Quantum Entanglement

Victorian Summer School

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Outline

1. **Non-locality and quantum mechanics**
   - Einstein's (EPR) spooky action at a distance 1935
   - Schrodinger's cat 1935
   - Bell's theorem 1965
     - - Bell and EPR experiments
   - GHZ's extreme multiparticle quantum nonlocality
2. **Introduce formalism of entanglement**
   - Density operator – mixed states
   - Inseparability of density matrix
   - Pauli spin examples
   - Werner states
   - Peres PPT criterion and concurrence
   - Quadrature squeezing and spin squeezing
   - CV Variance and spin squeezing criteria for entanglement
3. **Applications**
   - Quantum cryptography and quantum teleportation

Outline: Lectures 1-2

1. The beginnings of quantum entanglement

Non-locality, reality and quantum mechanics

EINSTEIN'S (EPR) SPOOKY ACTION AT A DISTANCE 1935

Schrodinger's cat 1935 – introducing entanglement

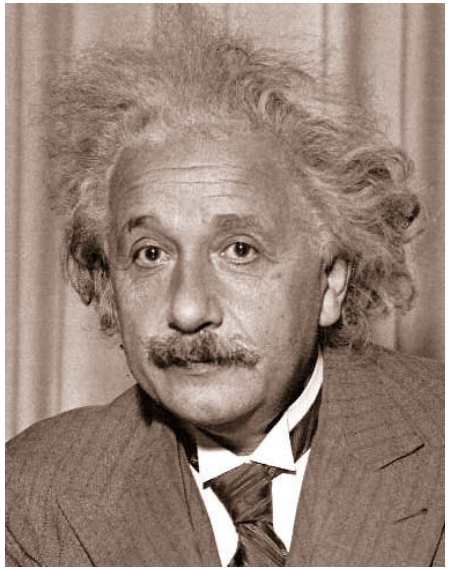
Bell's theorem 1965

– experiments

Greenberger-Horne-Zeilinger's (GHZ) theorem

extreme multiparticle quantum nonlocality 1990's

EPR paradox 1935 : Physical Review



- Einstein, Podolsky and Rosen

- Einstein was unhappy about quantum mechanics

- Believed it was correct but *incomplete:*

  formulated a powerful argument in favour of this

Quantum mechanics and reality-a problem?

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle + |x'\rangle\right)$$

•Principle of superposition

•Not one or the other until measured: *Dirac*

•Cannot view properties as existing until they are measured?

•Indeterminacy in predetermined value of x: wave function conveys a fundamental uncertainty? Measurement apparatus interacts with system?

(???? But there is more than this, as Einstein showed)

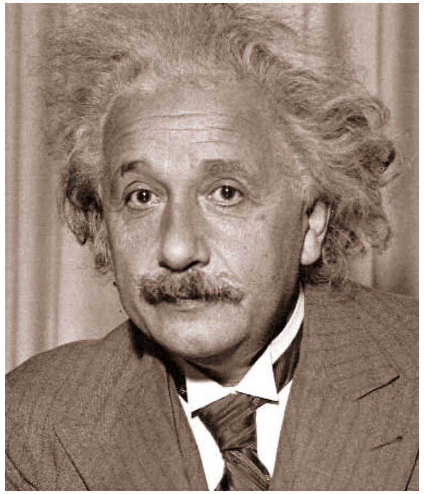
Einstein-Podolsky-Rosen argument (EPR paradox)

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

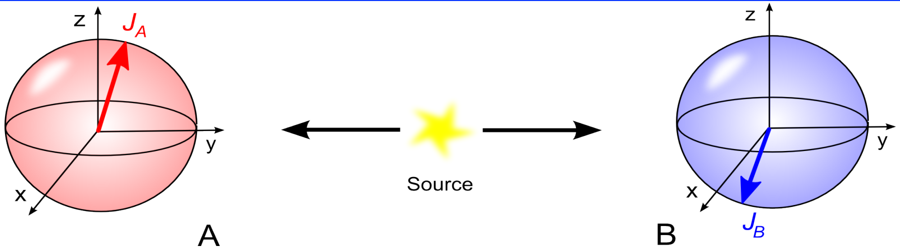
A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

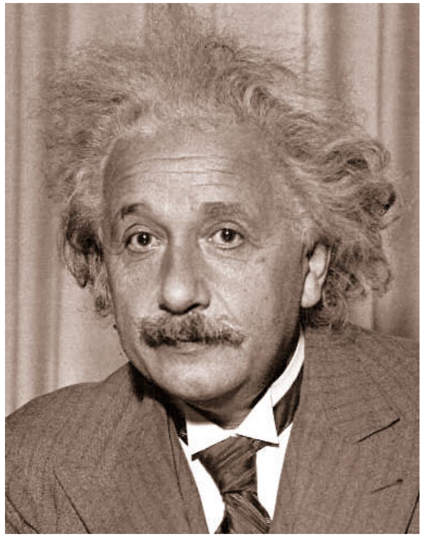
In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.



Einstein-Podolsky-Rosen argument (EPR paradox)

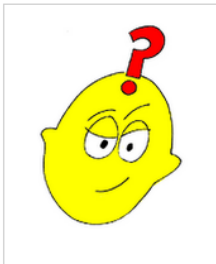


Bohm's version



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B\right)$$

Four steps to EPR's argument: section1.2 notes
STEP 1:
For singlet state, ALL spin components are correlated



Exercise 1: Show that all spin components are correlated.

$$J_\theta = J_Z\cos\theta + J_X\sin\theta$$

Understanding EPR correlation: Dr Bertlmann's socks

Words of John Bell to explain EPR correlation:

Dr. Bertlmann likes to wear two socks of different colours. Which colour he will have on a given foot on a given day is quite unpredictable

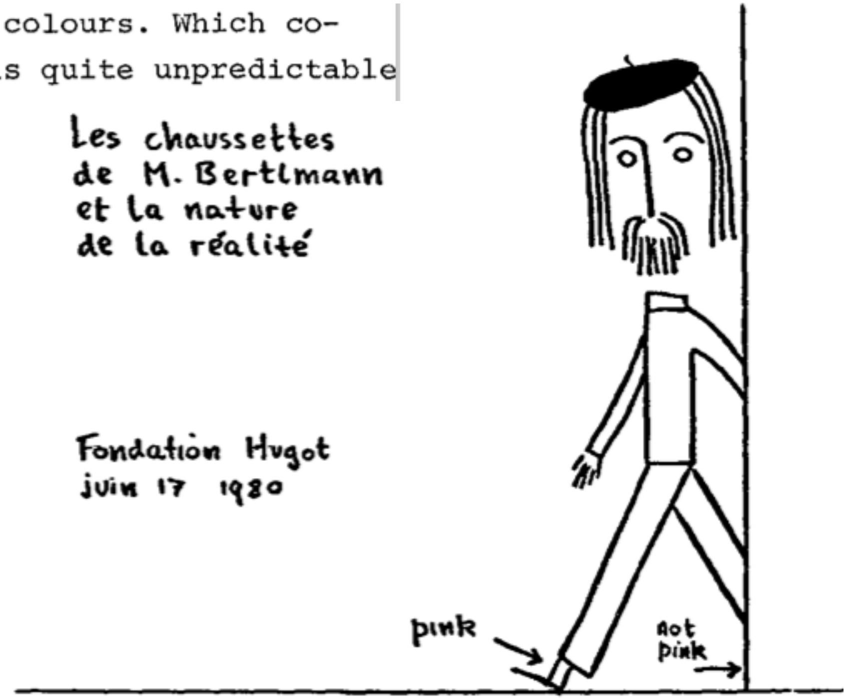


Fig. 1.

But when you see (Fig. 1) that the first sock is pink you can be already sure that the second sock will not be pink. Observation of the first, and experience of Bertlmann, gives immediate information about the second. There is no accounting for tastes, but apart from that there is no mystery here. And is not the EPR business just the same ?

Bertlmann socks and correlation

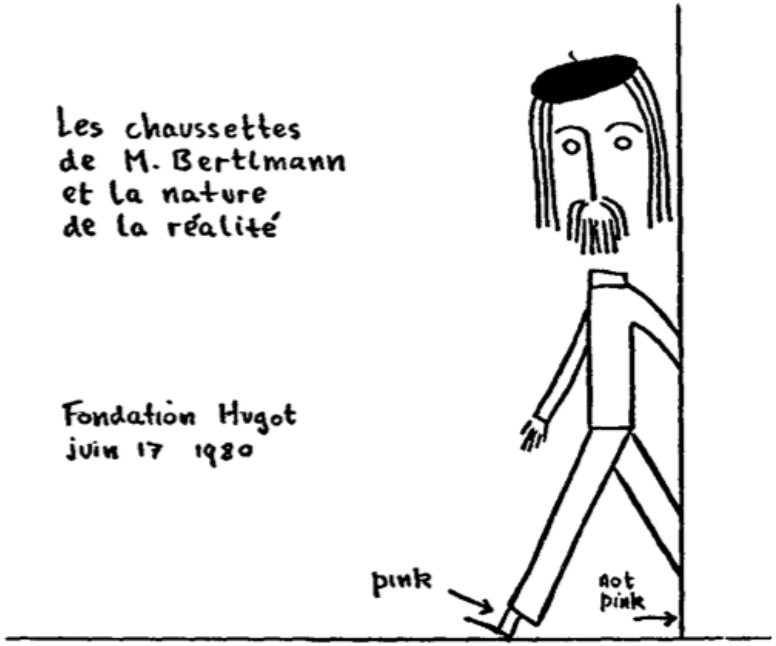


Fig. 1.

Question: Was the second sock "not pink" *before* the observer saw the first sock?

Or

Did the action of observing the first sock cause the second sock to be "not pink"?

Einstein-Podolsky-Rosen (EPR) argument:
“Elements of reality”

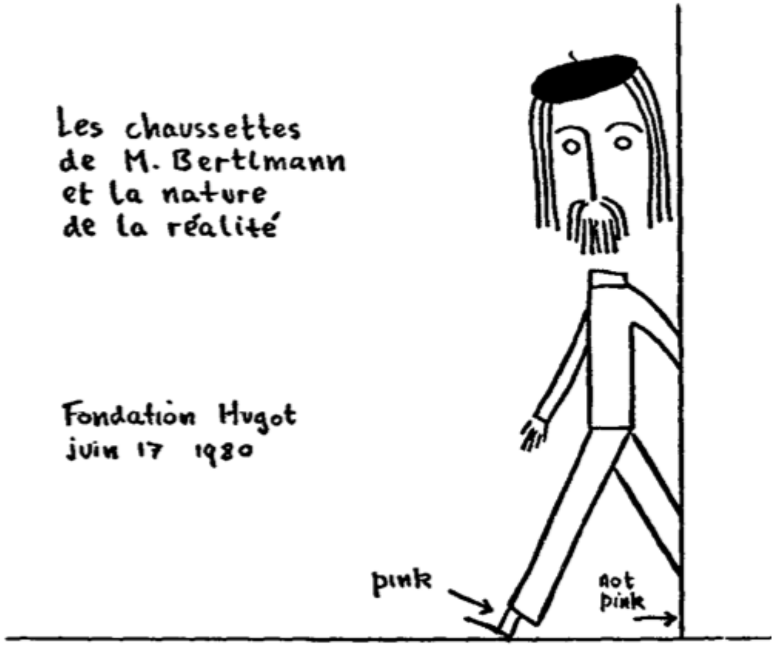


Fig. 1.

The colour of the second sock was predetermined – why?

because we can predict the colour by measurement on another spatially separated system

EPR call the colour of the second sock an “element of reality” of the system

Answer: The second sock had its colour ***before*** the observer saw the first sock

Yes, because of past interactions, there is a correlation.

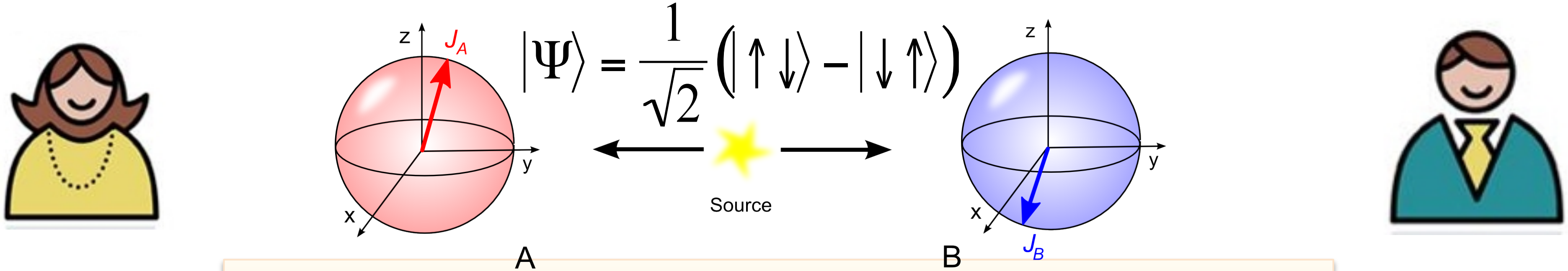
BUT The action of observing the first sock does not cause the colour of the second sock to change

Step (2) EPR make the argument stronger: Introduce Alice and Bob

Two spatially separated measurements
Alice looks at one sock, Bob the other

- Suppose Alice measures one sock to be pink
- She predicts with certainty that Bob will measure his sock to be "not pink"
- Her measurement did not cause Bob's sock to change colour
- Einstein said, that would be like "spooky action at a distance"

Step (2) now look at spin and EPR's "locality"



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

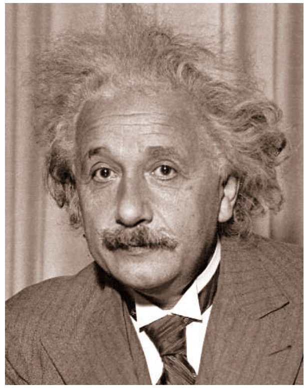
Spin measurement events are spacelike separated!
Alice cannot signal her outcome to Bob

- Suppose Alice measures one spin to be "up"
- She knows Bob will measure his spin to be "down"
- *Assume no "spooky action at a distance" – "locality"*
  Alice's measurement does not change Bob's system
- *Then Bob's spin (like the colour of the sock) is predetermined*
- *Bob's spin particle has a definite value (hmmm?)-*
  an "Element of Reality" or "hidden variable"

EPR argument step 2 : EPR are rigorous Assume premise of local realism

EPR's words PRA,1935

> hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system.

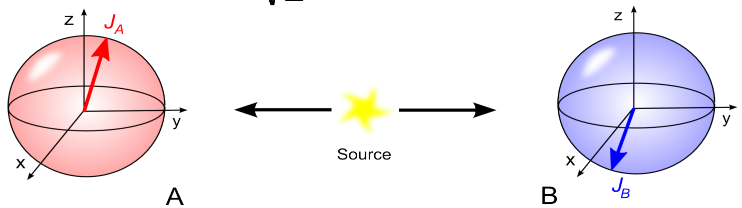


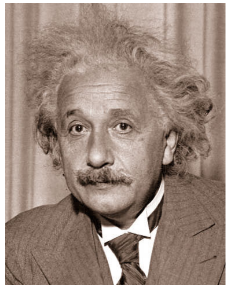
- EPR introduce *local realism*
- Measurement by Alice doesn't change Bob's system *"locality"*
- *If the result of measurement can be predicted with absolute certainty, without disturbing the system, then that result was a predetermined property of the system- "realism"*
- *Local realism implies*

    Bob's z-spin component is predetermined (hidden variable)

But ALL spin components are correlated! (step 3)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

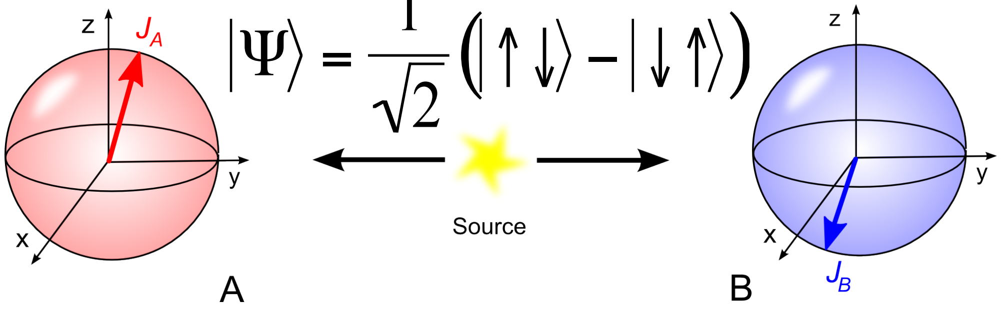




- EPR's argument: assume local realism
- Alice and Bob's X- spin components are *also* perfectly correlated
- So, carry EPR argument through again- and again
- Conclude: *All* of Bob's spin components are *completely predetermined* - hidden variables for each exist
- All his spins at any given time are *either* "up" *or* "down"

Delayed measurement choice

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

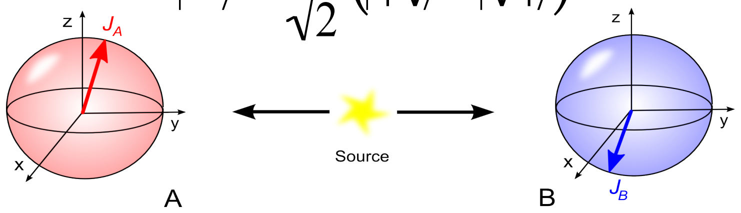


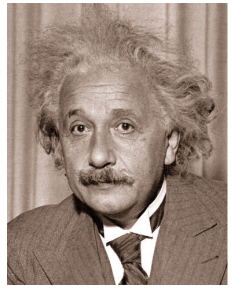
Spin measurement events are spacelike separated!
Alice cannot signal her outcome to Bob

- Remember, Alice can delay her measurement choice
  until the particles are no longer interacting and are in flight
- She can predict with certainty any of his spin components
  without disturbing his system (assuming Local Realism)
- *ALL of Bob's spin components are predetermined*

Quantum mechanics is incomplete (step 4)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$





EPR's argument : assume local realism

- Conclude: *All* of Bob's spin components are *completely predetermined -* hidden variables ("elements of reality") for each exist

- BUT this contradicts any quantum description for Bob's system! Why?

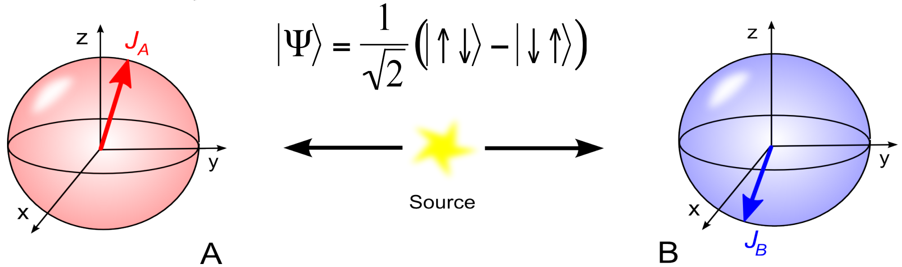
- EPR conclude: **Quantum mechanics is incomplete!**

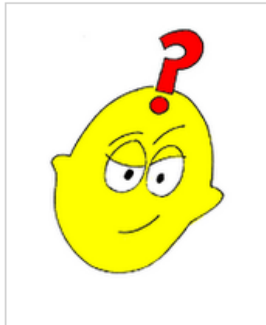
EPR's hopes of a local hidden variable (LHV) theory

While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.

EPR argument from today's perspective

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$





EPR's argument: **assumed local realism**

- Existence of EPR correlated states implies
  Quantum mechanics is not complete!

- The argument reveals the inconsistency between *premise of local realism* and *completeness of quantum mechanics*

- Later work of **BELL** showed **there can be no (local realistic) completion**

- Bell's theorem indicates either **local realism** or **quantum mechanics** is wrong! - we will take a look .....but first

Outline: Lecture 1

1. **The beginnings of quantum entanglement**

Non-locality, reality and quantum mechanics

Einstein's (EPR) spooky action at a distance 1935

SCHRODINGER'S CAT 1935-
ENTANGLEMENT

Bell's theorem 1965 - experiments

Greenberger-Horne-Zeilinger's (GHZ) theorem

extreme multiparticle quantum
nonlocality 1990's

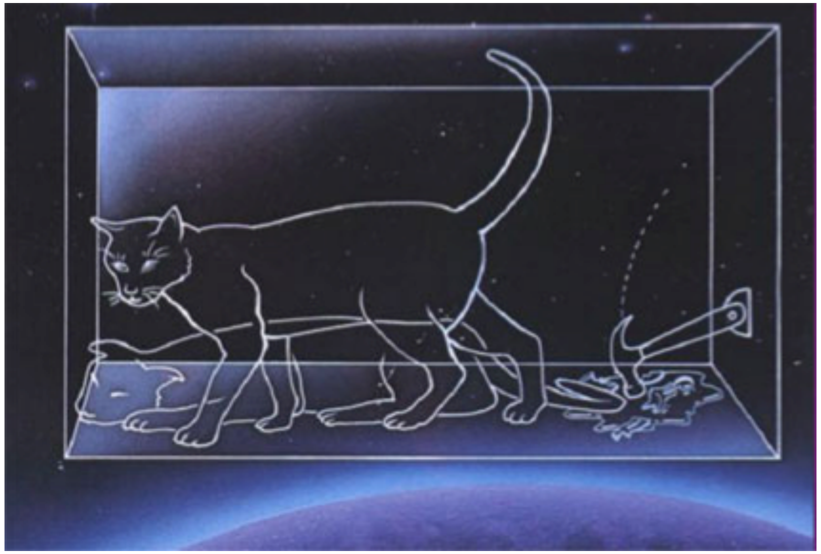
Schrodinger's response to EPR
- "entangled" states

Quantum mechanics and reality-a problem?

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle + |x'\rangle\right)$$

- Principle of superposition
- Not one or the other until measured: *Dirac*
- Cannot view things as existing until they are measured?

Schrodinger's cat: quantum mechanics and a macroscopic "unreality"?



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|dead\rangle + |alive\rangle\right)$$

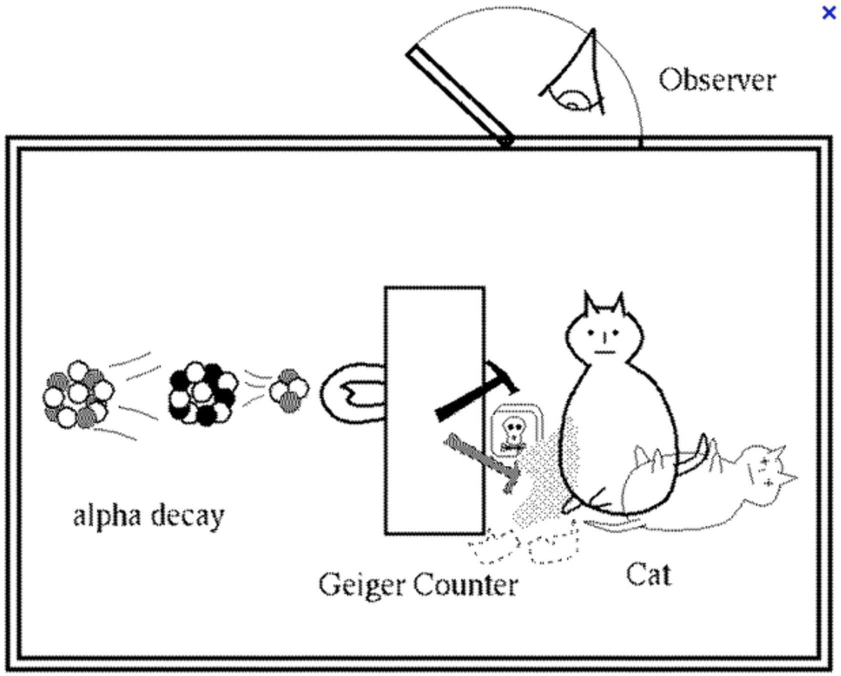
- Is the moon there when nobody looks?

- Quantum mechanics predicts macroscopic superpositions

- How does "not one or the other until measured" work for macroscopic superpositions? *Do we say "dead and alive"?*

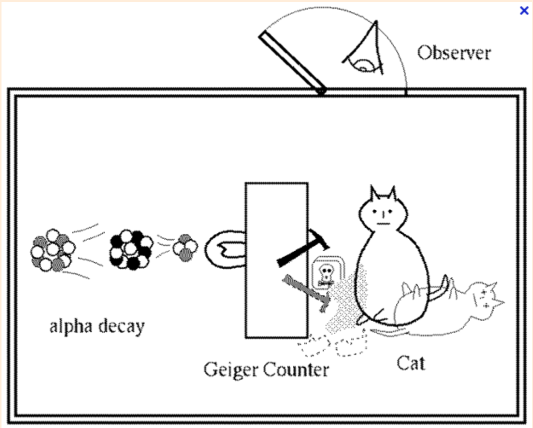
Schrodinger's cat - how is it created?

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$



- Microscopic decay - superposition
- Interaction with measurement device that releases poison to kill cat if result is "down"
- Cat itself ends up in a superposition of dead / alive states

Schrodinger's cat- how is it created?



Interaction of micro- system with the detector described by Hamiltonian H

If the initial state is $|\uparrow\rangle$ and that of detector is $|0\rangle$

then the final combined state is $|\uparrow \mathit{needle}\rangle|\uparrow\rangle$

If initial state is $|\downarrow\rangle$, then final state is $|\downarrow \mathit{needle}\rangle|\downarrow\rangle$

If initial state is the spin superposition, so that the overall initial state is

$$|\Psi\rangle_{initial} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)|0\rangle$$

Then the final state is (Schrodinger equation is linear)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow \mathit{needle}\rangle|\uparrow\rangle + |\downarrow \mathit{needle}\rangle|\downarrow\rangle\right)$$

Then consider the interaction with the detector and the cat, similarly, we get

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\mathit{alive}\rangle|\uparrow\rangle + |\mathit{dead}\rangle|\downarrow\rangle\right)$$

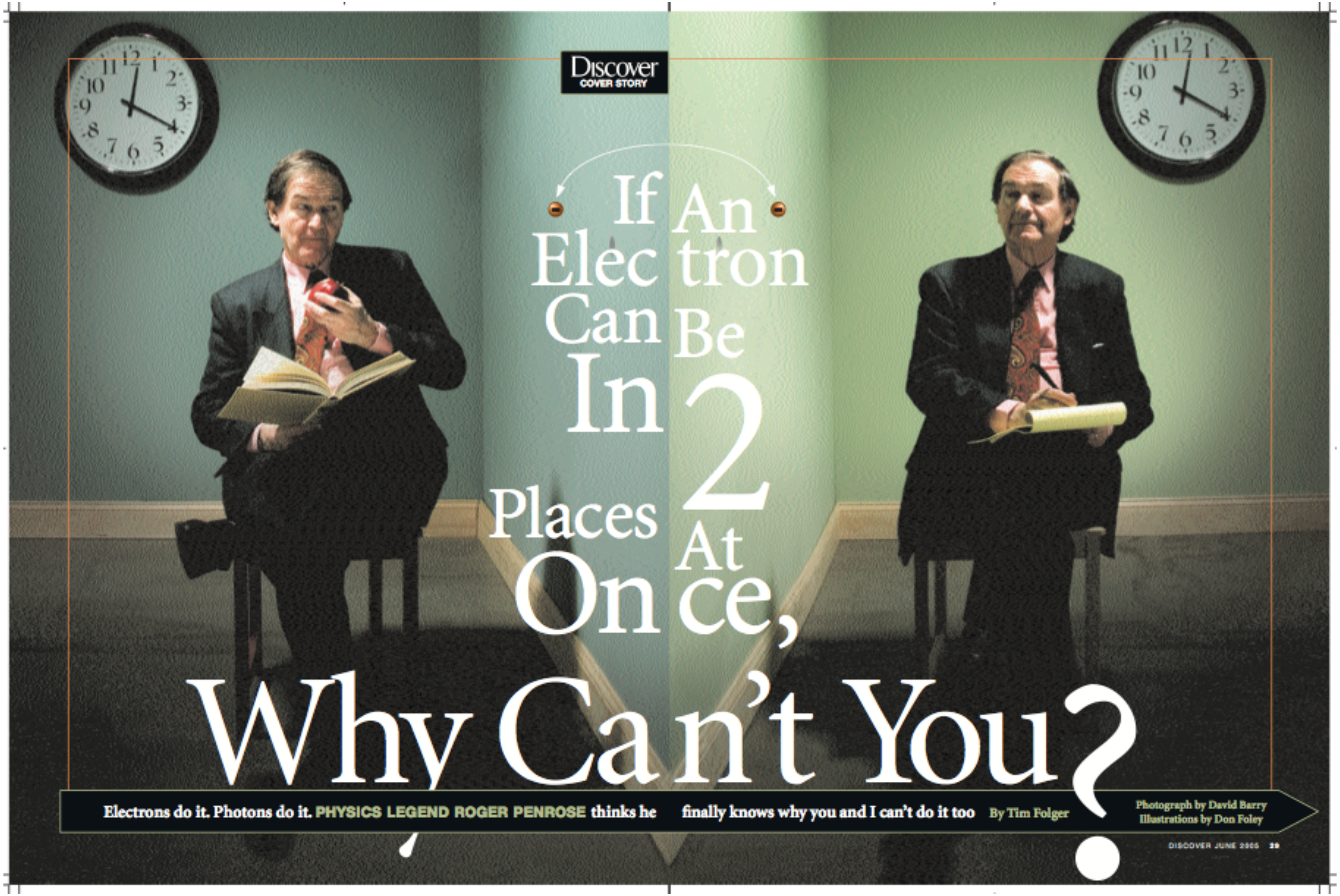
Schrodinger cat : Entanglement

Spatial separation

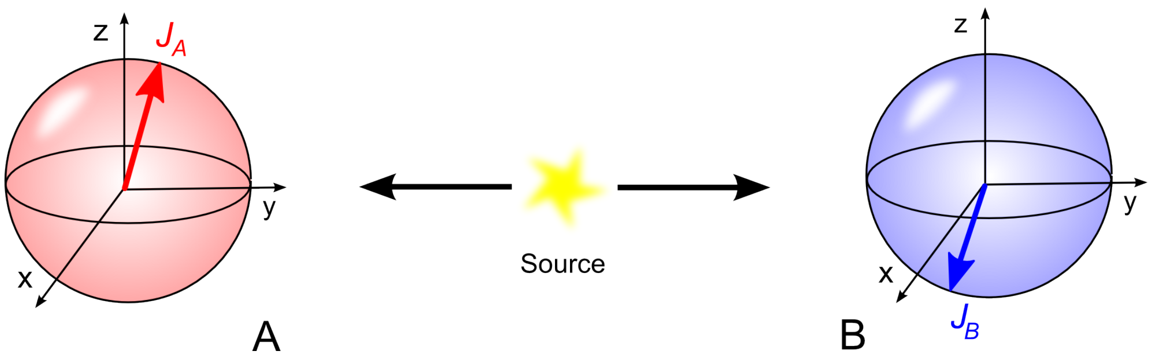
$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|alive\rangle|\uparrow\rangle + |dead\rangle|\downarrow\rangle\right)$$

- *Entangled* states
- Action of observer Alice *reduces* state of Bob
- Unless we accept a predetermined underlying correlation between A and B, this seems like spooky *action at a distance ("steering")*
- So the cat was dead *or* alive *before* measured *by Alice?*
- *If so, this isn't in the quantum description- hidden variables?*

Alternative theories for massive objects: Penrose, Diosi...



So - Schrodinger's Entangled States

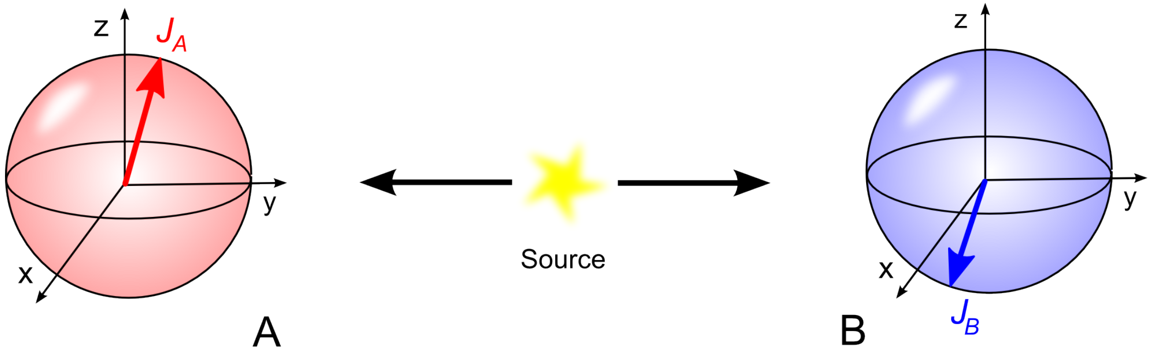


$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$$

A **(pure) entangled** state is one that cannot be written in any factorised form i.e.

$$|\phi\rangle \neq |\psi_A\rangle|\psi_B\rangle$$

Separable Quantum States



$$\rho = \sum_R P_R |\Phi_A\rangle|\Phi_B\rangle\langle\Phi_A|\langle\Phi_B|$$

P_R probability

ρ density operator

- Separable states are mixtures of factorised states
  "unentangled" states

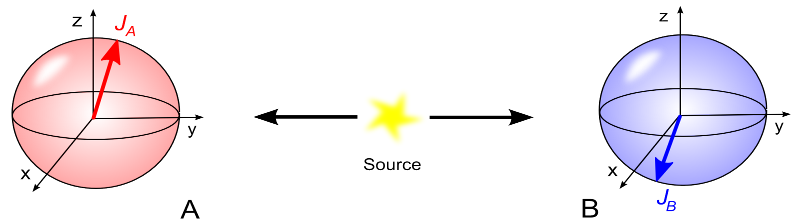
- Local density operators incorporate uncertainty principle
  $\Longrightarrow$ *local fuzziness*

- Reduces correlations between A and B - *can't get EPR*

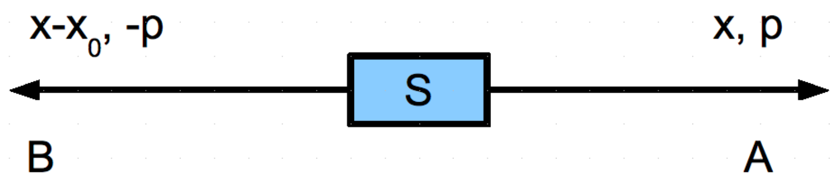
Entangled states: let's look at them

Entangled states are non-separable: 2 classic examples

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$$



$$\delta(x_A - x_B)\delta(p_A + p_B)$$



- Entangled states - greater correlation than separable states for *both conjugate* (non-commuting) observables
- Alice can predict Bob's x and p with no fuzziness - despite uncertainty relation!
- Both conditional variances are zero: $\Delta^2(x_B \mid x_A) \to 0$

  $\Delta^2(p_B \mid p_A) \to 0$

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EPR's hopes of a local hidden variable (LHV) theory

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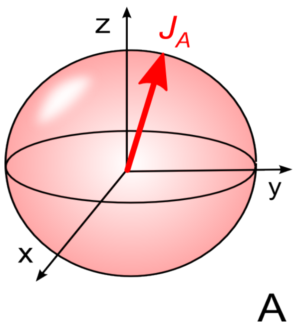
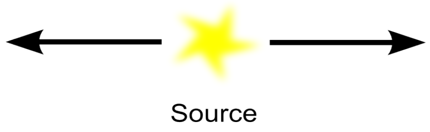
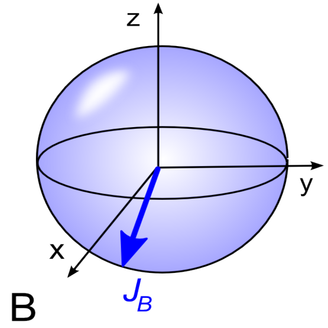
Bell's theorem **destroys** Einstein's hopes for local hidden variables 1965-6



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle$$



Pauli spins +1, -1

Measure Alice and Bob's spin product for different angle settings:

Construct

$$B = E(\theta,\phi) - E(\theta',\phi) + E(\theta,\phi') + E(\theta',\phi')$$

***IF** we assign local hidden variables λ to each spin:*

$$\Rightarrow |B| \le 2$$

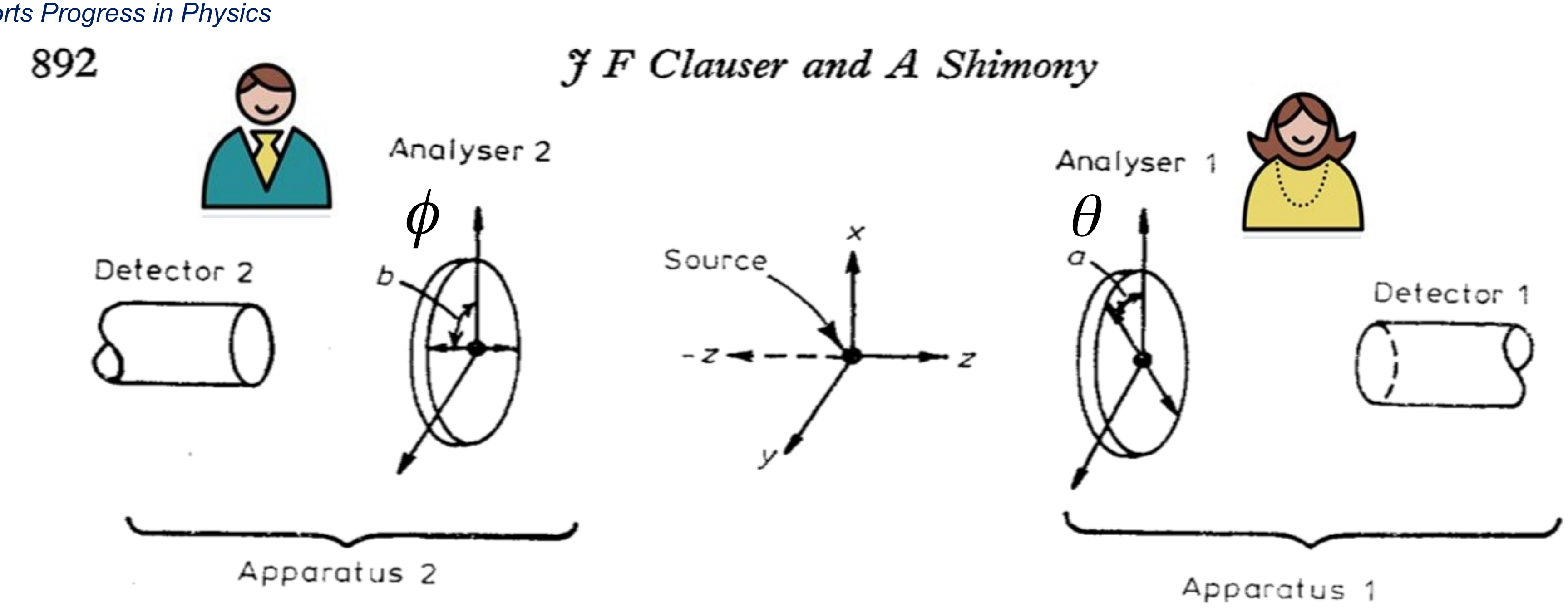
***IF** we use quantum mechanics*

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$\Rightarrow B = 2\sqrt{2}$$

BELL'S THEOREM: let's take a closer look
The local hidden variable (LR) prediction

From a review article 1970's
Reports Progress in Physics

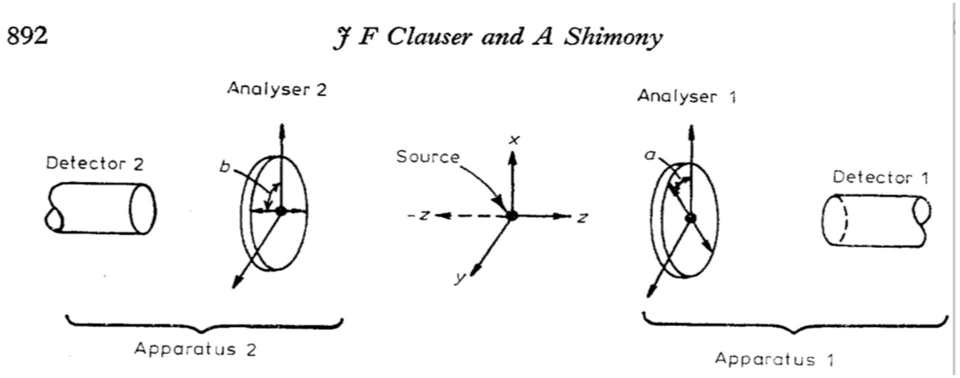


$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle$$

- Consider two (noncompatible) settings per site:
- Alice selects *either* θ *or* θ', Bob selects *either* ϕ *or* ϕ'

(note a=θ, b=ϕ in diagram)

Bell's theorem: let's take a closer look
The local hidden variable (LR) prediction



Recall: There is perfect correlation between Alice and Bob's spin θ components, and spin ϕ components

Then suppose EPR are right ie local realism is right, and there exist hidden parameters λ_θ^k to describe the spins for Bob ($k = B$) and for Alice ($k = A$). For simplicity, we can use Pauli spins, so the outcome for "spin" measurement is $+1$ or -1.

Then the value of λ_θ^A and λ_ϕ^B is *always either* $+1$ or -1.

The **LOCAL HIDDEN VARIABLE** (Local Realism) prediction

Now consider the following construction for a two-setting experiment: ie two angles at each location

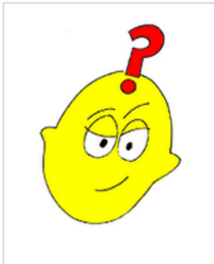
$$B = E(\theta,\phi) - E(\theta',\phi) + E(\theta,\phi') + E(\theta',\phi')$$

Exercise 3:

Constuct a Table of *all* possibilities for the LR (LHV) prediction. If spins predetermined:

$$\begin{aligned} B &= \langle\lambda_\theta\lambda_\phi\rangle - \langle\lambda_{\theta'}\lambda_\phi\rangle + \langle\lambda_\theta\lambda_{\phi'}\rangle + \langle\lambda_{\theta'}\lambda_{\phi'}\rangle \\ &= \langle\lambda_\theta\lambda_\phi - \lambda_{\theta'}\lambda_\phi + \lambda_\theta\lambda_{\phi'} + \lambda_{\theta'}\lambda_{\phi'}\rangle \equiv \langle B_\lambda\rangle \end{aligned}$$

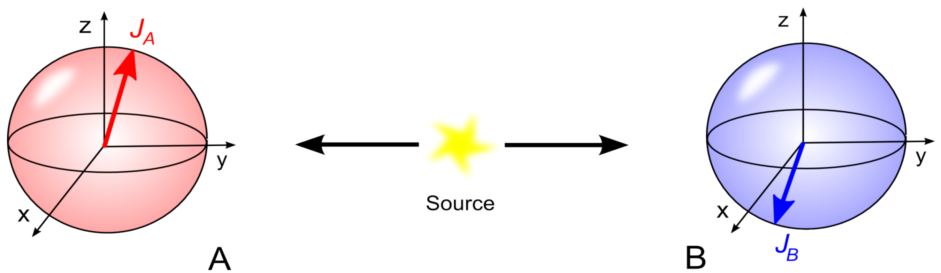
Exercise 2:



Outcomes for B according to LR:

λ_θ	$\lambda_{\theta'}$	λ_ϕ	$\lambda_{\phi'}$	$Prod(\theta,\phi)$	$Prod(\theta',\phi)$	$Prod(\theta,\phi')$	$Prod(\theta',\phi')$	B_λ
+1	+1	+1	+1	+1	+1	+1	+1	2
+1	+1	+1	-1	+1	+1	-1	-1	-2

Local Hidden Variables implies Bell's Inequality



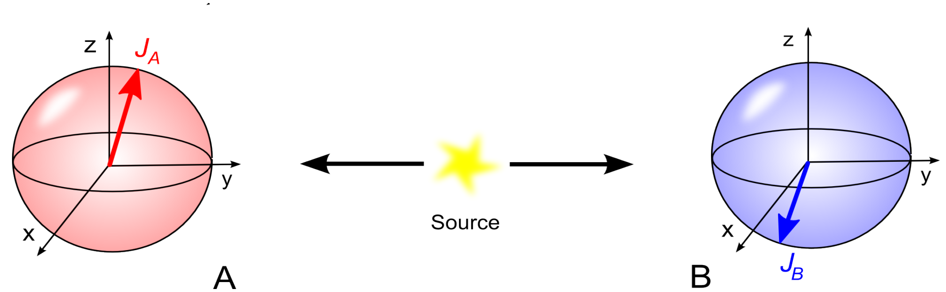
$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle = ?$$

Local hidden variables $\Longrightarrow$ *Clauser-Horne-Shimony-Holt (CHSH) Bell inequality*

$$|B| = |E(\theta,\phi) - E(\theta',\phi) + E(\theta,\phi') + E(\theta',\phi')| \le 2$$

We assumed perfect EPR correlation, so the values of hidden variables λ were +1, or -1
CHSH Bell inequality still holds in presence of arbitrary correlation

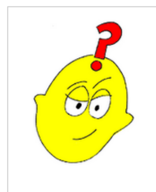
What does quantum mechanics say?



$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle = ?$$

Quantum mechanics: four Bell states violate CHSH Bell Inequality

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right), |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle\right)$$

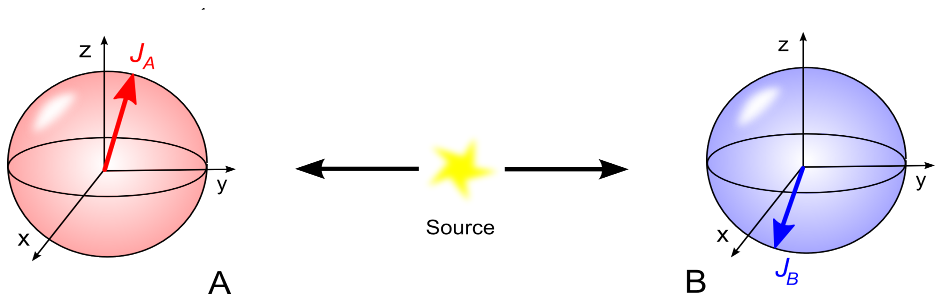


$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle = -\cos(\phi - \theta)$$

$$\theta = 0,\ \theta' = \pi/2, \quad \phi = \pi/4,\ \phi' = 3\pi/4 \qquad \Rightarrow |B| = 2\sqrt{2}$$

Exercise 3: calculate this prediction

Quantum mechanics violates Bell inequality



$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle = ?$$

Local hidden variables $\Longrightarrow$ *Clauser-Horne-Shimony-Holt (CHSH) Bell inequality*

$$|B| = |E(\theta,\phi) - E(\theta',\phi) + E(\theta,\phi') + E(\theta',\phi')| \leq 2$$

Quantum mechanics: four Bell states maximally violate CHSH Bell Inequality

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right), |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle\right)$$

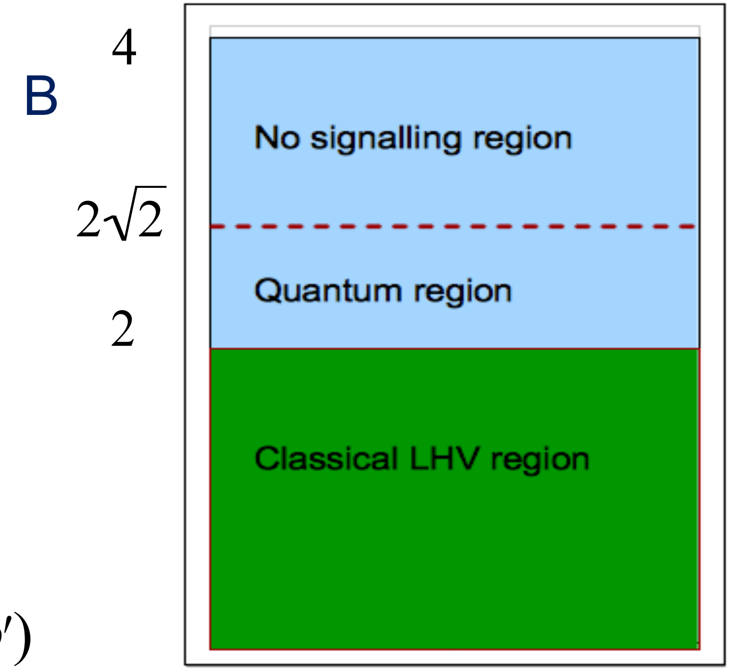
$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle = -\cos(\phi - \theta) \quad \theta = 0,\ \theta' = \pi/2, \quad \phi = \pi/4,\ \phi' = 3\pi/4$$

$$\Rightarrow |B| = 2\sqrt{2}$$

What is the QUANTUM inequality for B? Tsirelson bound

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

The Bell states give the maximum possible B within quantum mechanics



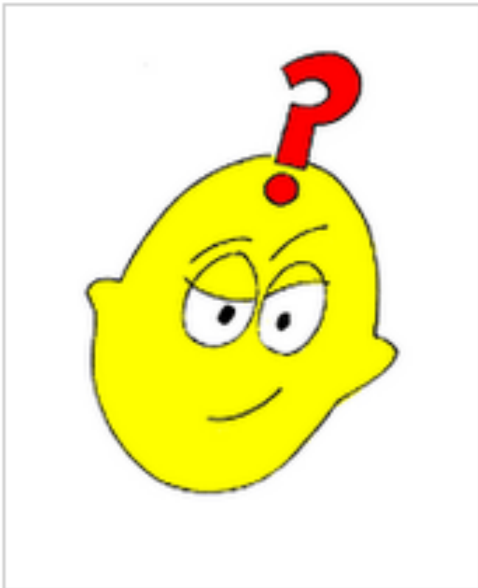
$$B = E(\theta,\phi) - E(\theta',\phi) + E(\theta,\phi') + E(\theta',\phi')$$

Local hidden variables LHV $\Rightarrow |B| \leq 2$

Quantum mechanics QM $\Rightarrow B \leq 2\sqrt{2}$ ***Result proved by Tsirelson***

Interesting question: Why is quantum mechanics not more nonlocal?
("No-signalling" theories can have an even greater correlation- up to 4)

Exercises



Outline: Lectures 1-2

1. The beginnings of quantum entanglement

Non-locality, reality and quantum mechanics

Einstein's (EPR) spooky action at a distance 1935

Schrodinger's cat 1935 – introducing entanglement

BELL'S THEOREM 1965

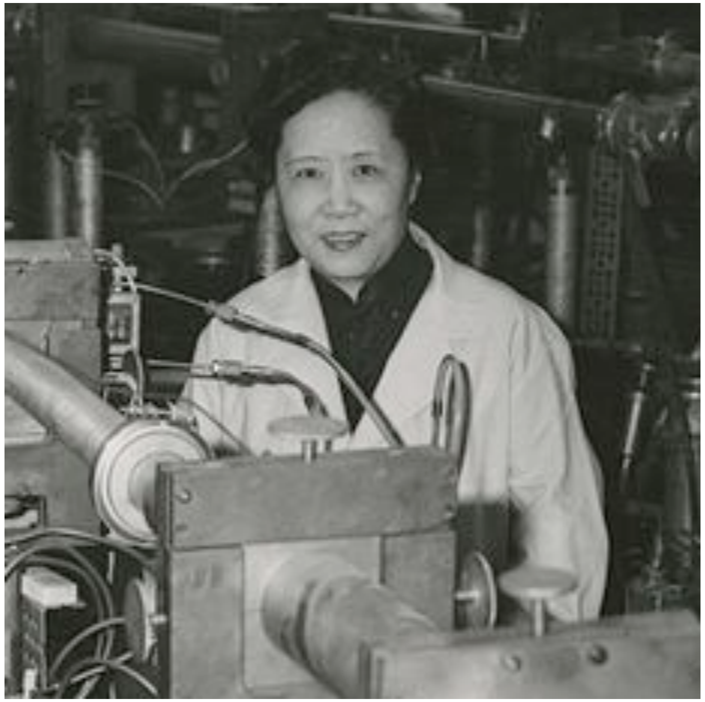
BELL AND EPR EXPERIMENTS

Greenberger-Horne-Zeilinger's (GHZ) theorem

extreme multiparticle quantum nonlocality 1990's

Experiments: Early EPR experiment photons

Wu and Shaknov, PRA 1950
Columbia University

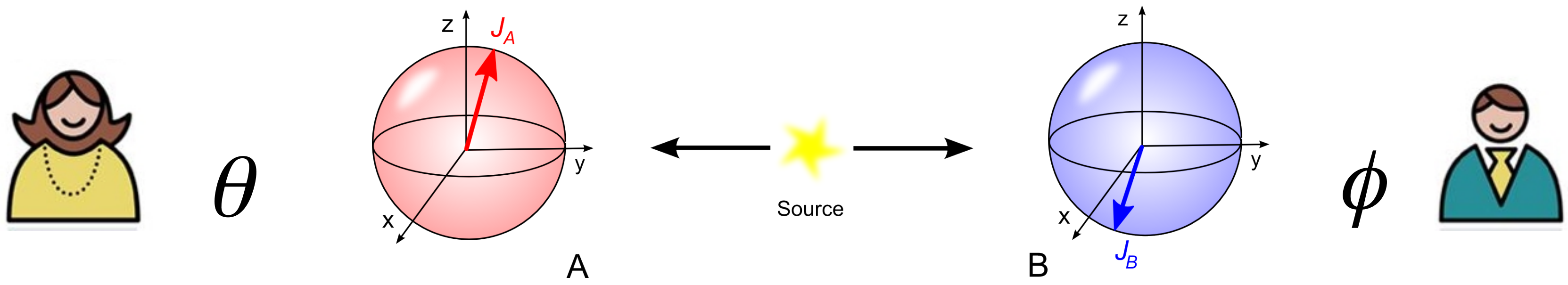


EPR correlation of polarisation of
two photons propagating in opposite directions

Experiments: Quantum mechanics OR local realism?
Which one is right?

Testing Bell's theorem

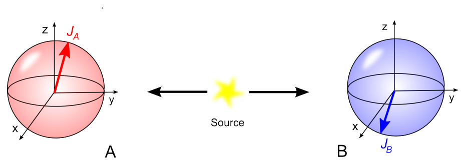
$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle = -\cos(\phi - \theta)$$



Clauser, Aspect, Zeilinger et al

Experiments: Quantum mechanics OR local realism? Which one is right?

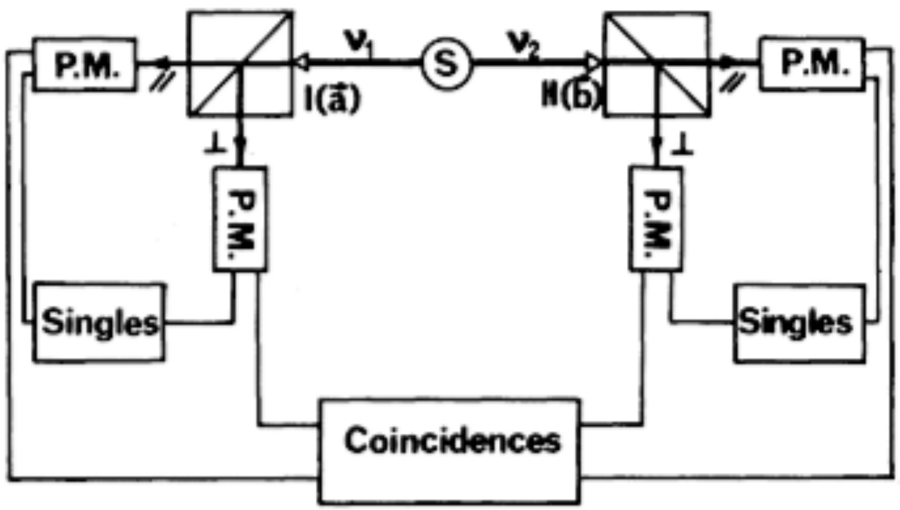
Testing Bell theorem



Clauser, Aspect, Zeilinger et al

θ

ϕ



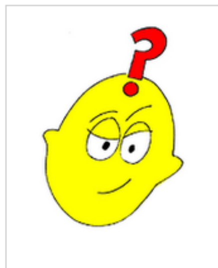
<u>Polarised photons</u>

Photon pairs a,b

Polarised + or – (qubit)

Polarisation of each pair is correlated

$$|\Psi\rangle_{source} = \frac{1}{\sqrt{2}}\left(|+\rangle_a|+\rangle_b + |-\rangle_a|-\rangle_b\right)$$



First, we should understand the notation and predictions for this source!

Need a little formalism: harmonic oscillator

Quantisation of the radiation field / harmonic oscillator
A mode of the field is quantised as a harmonic oscillator:

$$H = \hbar\omega(a^\dagger a + 1/2)$$

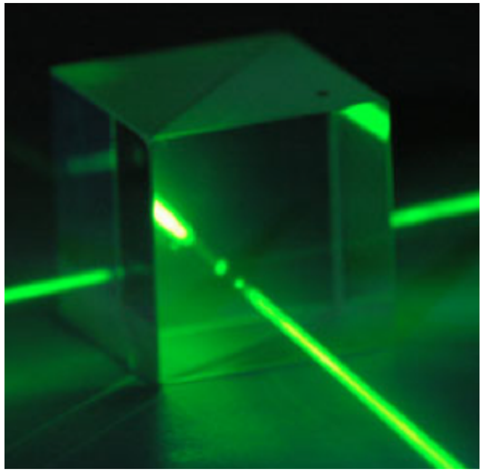
where $a^\dagger$,a are creation and destruction operators $[a^\dagger, a] = 1$. The $n = a^\dagger a$ is the (photon) number operator (we sometimes drop the "hat" if meaning of operator is clear) , and we can define eigenstates of this number operator $\hat{n}|n\rangle = n|n\rangle$. The vacuum state is $|0\rangle$ and raising lowering operator rules apply: $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ $a|n\rangle = \sqrt{n}|n-1\rangle$. So, we can use symbols, $|0\rangle$, $|1\rangle$ to refer to spin Up or Down, OR a single or zero excitation of a mode OR whether a photon occupies polarisation mode $+$ or $-$. The most common qubit is a photon in $+$ polarised mode (bit value $+1$) versus photon in $-$ polarised mode (bit value 0).

Common simple approach: to describe light through a **beam splitter** (50/50 mirror) OR **polariser** : creation of rotated modes

$$\begin{aligned} a_{out,+} &= \cos\theta a_+ + \sin\theta a_- \\ a_{out,-} &= -\sin\theta a_+ + \cos\theta a_- \end{aligned}$$

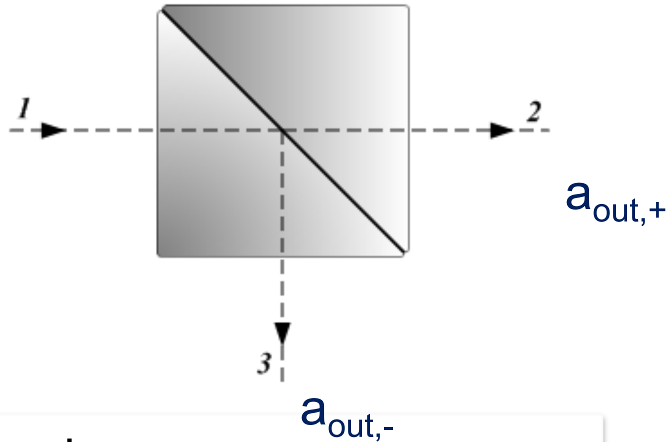
Check that the photon number conserved-

Beam splitter – polariser measurement



$$\begin{aligned} a_{out,+} &= \cos\theta a_+ + \sin\theta a_- \\ a_{out,-} &= -\sin\theta a_+ + \cos\theta a_- \end{aligned}$$

Polarising beam splitter



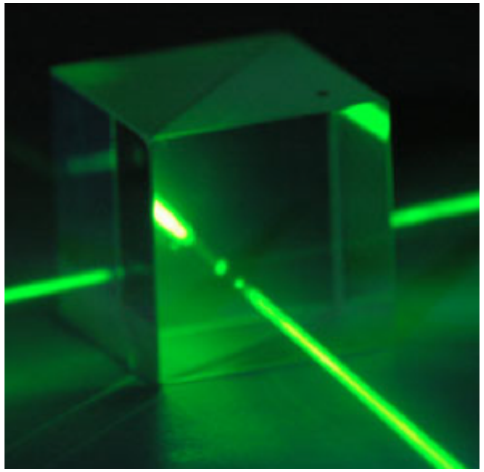
INPUT polarised +,- along axis a_+, a_-

Light coming out is split into two beams:
polarised parallel and orthogonal to the polariser axis rotated by angle θ

Consider a single photon incident :what happens?

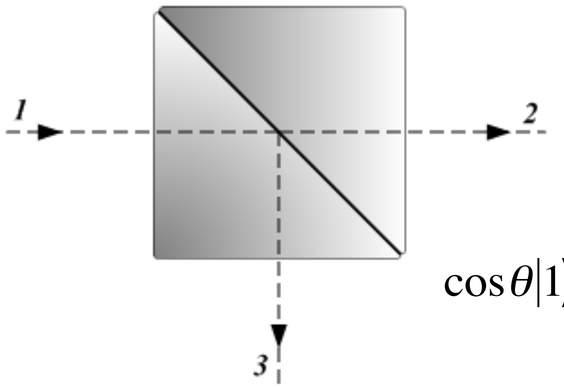
Beam splitter – polariser measurement

The photon acts like a particle – detected at 2 or 3



$$a_{out,+} = \cos\theta a_+ + \sin\theta a_-$$
$$a_{out,-} = -\sin\theta a_+ + \cos\theta a_-$$

Polarising beam splitter



Spin +1

$a_{out,+}$ **"Qubit" outcome**

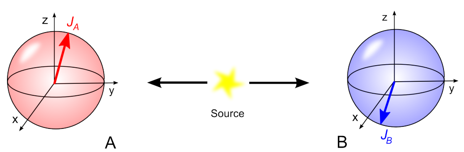
$$\cos\theta|1\rangle_{a,out+}|0\rangle_{a,out-} - \sin\theta|0\rangle_{a,out+}|1\rangle_{a,out-}$$

$a_{out,-}$ Spin -1

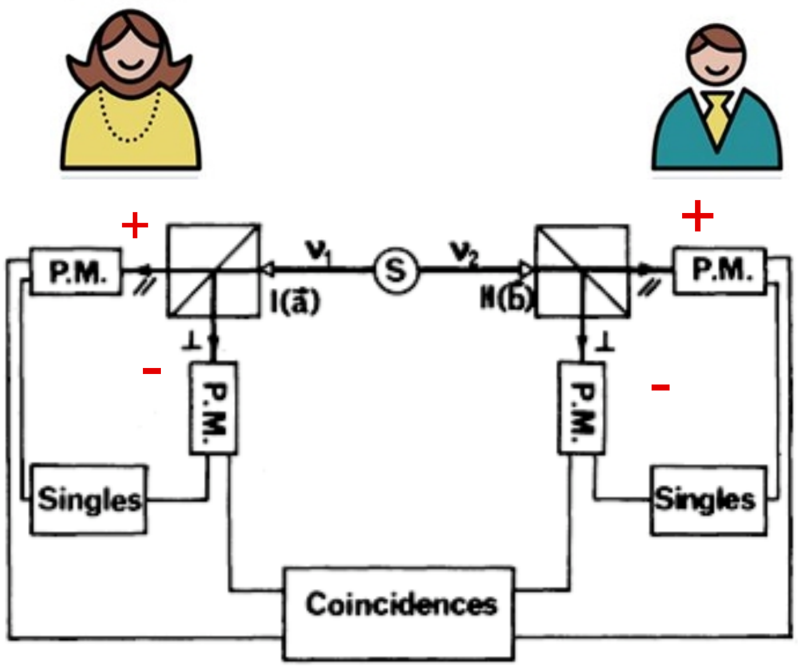
***Consider a single photon incident (mode a_+): detected at either* the + *or* – location**
Call result spin J = +1 or -1 (photon is in the superposition state)

Experiments: Quantum mechanics OR local realism? Which one is right?

Testing Bell theorem

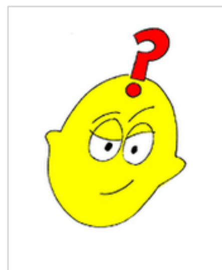


Clauser, Aspect, Zeilinger et al



<u>Polarised photons</u>
Photon pairs a,b
Polarised + or – (qubit)
Polarisation of each pair is correlated

$$|\Psi\rangle_{source} = \frac{1}{\sqrt{2}}\left(|+\rangle_a|+\rangle_b + |-\rangle_a|-\rangle_b\right)$$



We detect correlated photon clicks, just like the spin ½ particles!

Bell test – with photons and polarisers

Input to two polarising beam splitters: four modes

$$\frac{1}{\sqrt{2}}\left\{|1\rangle_{a+}|0\rangle_{a-}|1\rangle_{b+}|0\rangle_{b-}+|0\rangle_{a+}|1\rangle_{a-}|0\rangle_{b+}|1\rangle_{b-}\right\}\equiv\frac{1}{\sqrt{2}}\left\{|+\rangle_a|+\rangle_b+|-\rangle_a|-\rangle_b\right\}$$

Correlated "qubits"

Alice rotates her polariser

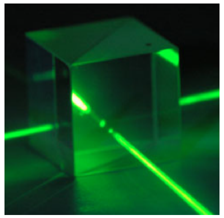
θ

Bob rotates his polariser

ϕ

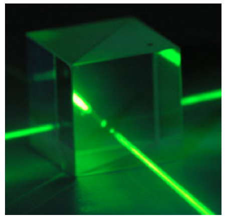
Source

$a_{out,+}$ +



- $a_{out,-}$

$$\leftarrow \quad \frac{1}{\sqrt{2}}\left\{|+\rangle_a|+\rangle_b+|-\rangle_a|-\rangle_b\right\} \quad \rightarrow$$



+ $b_{out,+}$

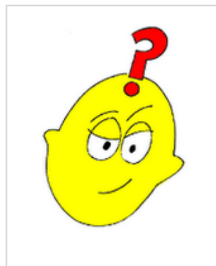
Single detection "clicks"
Photon either + or -

- $b_{out,-}$

Output of polarisers: calculate

$$|\Phi\rangle=\cos(\theta-\phi)\frac{1}{\sqrt{2}}\left\{|+\rangle_a|+\rangle_b+|-\rangle_a|-\rangle_b\right\}+\sin(\theta-\phi)\frac{1}{\sqrt{2}}\left\{|+\rangle_a|-\rangle_b-|-\rangle_a|+\rangle_b\right\}$$

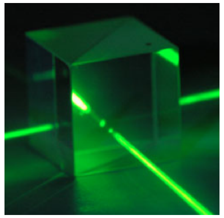
Correlated qubits *Anti-correlated qubits*



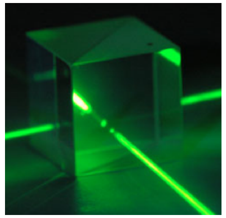
Bell test – with photons and polarisers

θ ϕ

Source



$$+ \quad \leftarrow \quad \frac{1}{\sqrt{2}}\left\{|+\rangle_a|+\rangle_b + |-\rangle_a|-\rangle_b\right\} \quad \rightarrow$$

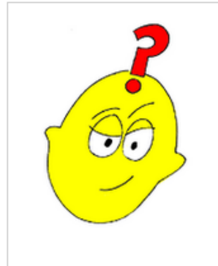


Single detection "clicks"
Photon either + or -

+

-

-



Output of polarisers: calculate

$$|\Phi\rangle = \cos(\theta-\phi)\frac{1}{\sqrt{2}}\left\{|+\rangle_a|+\rangle_b + |-\rangle_a|-\rangle_b\right\} + \sin(\theta-\phi)\frac{1}{\sqrt{2}}\left\{|+\rangle_a|-\rangle_b - |-\rangle_a|+\rangle_b\right\}$$

Correlated qubits *Anti-correlated qubits*

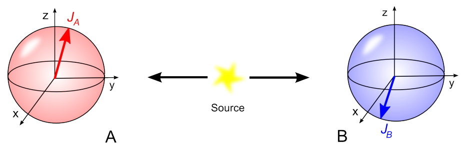
Quantum prediction is:

$$\Pr(+,+) = \Pr(-,-) = \cos^2(\theta-\phi); \quad \Pr(-,+) = \Pr(+,-) = \sin^2(\theta-\phi)$$

$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle = \cos 2(\phi-\theta)$$

Experiments: Quantum mechanics OR local realism?
So, which one is right?

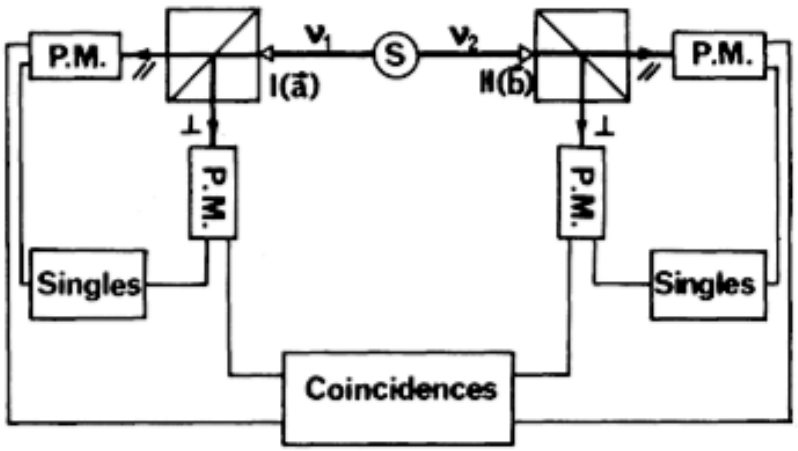
Testing Bell theorem



Will violate the Bell inequality like before- (use same angles divided by 2!)

$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle = \cos 2(\phi - \theta)$$

θ ϕ

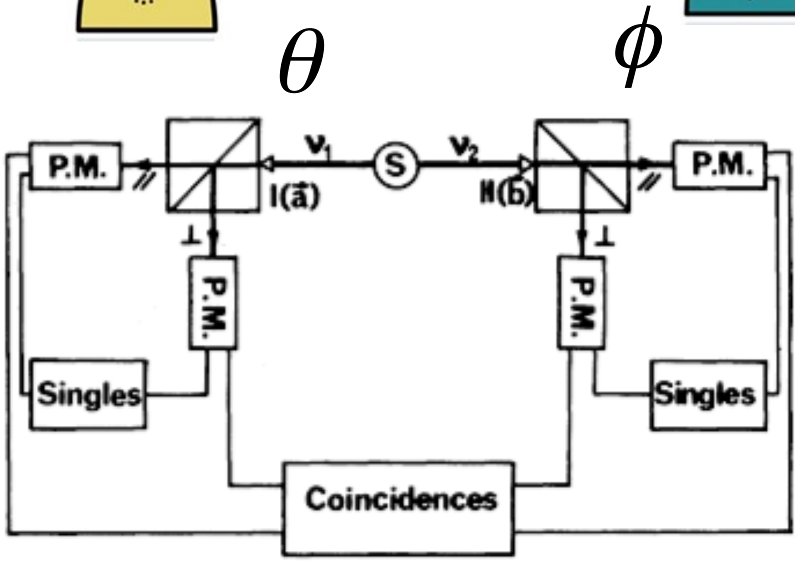


Just one photon pair incident at a time
Alice and Bob get “click” at one of their detectors
+ or - “spin”

Experiments: Quantum mechanics OR local realism?
Which one is right? B=2.70!

Testing Bell theorem

Clauser, Aspect, Zeilinger et al



$$E(\theta,\phi) = \left\langle J_\theta^A J_\phi^B \right\rangle = \cos 2(\phi - \theta)$$

$$\equiv \cos 2(b - a)$$

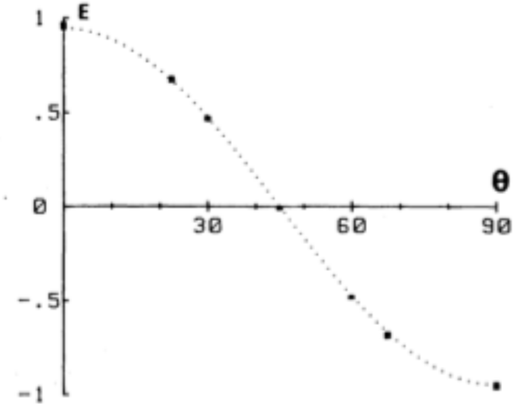


FIG. 3. Correlation of polarizations as a function of the relative angle of the polarimeters. The indicated errors are ±2 standard deviations. The dotted curve is not a fit to the data, but quantum mechanical predictions for the actual experiment. For ideal polarizers, the curve would reach the values ±1.

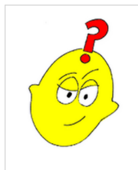
anics predicts

$$E(\vec{a},\vec{b}) = F\,\frac{(T_1^{\parallel} - T_1^{\perp})(T_2^{\parallel} - T_2^{\perp})}{(T_1^{\parallel} + T_1^{\perp})(T_2^{\parallel} + T_2^{\perp})}\cos 2(\vec{a},\vec{b}) . \tag{5}$$

($F = 0.984$ in our case; it accounts for the finite solid angles of detection.) Thus, for our experiment,

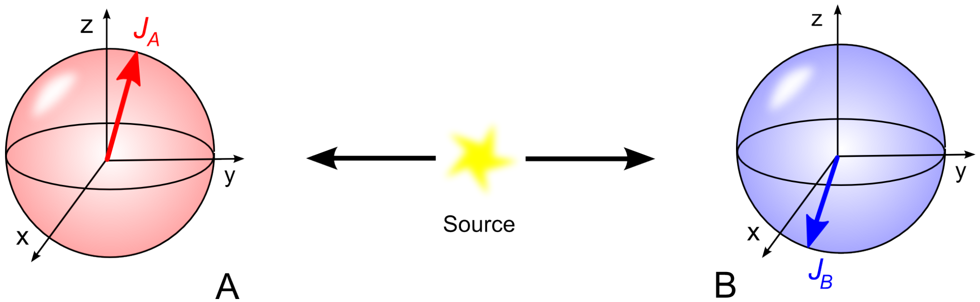
$$S_{QM} = 2.70 \pm 0.05 .$$

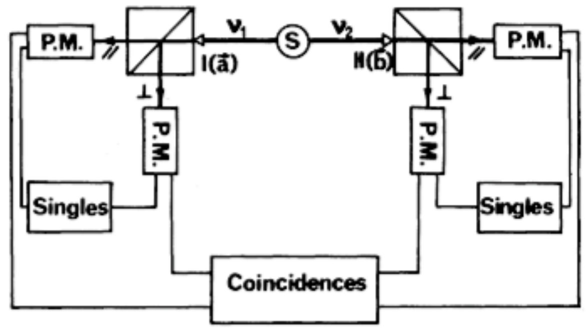
This is our "B"



Experiments: two qubit (particle) case

Testing Bell theorem





$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Photons: all support quantum mechanics

BUT None overcome detection efficiency loophole η>0.8, (lower for non-maximally entangled states) *(so far)*

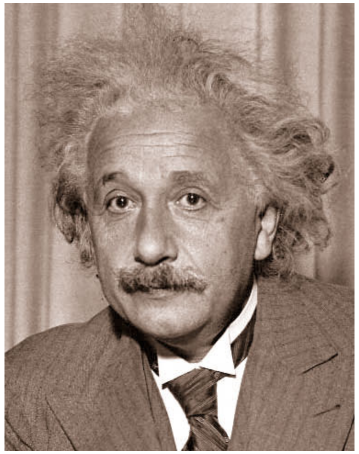
but spacelike separations

Massive particles:

Ions Wineland et al, excellent efficiency but poor separation

BUT can't exclude that there has been subluminal communication

Nonlocality with "position-momentum" (continuous variables CV)?



- Is there quantum nonlocality / EPR / entanglement for **continuous variable observables**?
- Eg where conjugate observables are – **position, momentum**
  - **YES!**

- *EPR's original argument was with x, p*
- *This has been realised experimentally for optical amplitudes*

Squeezing (continuous variable CV)

2.1 Continuous variable (cv) squeezing

Consider harmonic oscillator:

$$
\begin{aligned}
X &= a + a^\dagger \\
P &= (a^\dagger - a)/i
\end{aligned}
$$

Then the uncertainty relation follows (use $[a, a^\dagger] = 1$)

$$\Delta X \Delta P \geq 1$$

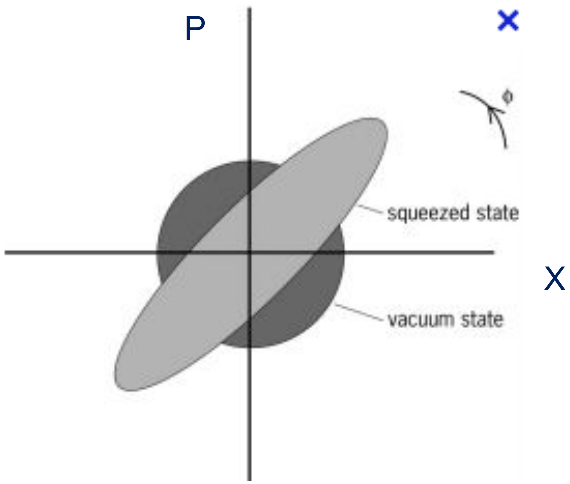
The *minimum uncertainty states* are the *coherent* states $|\alpha\rangle$, which are the eigenstates $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$, and these give $\Delta X = \Delta P = 1$ and the "*squeezed states*" for which $\Delta X = e^{-r}$, $\Delta P = e^{r}$.

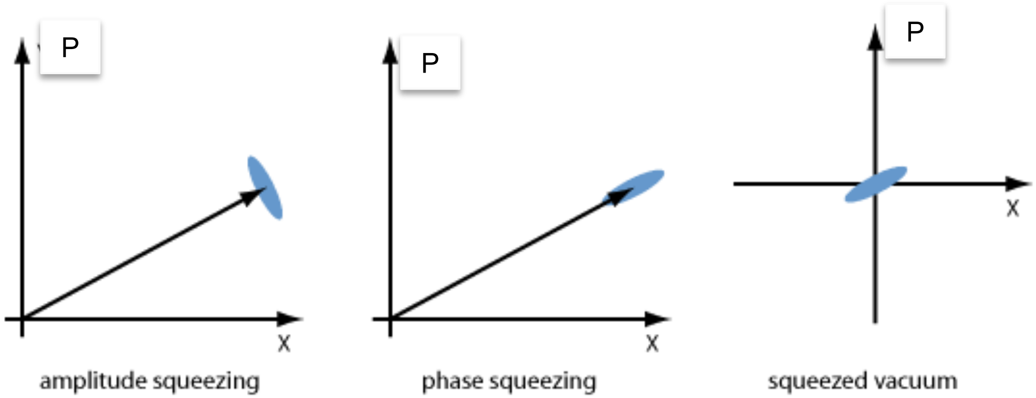
We have "squeezing" when

$$\Delta X < 1$$

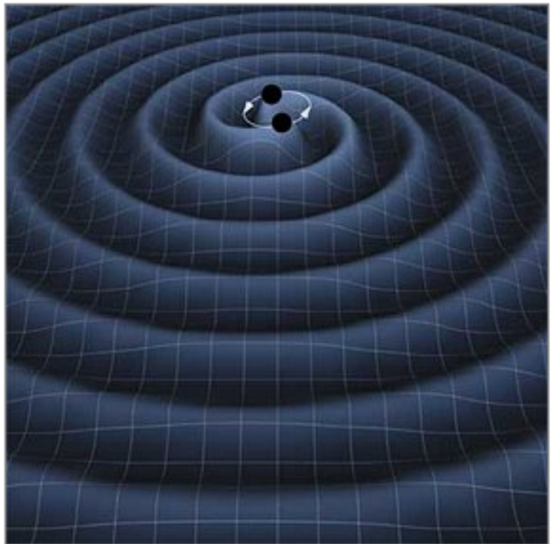
Squeezing was first observed for light for X (quadrature phase amplitudes) in the 1980's.

Squeezing (cv)





Squeezed light /gravity wave detection

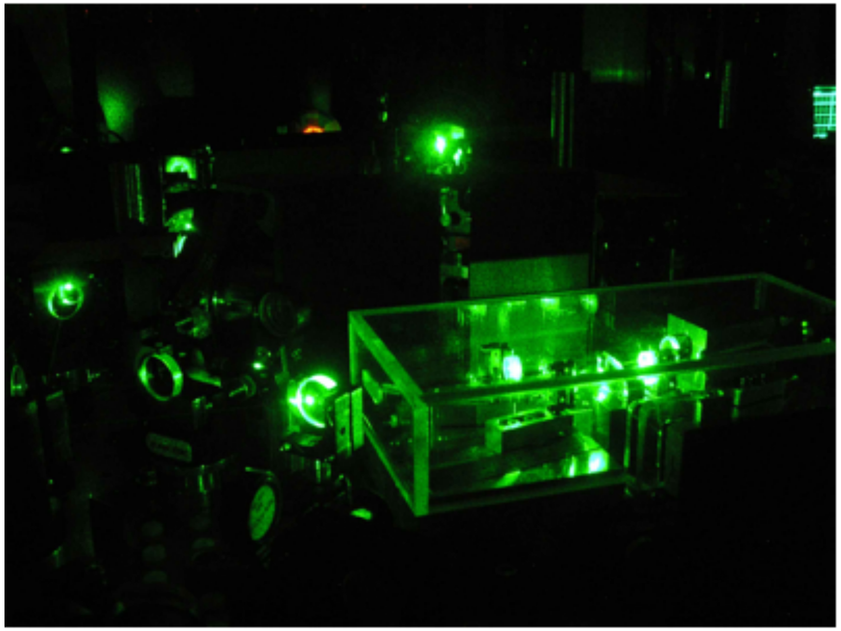


New light squeezing detectors may allow scientists to finally see gravitational waves such as those produced by colliding black holes (Source: T. Carnahan/NASA GSFC/LIGO)

Aerial view of the LIGO interferometer in Hanford, Washington. Photo courtesy LIGO Laboratory.

A research collaboration has taken steps toward improving the sensitivity of gravitational wave detectors, devices designed to measure distance changes as minute as one-thousandth the diameter of a proton. Scientists hope these detectors can one day further verify Einstein's theory of general relativity and even open a new window into the strange workings of the universe.

How is squeezing generated?



Optical parametric down conversion (OPO)

Quadratic Hamiltonian

$$H = \kappa E(a^{+^2} + a^2)$$

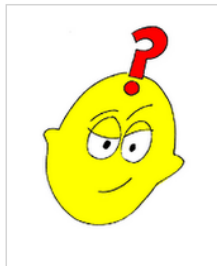
$$X_\theta = X\cos\theta + P\sin\theta$$

Solutions: solve for X, P as function of time,
then calculate the variances for a vacuum initial state
For some θ:

$$(\Delta X_\theta)^2 = e^{-\kappa' t}$$

$$(\Delta P_\theta)^2 = e^{\kappa' t}$$

SQUEEZING!



Squeezing (cv) – how measured?

How is this squeezing measured?
Combine with large coherent field (laser) using a beam splitter (50/50 mirror) to get a measure of this fluctuation eg

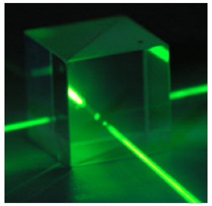
$$
\begin{aligned}
a_{out,+} &= [a_+ + a_-]/\sqrt{2} \\
a_{out,-} &= [-a_+ + a_-]/\sqrt{2}
\end{aligned}
$$

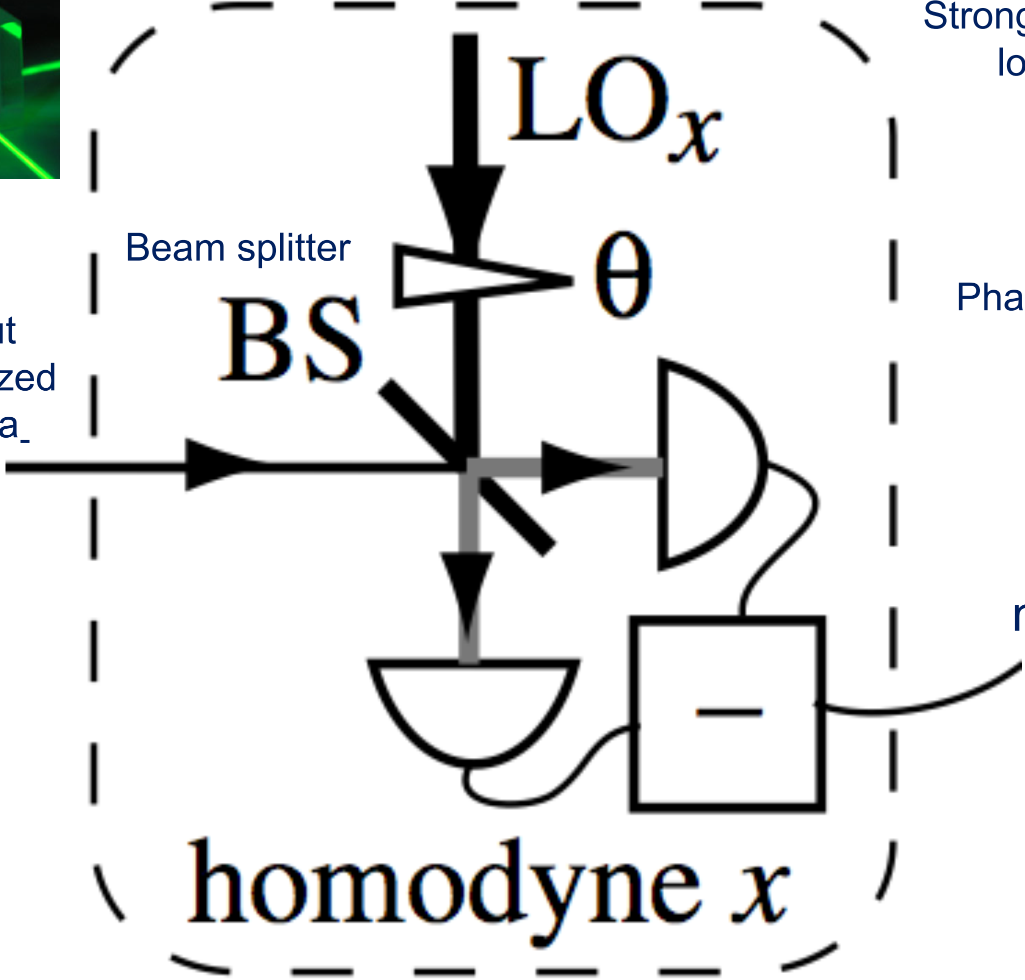
but if a_+ is very large, it can be classical amplitude $Ee^{-i\theta}$- then the photon number difference between the two arms of the beam splitter is

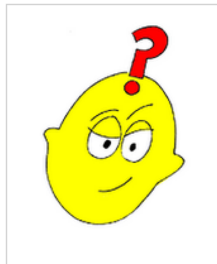
$a^\dagger_{out,+} a_{out,+} - a^\dagger_{in+} a_{in,+} = E(a^\dagger_- e^{i\theta} + a_- e^{-i\theta})$... this becomes X or P depending on the choice of phase θ.

Need to identify the "quantum limit": defined as that for a coherent state, best to take vacuum $|0\rangle$: so measure noise levels with a_- a vacuum, then compare with noise levels when a_- is the squeezed light source.

Squeezing measurement (cv)-optical



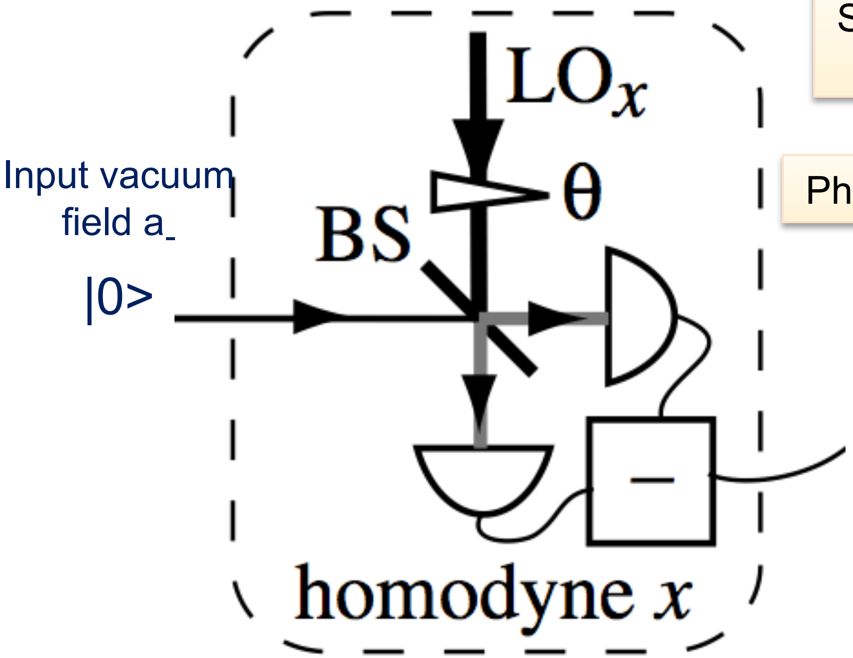




Question: what if the second input port a_- has a vacuum state |0> input?
How does variance of number difference vary with θ?

See notes on this

Squeezing measurement (cv)-optical



Strong laser field E a_+: local oscillator

Phase difference that selects X or P

Photon number difference measurement - corresponds to E x_θ :

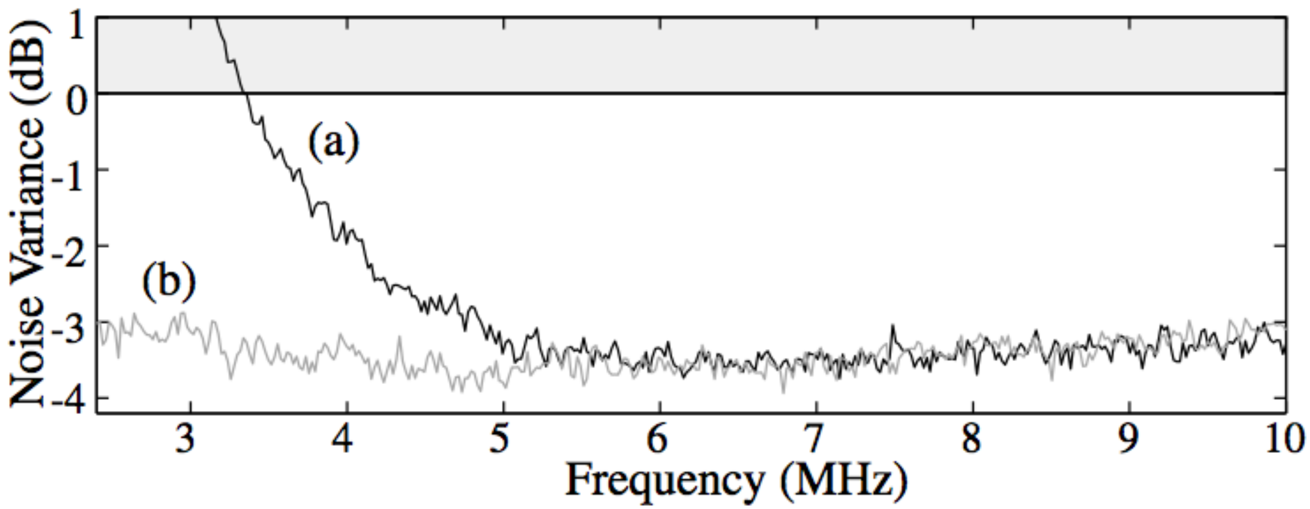
Fluctuation in number difference is a measure of the variance in X_θ

Question: what if the second input port a_- has a vacuum state |0> input?
How does variance of number difference vary with θ?

Answer: |0> is a coherent state- variance doesn't change with θ
This "noise" is called the "standard quantum limit", "shot noise level", "vacuum noise level"

Squeezing shows as noise reduction in photon number difference

Optical Parametric Oscillator (OPO or OPA)



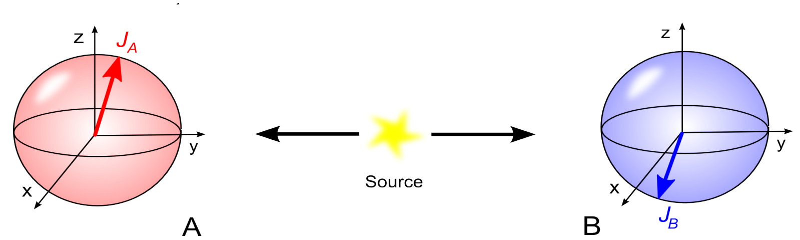
Vacuum noise level (coherent state)

Squeezed noise level

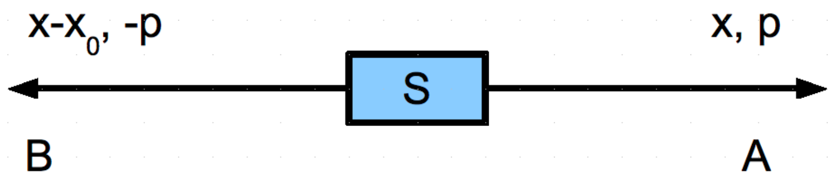
EPR entanglement and squeezing

Entangled states are non-separable: 2 classic examples

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



$$\delta(x_A - x_B)\delta(p_A + p_B)$$



•Entangled states - greater correlation than separable states for *both conjugate* (non-commuting) observables

•*Variances* of **SUMS OF MOMENTA** and **DIFFERENCES OF POSITION** are zero ie they are squeezed

•How to detect such "EPR" entanglement?.....use squeezing!

EPR entanglement and squeezing

- Entangled states - greater correlation than separable states for *both conjugate* (non-commuting) observables

$$\Delta^2(p_B \mid p_A) \to 0 \qquad \Delta^2(x_B \mid x_A) \to 0$$

EPR paradox when: $\Delta^2(X_B \mid X_A)\Delta^2(P_B \mid P_A) < 1$

Because, then the Elements of reality for Bob's X and P "violate" the Heisenberg Uncertainty Principle
$\Delta^2 X_B \Delta^2 P_B < 1$

- *Variances* of sums of momenta and differences of position are zero ie they are squeezed

Can also measure $\Delta^2(X_A - X_B) + \Delta^2(P_A + P_B) \to 0$

EPR entanglement using squeezing

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Realization of the Einstein-Podolsky-Rosen Paradox for Continuous Variables

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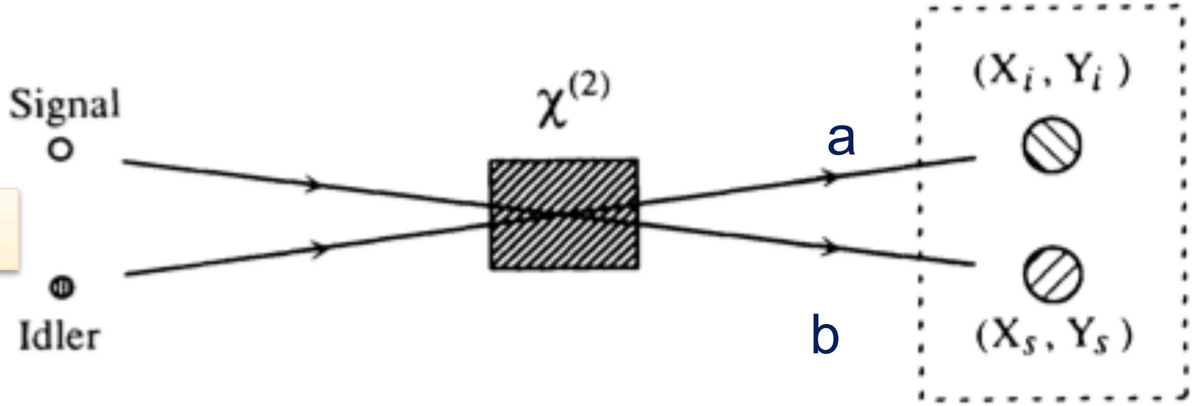
(Received 20 February 1992)

E is a pump field



$$H = \kappa E(a^+ b^+ + ab)$$

EPR correlated quadrature amplitudes generated using a **two-mode OPO Hamiltonian**
X^A correlated with X^B
P^A anticorrelated with P^B

Note: they use Y to mean P

EPR entanglement using squeezing

θ

X^A *or* P^A

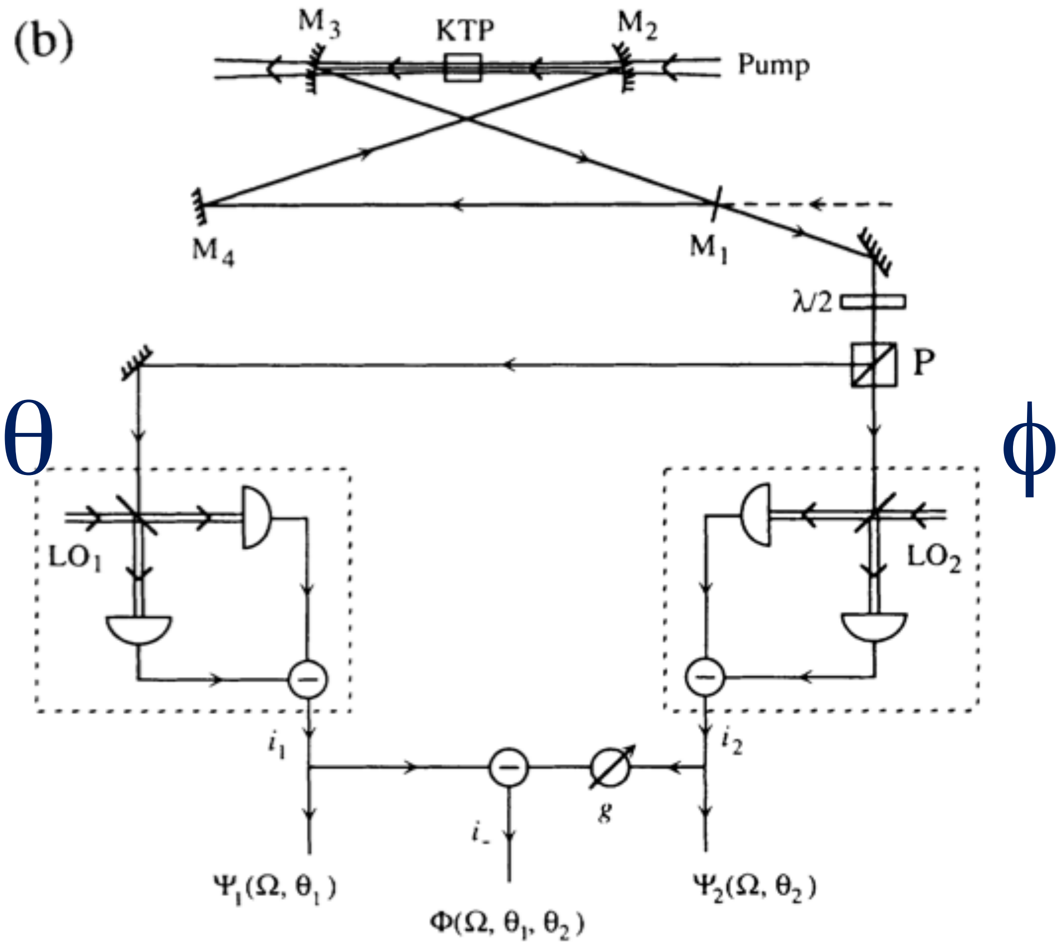


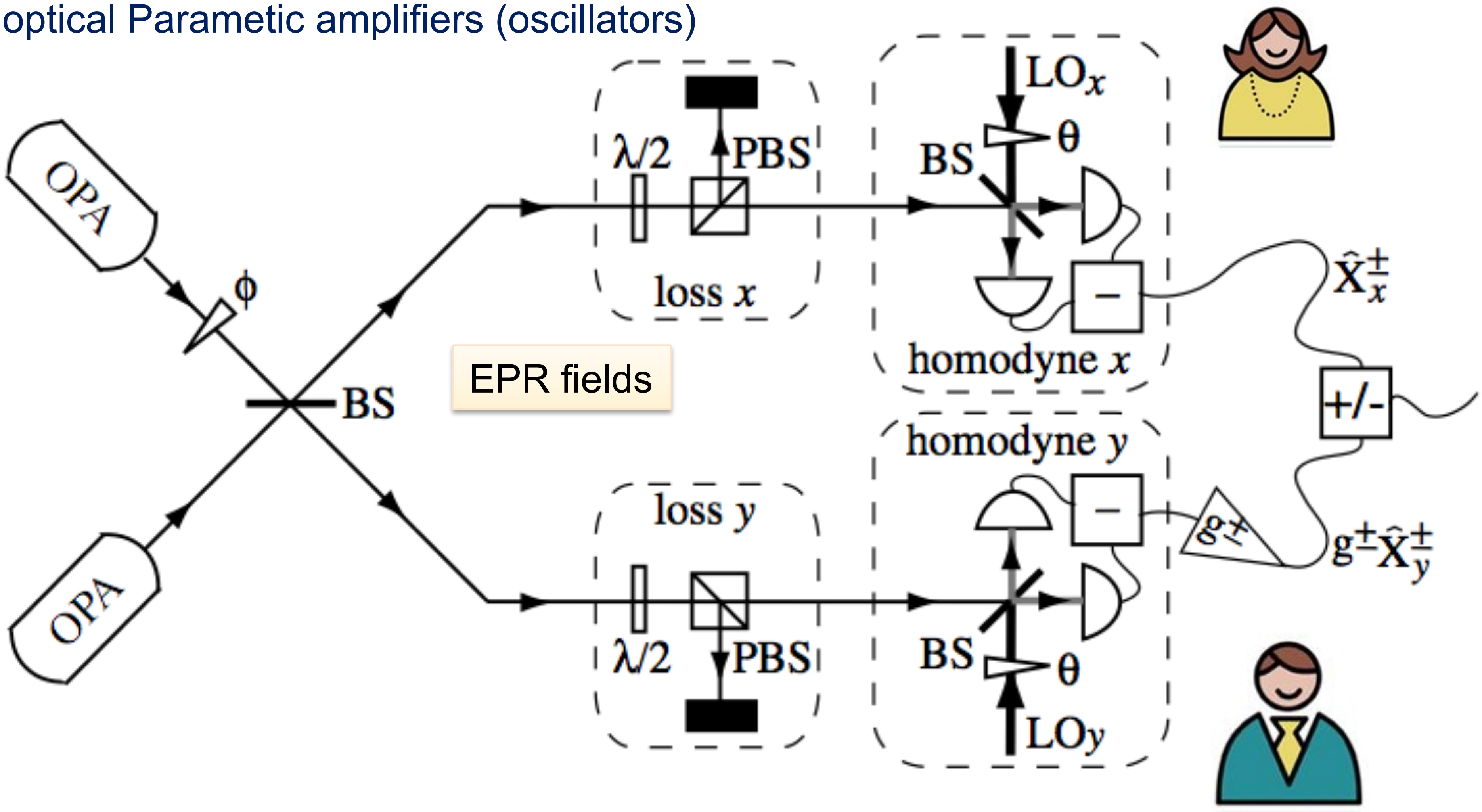
FIG. 1. (a) Scheme for realization of the EPR paradox by nondegenerate parametric amplification, with the optical amplitudes (X_s, Y_s) inferred in turn from (X_i, Y_i). (b) Principal components of the experiment.

ϕ

X^B *or* $-P^B$

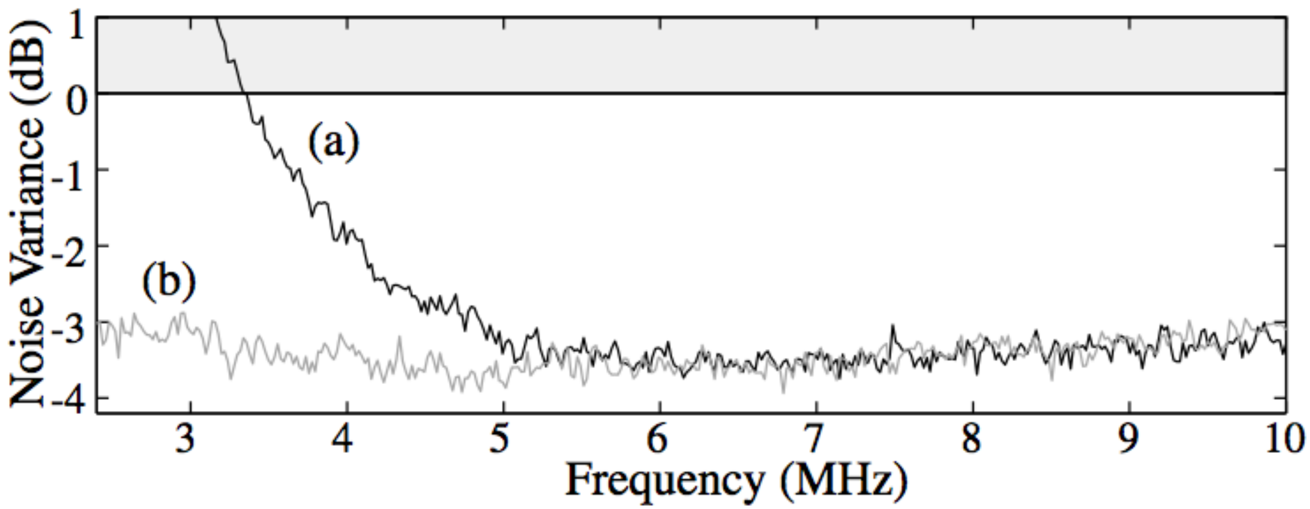
EPR entanglement using squeezing

2 optical Parametic amplifiers (oscillators)



EPR entanglement using squeezing

Noise for **BOTH** $X^A - X^B$ ***AND*** $P^A + P^B$ reduced below Bob's quantum limit



Confirms EPR paradox and entanglement for CV optical amplitudes

THIS IS NOT BELL'S THEOREM HOWEVER !

Outline: Lectures 1-2

1. The beginnings of quantum entanglement

Non-locality, reality and quantum mechanics

Einstein's (EPR) spooky action at a distance 1935

Schrodinger's cat 1935 – introducing entanglement

Bell's theorem 1965

Bell and EPR experiments

Greenberger-Horne-Zeilinger's (GHZ) theorem

extreme multiparticle quantum nonlocality 1990's

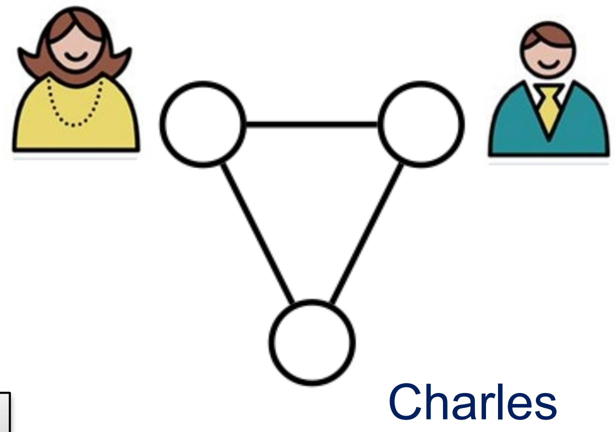
G H Z states

What is your guess? Are violations of local hidden variable theories possible as the number of particles increase?
How does quantum mechanics behave?
Need to look at LHV versus QM predictions

Zeilinger, Greenberger, Horne
Paradoxes
See Mermin article

Greenberger-Horne-Zeilinger GHZ multipartite extreme nonlocality

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\right)$$



$$\langle\sigma_x^A\sigma_y^B\sigma_y^C\rangle = \langle\sigma_y^A\sigma_x^B\sigma_y^C\rangle = \langle\sigma_y^A\sigma_y^B\sigma_x^C\rangle = +1$$

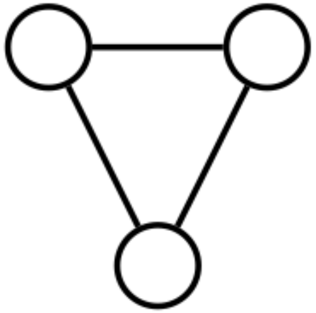
What does EPR's Local realism say about this?

- Can predict *any* spin, by measuring other two
- Hence, LR no "spooky action at a distance" tells us **each** spin is predetermined
- The spins are described by hidden variables λ_θ (value +1 or -1 ….so always $\lambda_\theta^2=1$, also $\lambda_x^A\lambda_y^B\lambda_y^C = +1$ etc)

$$\langle\sigma_x^A\sigma_x^B\sigma_x^C\rangle = \langle\lambda_x^A\lambda_x^B\lambda_x^C\rangle = \langle\lambda_x^A\lambda_x^B\lambda_x^C(\lambda_y^A)^2(\lambda_y^B)^2(\lambda_y^C)^2\rangle$$
$$= \langle\lambda_x^A\lambda_y^B\lambda_y^C\ \lambda_y^A\lambda_x^B\lambda_y^C\ \lambda_y^A\lambda_y^B\lambda_x^C\rangle$$
$$= +1$$

Greenberger-Horne-Zeilinger GHZ multipartite extreme nonlocality

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\right)$$



$$\langle \sigma_x^A \sigma_y^B \sigma_y^C \rangle = \langle \sigma_y^A \sigma_x^B \sigma_y^C \rangle = \langle \sigma_y^A \sigma_y^B \sigma_x^C \rangle = +1$$

What does QM say?

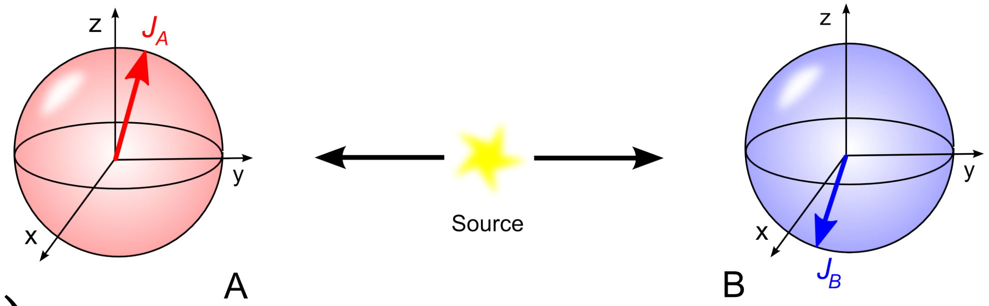
Mermin article

$$\langle \sigma_x^A \sigma_x^B \sigma_x^C \rangle = -1$$

The Quantum result is exactly opposite the local realism result!
Extreme violation – in one measurement!

Are entanglement and nonlocality equivalent?

Interesting results for two qubit case



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right)$$

Bell states

- All 2 qubit pure entangled states violate CHSH Bell inequality (Gisin)

- BUT there exist states (Werner) that are entangled
 but are consistent with LR (all measurements)
 ie cannot violate a Bell inequality
- Answer - no

Outline

1. **Non-locality and quantum mechanics**
   - Einstein's (EPR) spooky action at a distance 1935
   - Schrodinger's cat 1935
   - Bell's theorem 1965
     - Bell and EPR experiments
   - GHZ's extreme multiparticle quantum nonlocality
2. **Introduce formalism of entanglement**
   - Density operator – mixed states
   - Inseparability of density matrix
   - Pauli spin examples
   - Werner states
   - Peres PPT criterion and concurrence
   - Quadrature squeezing and spin squeezing
   - CV Variance and spin squeezing criteria for entanglement
3. **Applications**
   - Quantum cryptography and quantum teleportation