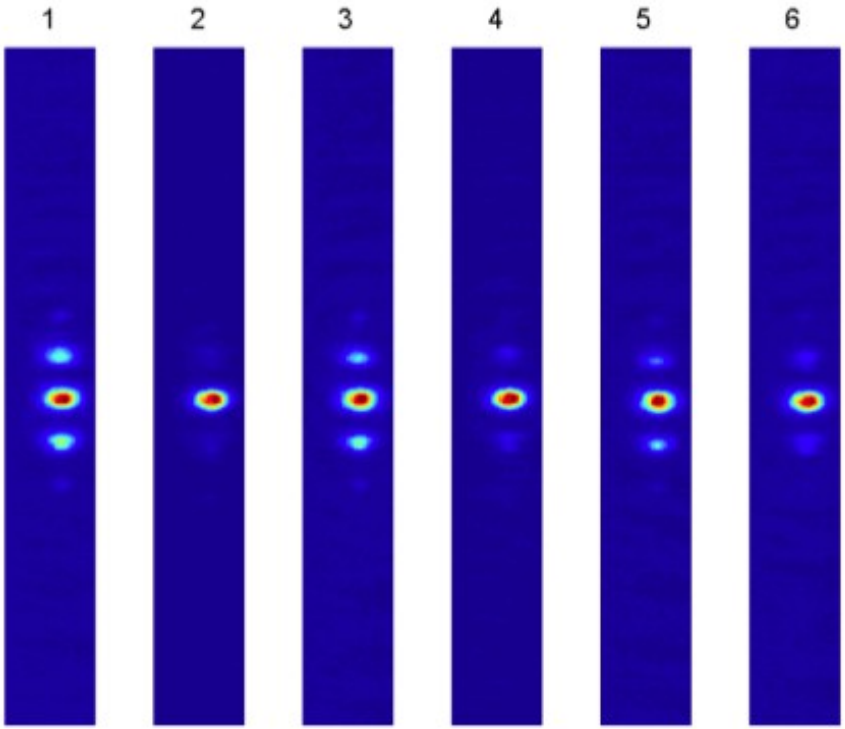
Kicked rotor setup



Talbot effect



$$T = \frac{\pi}{2\,\omega_r}$$



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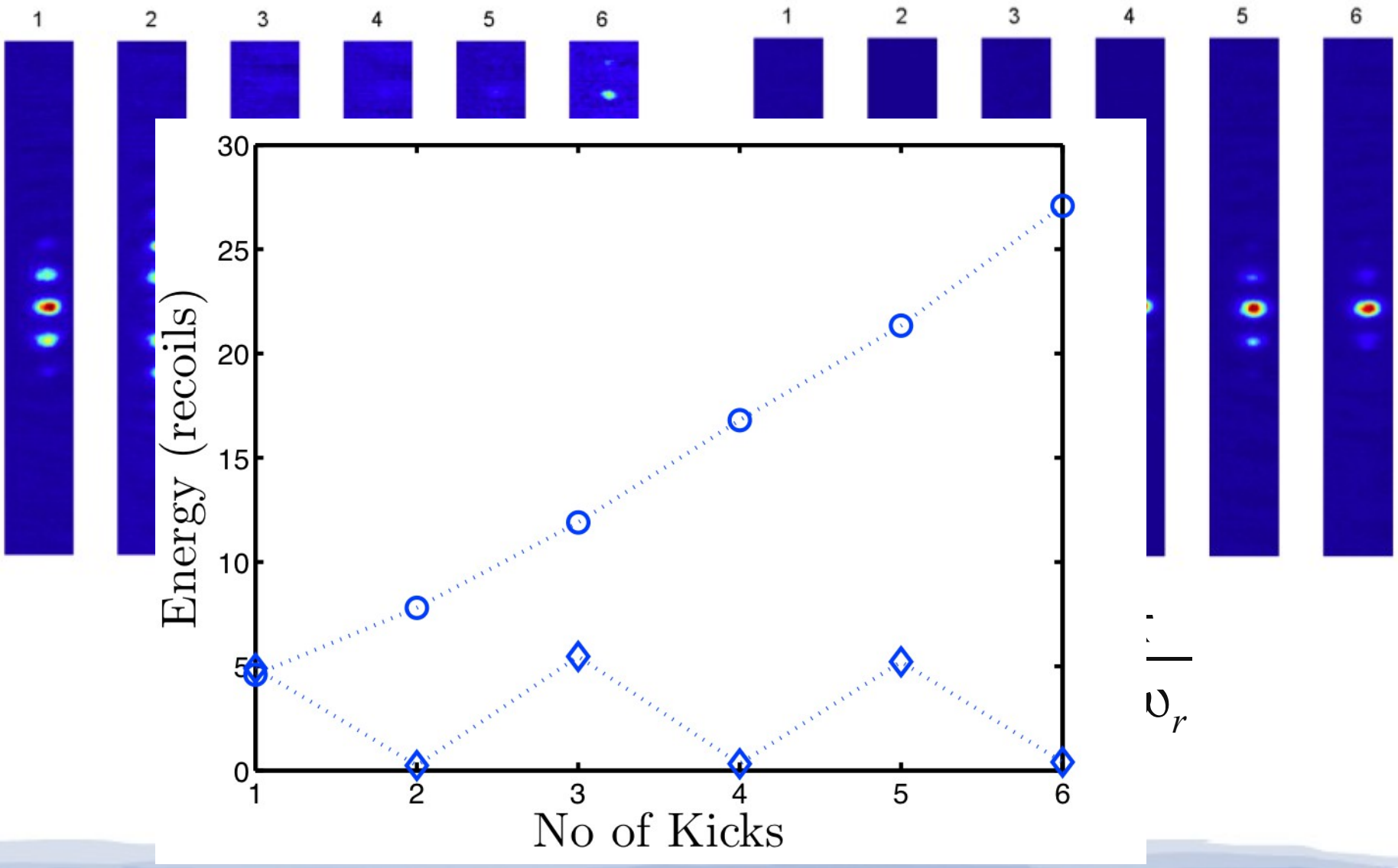

Simulations

- Start with wave packet

$$\psi = C \exp\left(\frac{-x^2}{2\sigma^2}\right) \exp\left(ik_a x\right)$$

- Time evolution

$$H = \frac{p^2}{2\mathrm{m}} + V_0 \cos\left(2\mathrm{k}_l x\right) \sum_n f\left(t - nT\right)$$

- Integrate k_a over momentum distribution BEC

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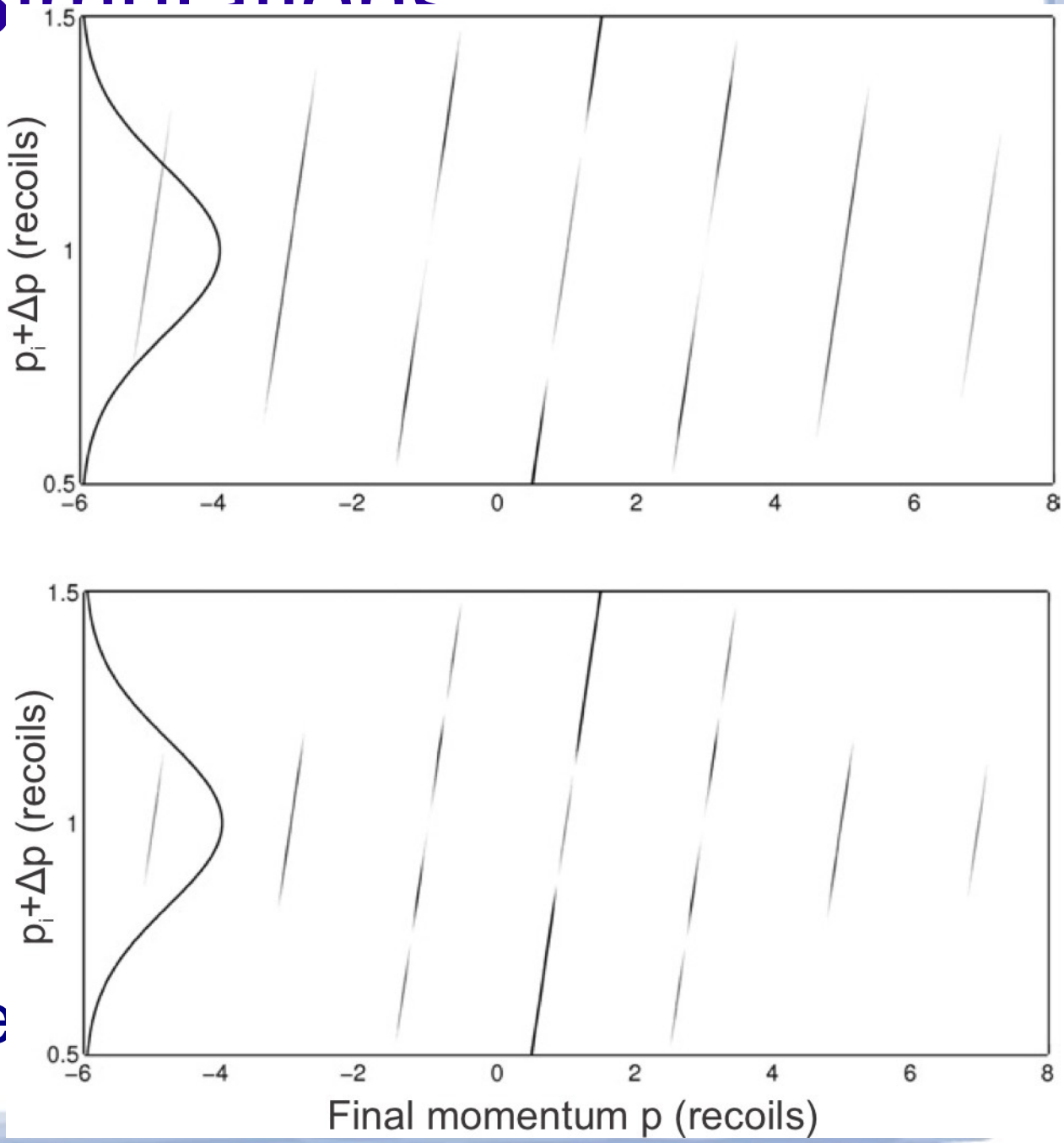
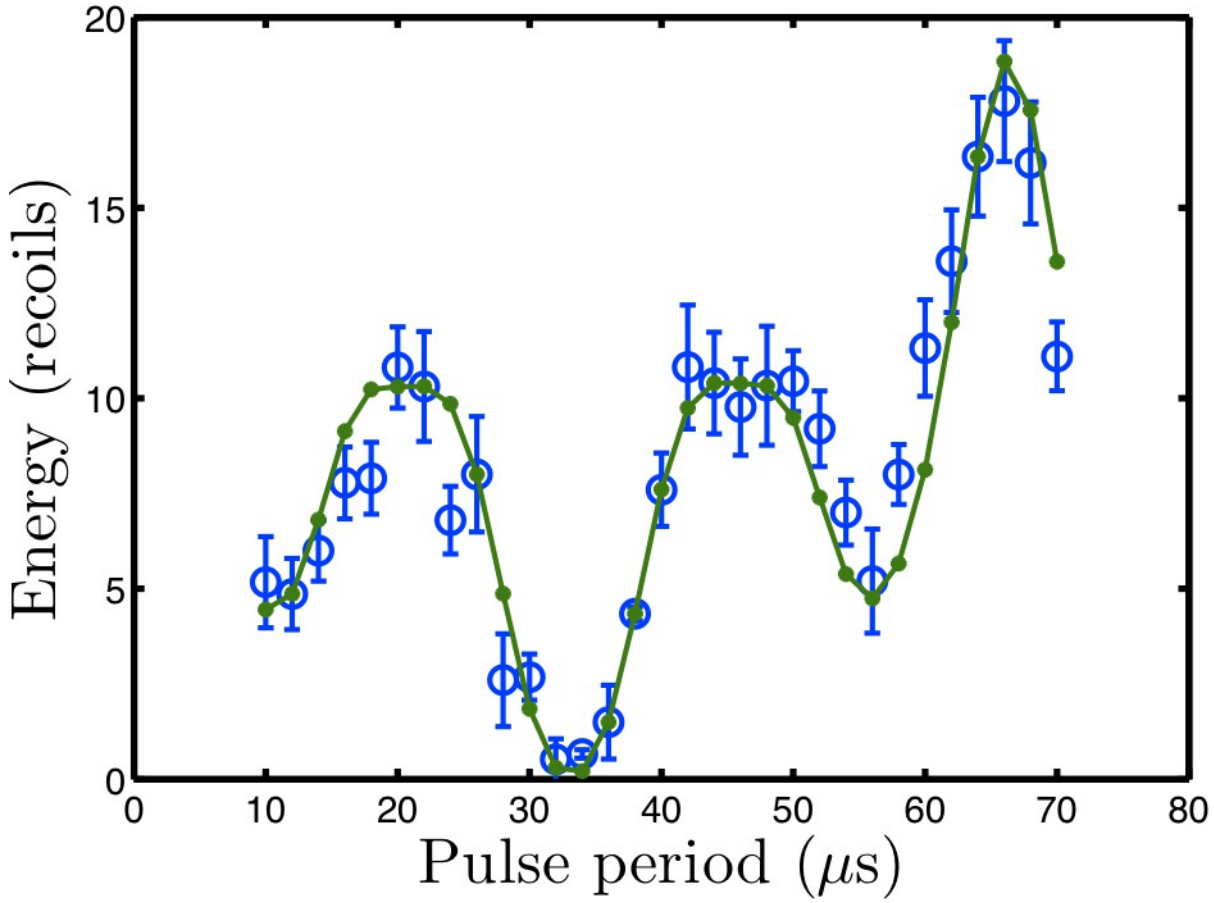


Varying period



Talbot effect conclusion

- Addition or cancellation on resonance
- In between not so clear