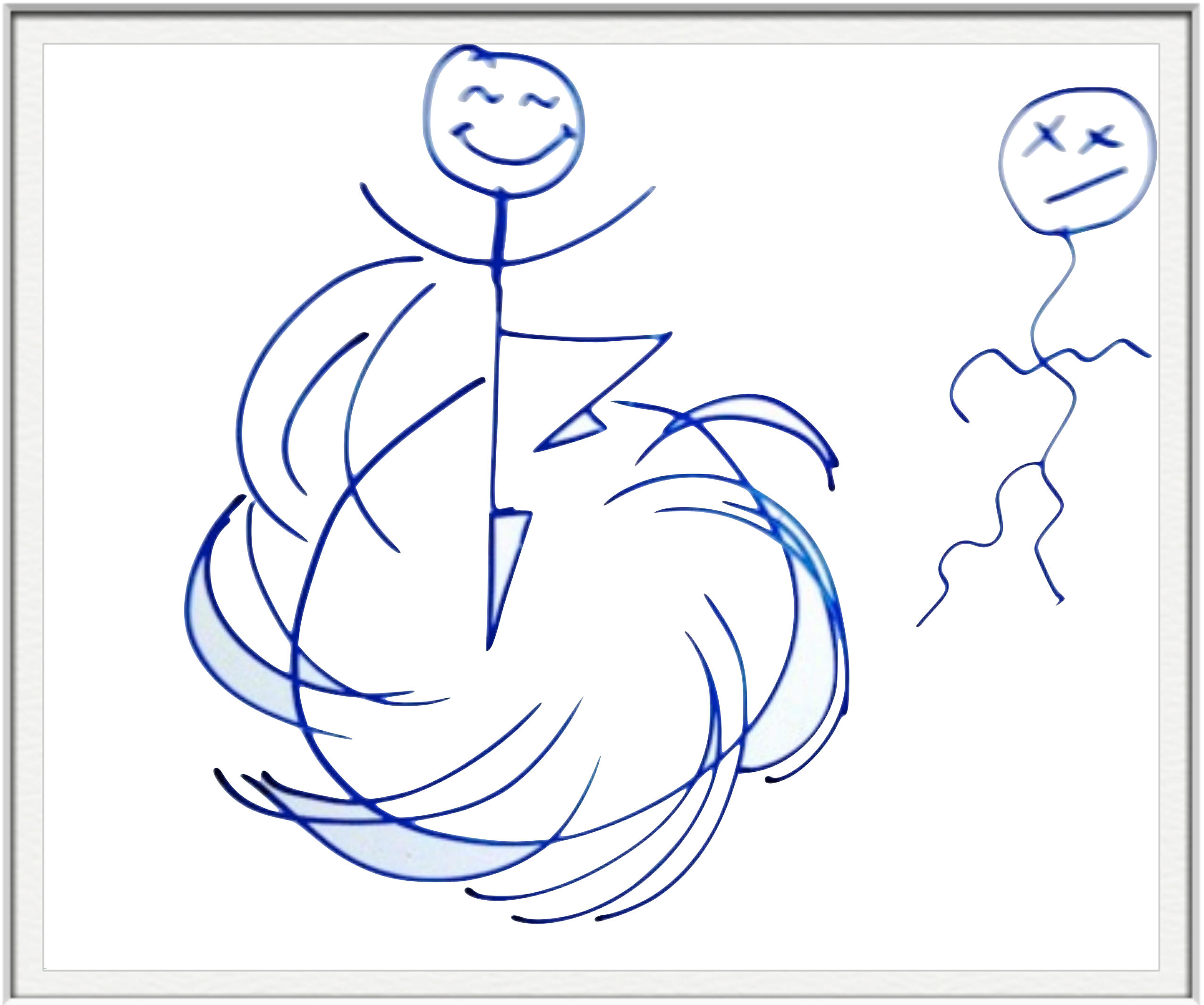
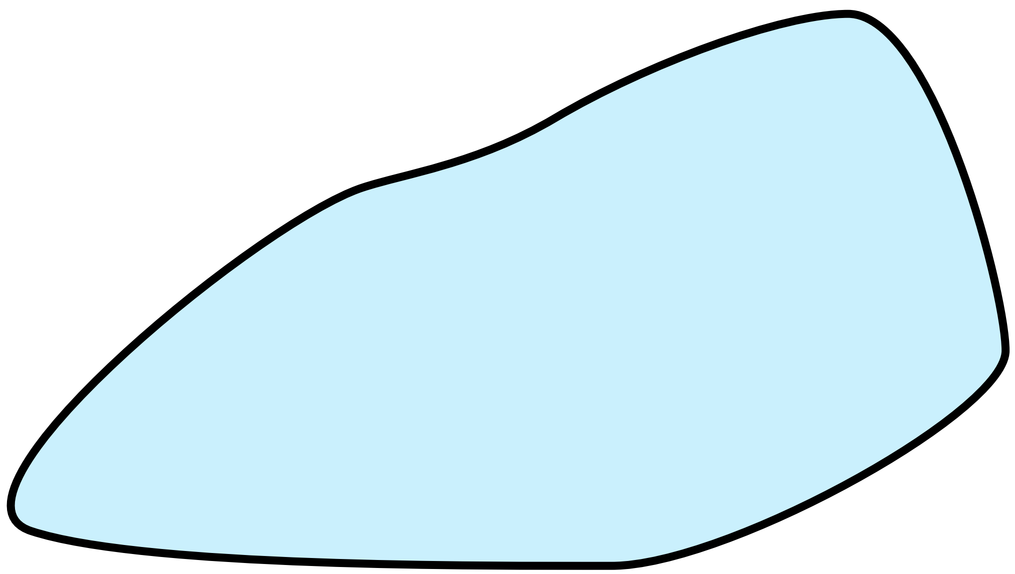
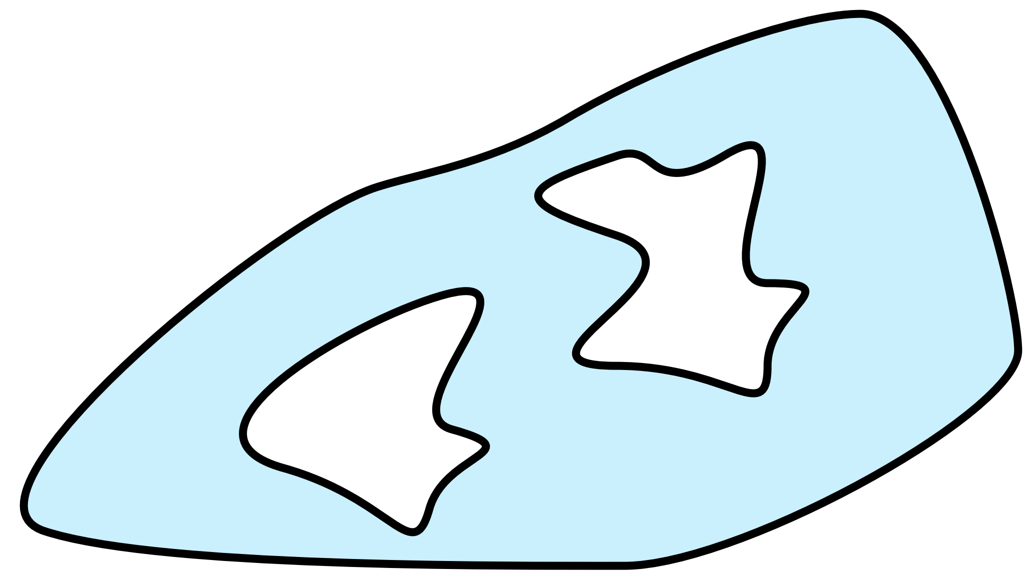
Lecture 3

where bosons make pirouettes and get dizzy



Topological (defects) excitations

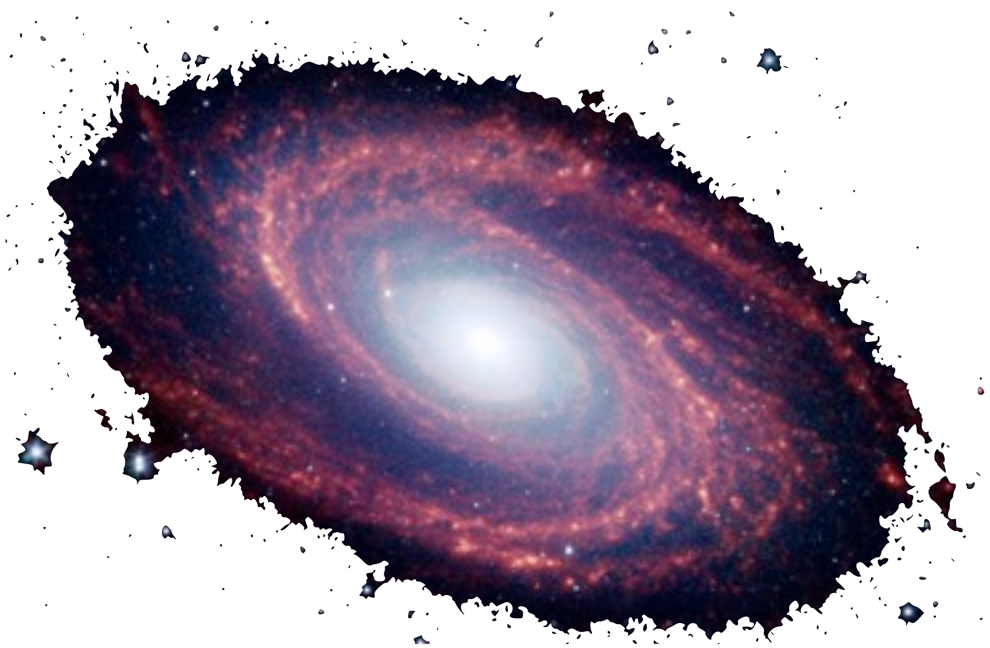




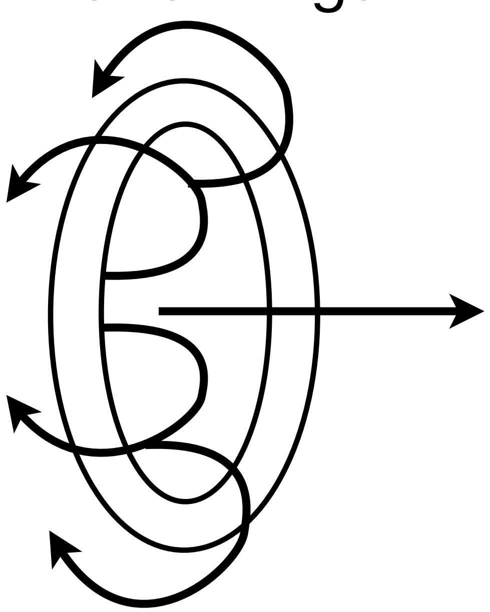
singly connected region

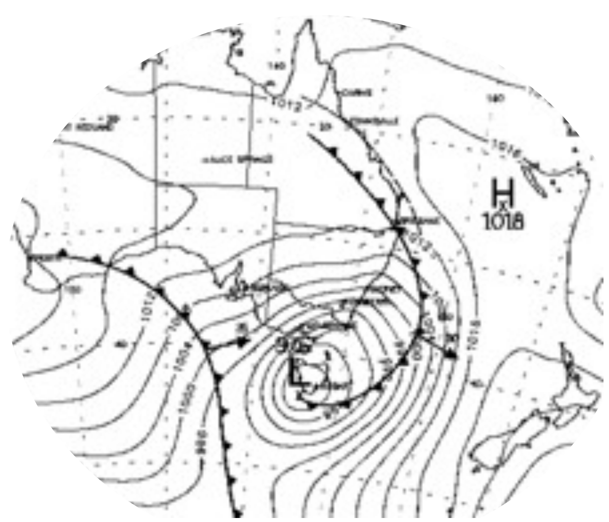
multiply connected region

one vortex, many vortices



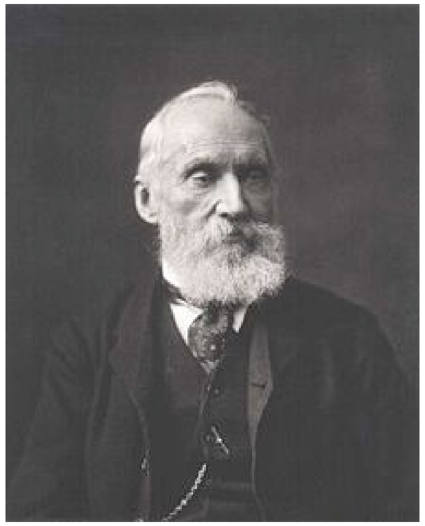
vortex rings





http://youtu.be/bT-fctr32pE

Lord Kelvin



"Vortices of pure energy can exist and, if my theories are right, can compose the bodily form of an intelligent species."

Of the Vortex nature of Atoms: "It is only a dream."

...string theory...?

Stable knot-like structures in classical field theory

L. Faddeev*† & Antti J. Niemi†‡

* St Petersburg Branch of Steklov Mathematical Institute, Russian Academy of Sciences, Fontanka 27, St Petersburg, Russia
† Helsinki Institute of Physics, PO Box 9, FIN-00014 University of Helsinki, Finland
‡ Department of Theoretical Physics, Uppsala University, PO Box 803, S-75108 Uppsala, Sweden

In 1867, Lord Kelvin proposed that atoms—then considered to be elementary particles—could be described as knotted vortex tubes in ether[1]. For almost two decades, this idea motivated an extensive study of the mathematical properties of knots, and the results obtained at that time by Tait[2] remain central to mathematical knot theory[3,4]. But despite the clear relevance of knots to a large number of physical, chemical and biological systems, the physical

387 | 1 MAY 1997

Nature 387, 58–61 (1997)

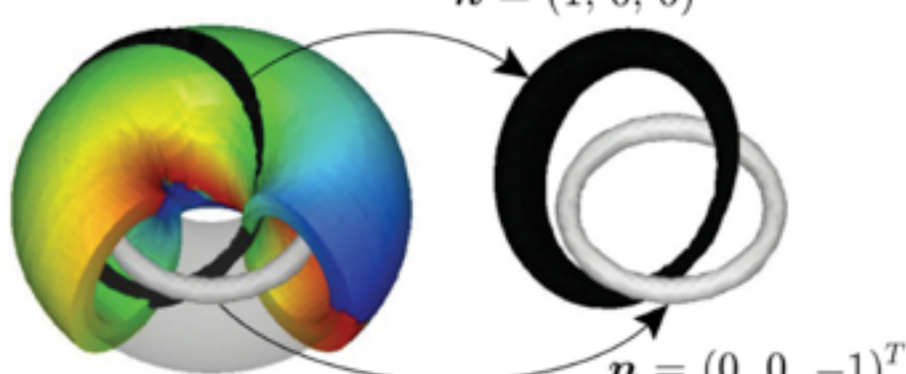
PRL 100, 180403 (2008) PHYSICAL REVIEW LETTERS week ending 9 MAY 2008

Knots in a Spinor Bose-Einstein Condensate

Yuki Kawaguchi,[1,*] Muneto Nitta,[2] and Masahito Ueda[1,3,*]

[1]Department of Physics, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8551, Japan
[2]Department of Physics, Keio University, Hiyoshi, Yokohama, Kanagawa 223-8521, Japan
[3]ERATO Macroscopic Quantum Control Project, JST, Tokyo 113-8656, Japan
(Received 14 February 2008; published 7 May 2008)

We show that knots of spin textures can be created in the polar phase of a spin-1 Bose-Einstein condensate, and discuss experimental schemes for their generation and probe, together with their lifetime.



Isolated optical vortex knots
Mark R. Dennis, Robert P. King, Barry Jack, Kevin O'Holleran and Miles J. Padgett
Nature Physics 6, 118 - 121 (2010)

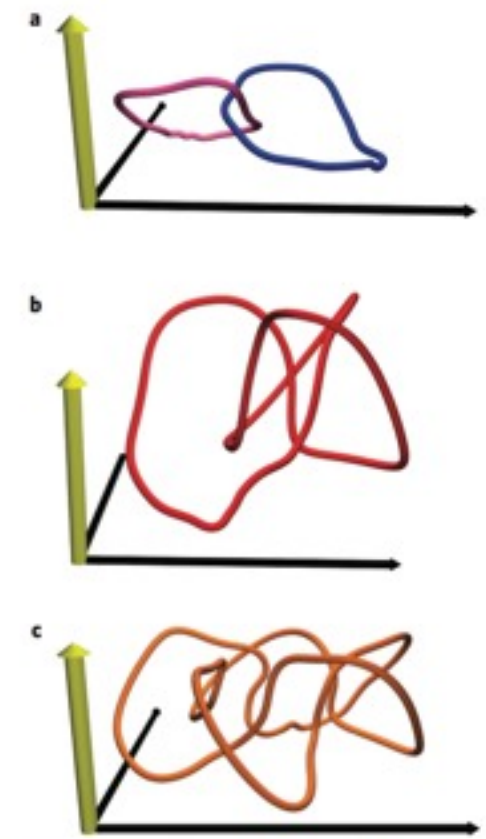


Figure 3 | Plots of experimental nodal knots and links. The curves

$$-\frac{\hbar^2}{2m}\nabla^2\phi(\mathbf{r}) + V_{\mathrm{ext}}(\mathbf{r})\phi(\mathbf{r}) + g|\phi(\mathbf{r})|^2\phi(\mathbf{r},t) = \mu\phi(\mathbf{r})$$

external potential assumed smooth on the local healing length scale

$$-\frac{\hbar^2}{2m}\nabla^2$$

quantum kinetic potential

$$g|\phi(\mathbf{r})|^2$$

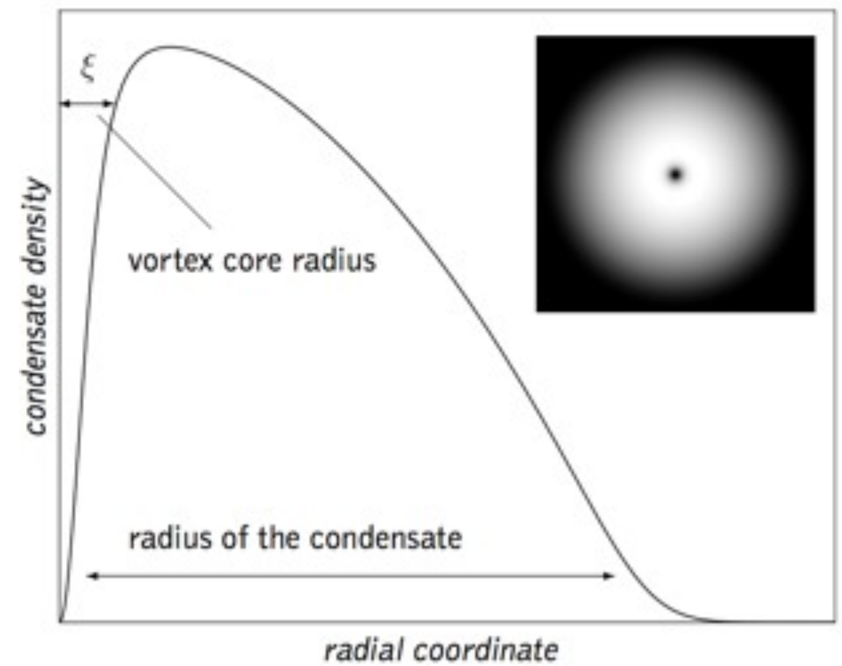
repulsive particle potential

these are balanced when

$$\frac{\hbar^2}{2m}\frac{1}{\xi^2} = gn(\mathbf{r})$$

healing length

$$\boxed{\xi = \frac{\hbar}{\sqrt{2m\mu}}}$$



healing length: characteristic small spatial scale in the system, length scale over which the wavefunction can "heal" when forced to zero at a point, typical size of a vortex core

vorticity is quantized

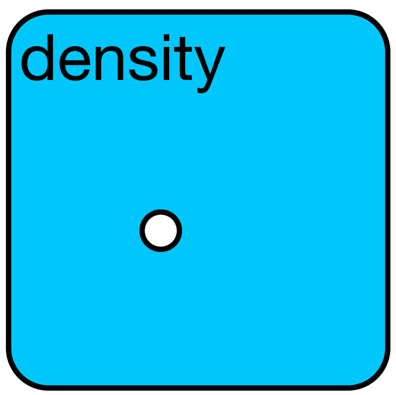
if v is smooth!

scalar field $\phi(\mathbf{r},t) = |\phi(\mathbf{r},t)| e^{i\theta(\mathbf{r},t)}$ $\quad \mathbf{v}(\mathbf{r},t) = \frac{\hbar}{m}\nabla\theta(\mathbf{r},t)$ $\quad \omega(\mathbf{r},t) = \nabla \times \mathbf{v} = 0$

must be single valued
(to prevent possibility of
inconsistent observations
by two different observers)!

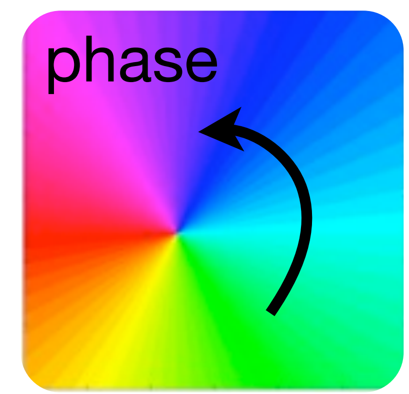
potential flow, irrotational

vorticity



must have a
zero in density!

if singularity
in the phase



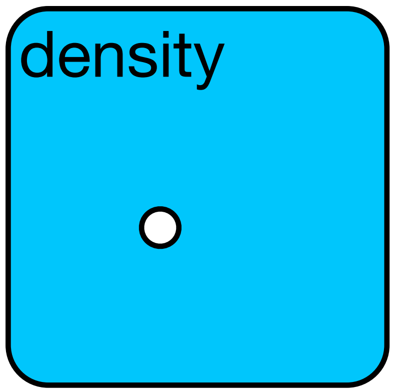
$$\kappa = \oint \mathbf{v} \cdot d\mathbf{l} = \frac{\hbar}{m} \oint \nabla\theta(\mathbf{r}) \cdot d\mathbf{l} = \ell 2\pi \frac{\hbar}{m} = \ell \frac{h}{m}$$

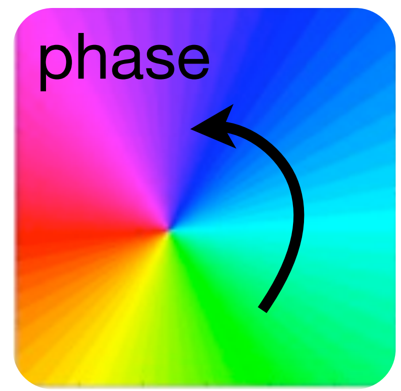
Bohr-Sommerfeld-Wilson-Dirac-Onsager-Feynman
quantization (of circulation) condition

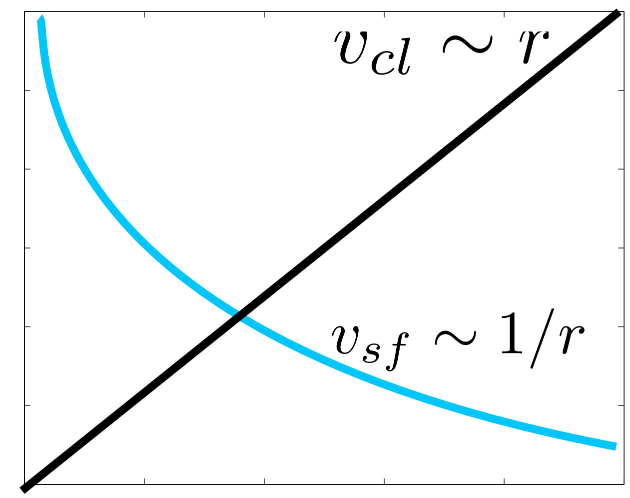
vorticity is quantized

scalar field $\phi(\mathbf{r},t) = |\phi(\mathbf{r},t)|e^{i\theta(\mathbf{r},t)}$

$$\mathbf{v}(\mathbf{r},t) = \frac{\hbar}{m}\nabla\theta(\mathbf{r},t)$$







solid body

vortex

assuming purely azimuthal flow $\phi = f(r)e^{i\ell\varphi}$ $\quad v = \frac{h}{m}\frac{\ell}{r}$

kinetic energy stored in a vortex

$$E_v = \int \frac{1}{2}m|\phi|^2 v^2 d\mathbf{r} = \frac{mn}{2}\int_{r_c}^{R}\frac{h^2\ell^2}{m^2r^2} r dr 2\pi L = \frac{\ell^2 n\hbar^2\pi L}{m}\ln\left(\frac{R}{|\ell| r_c}\right)$$

unlike in classical systems multiquantum vortices are in general energetically / dynamically unstable against splitting into multiple simple vortices which arrange into a vortex lattice (minimum energy principle!)

phase vortices are just nodal lines of complex functions!

vortices in electromagnetic fields

acoustic vortices

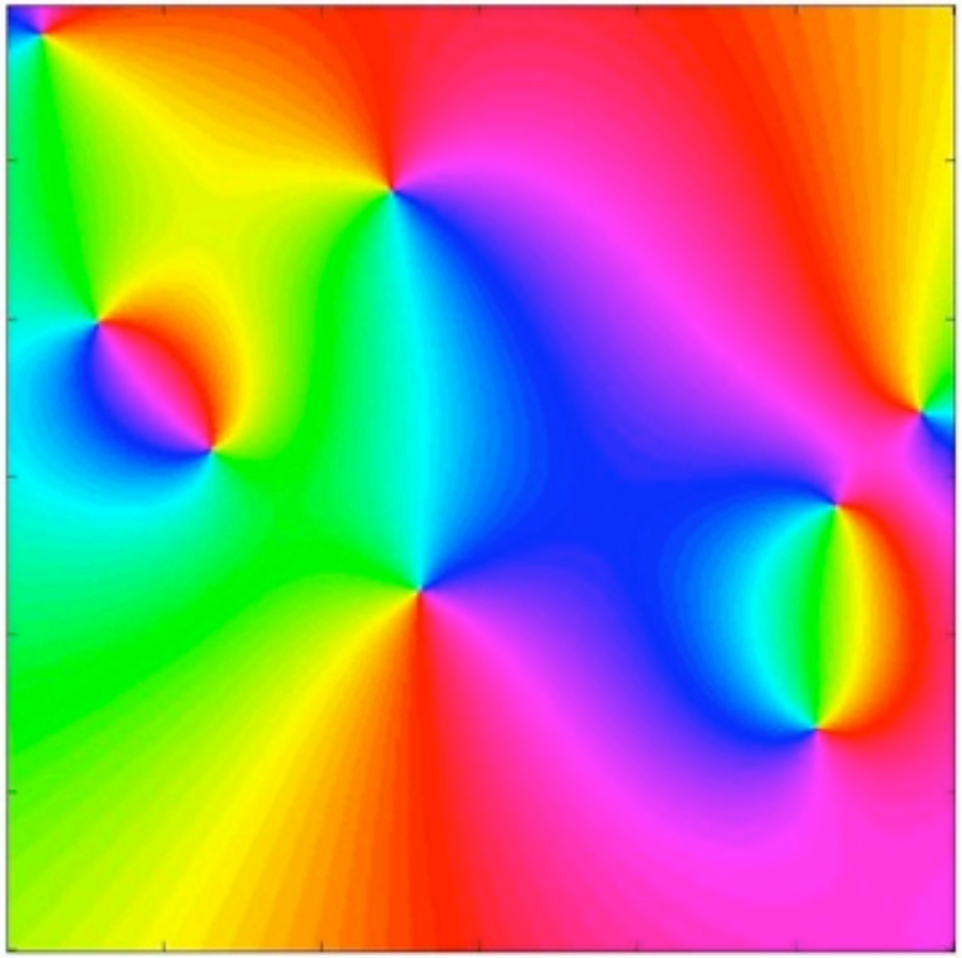
superfluid helium or any other BEC

type II superconductors

hydrogen atom angular momentum eigenstates

electron vortices

.
.
.

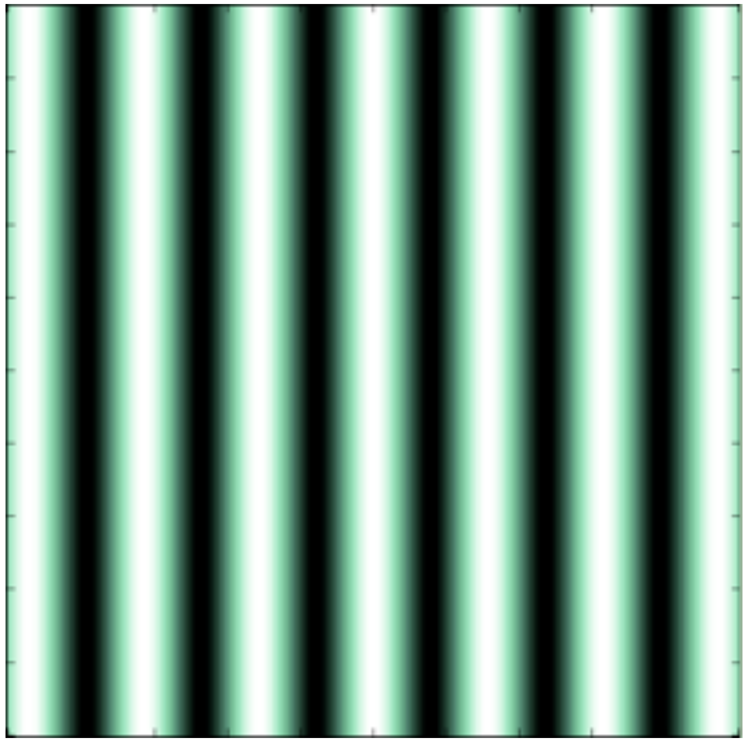


signature of a vortex

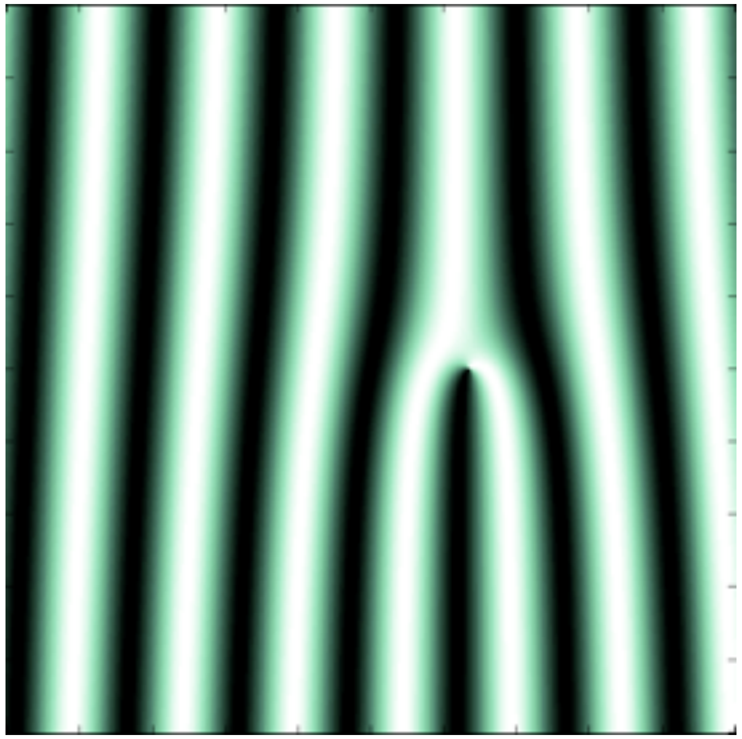
absolute value of phase is not directly observable, and an intensity zero is not a sufficient criterion for the existence of a vortex

measure phase difference via interference $\left|\phi_1(\mathbf{r}) + \phi_2(\mathbf{r})\right|^2$

$$\phi_1(\mathbf{r}) = e^{ikx}$$
$$\phi_2(\mathbf{r}) = 1$$



$$\phi_1(\mathbf{r}) = e^{ikx}$$
$$\phi_2(\mathbf{r}) = e^{i\theta}$$



fork in an interferogram marks the dislocation!

Phys. Rev. Lett. **87**, 080402 (2001)

how to create vortices in BECs?

B. Anderson, Resource Article: Experiments with Vortices in Superfluid Atomic Gases
Journal of Low Temperature Physics 161, 574 **(2010)**

coherent population transfer / phase imprinting

stirring / rotating bucket

dynamical instabilities

topological Berry phase engineering

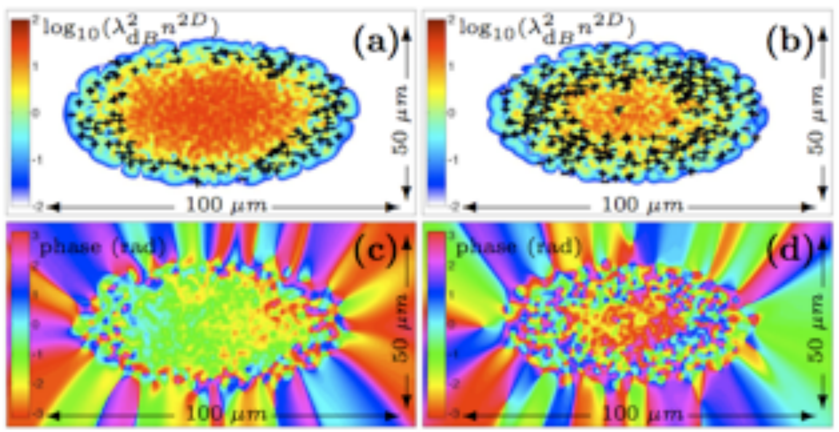
spontaneous formation

matterwave interference

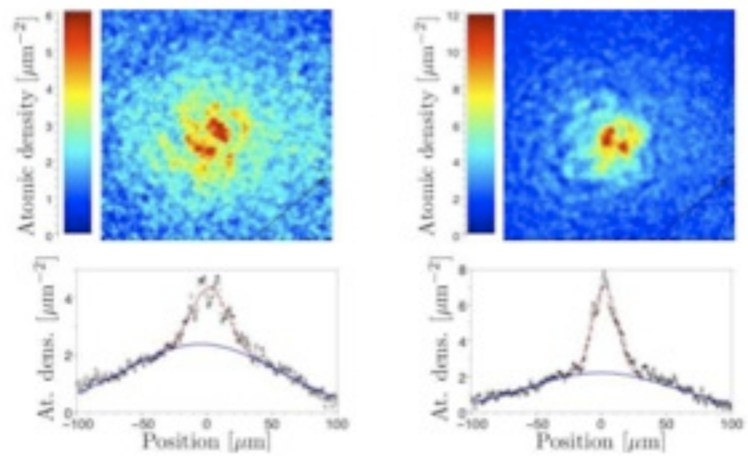
etc.

examples: **spontaneous** vortices

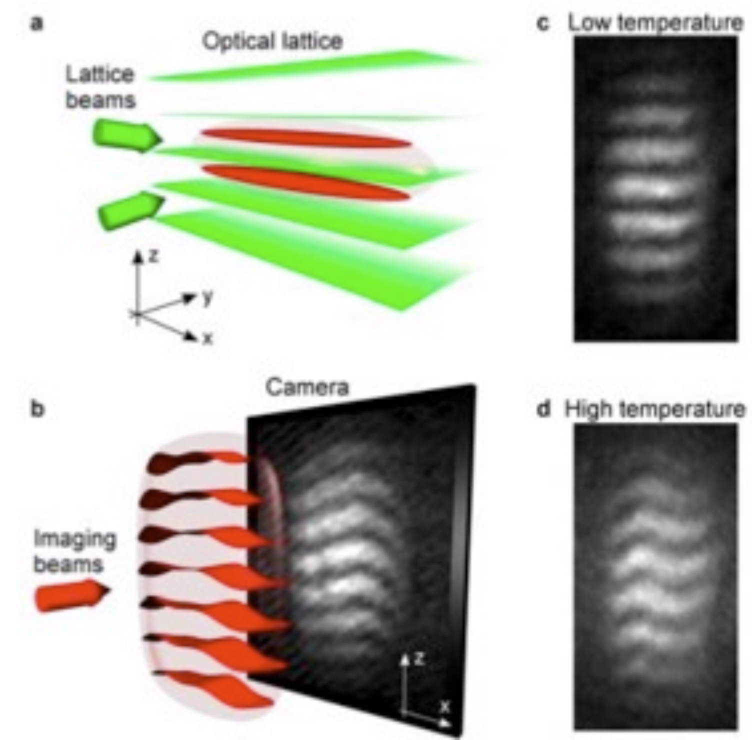
Berezinskii-Kosterlitz-Thouless mechanism



colder hotter

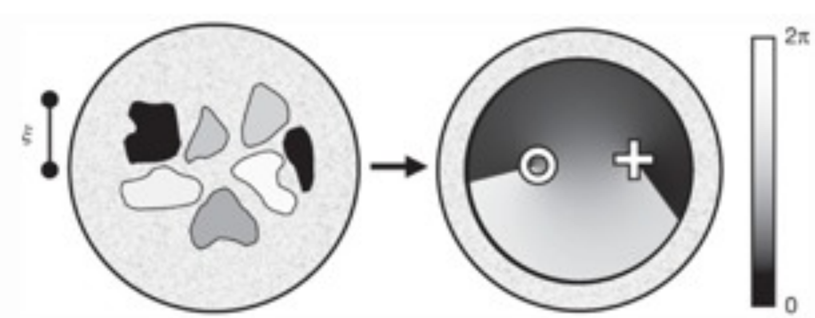


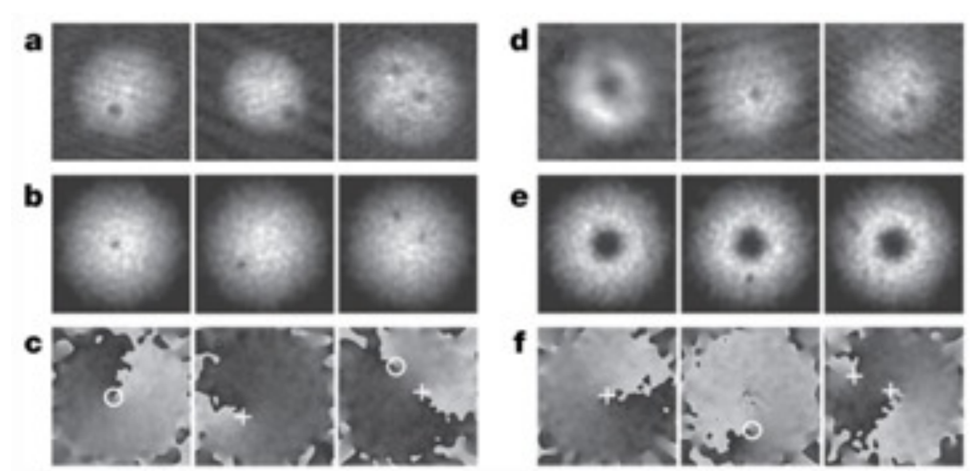
Phys. Rev. Lett. **102**, 170401 (2009)



Nature **441,** 1118 (2006)

Kibble-Zurek mechanism

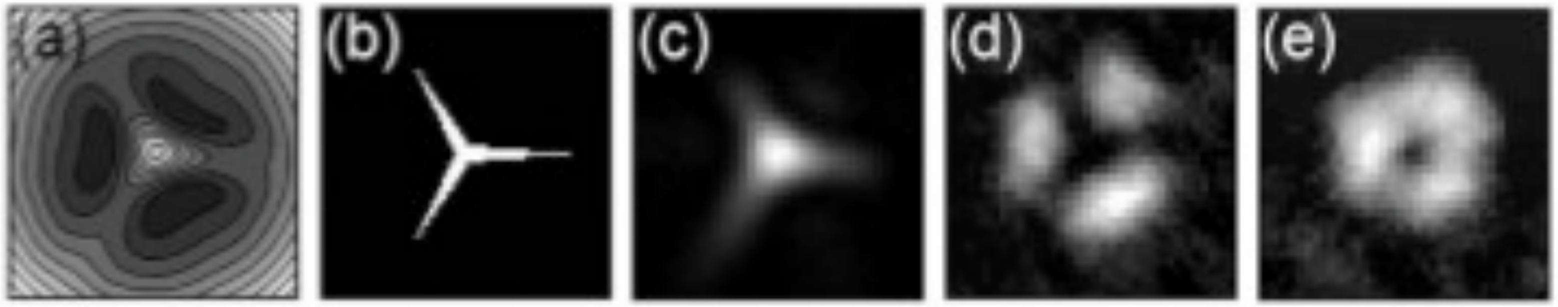




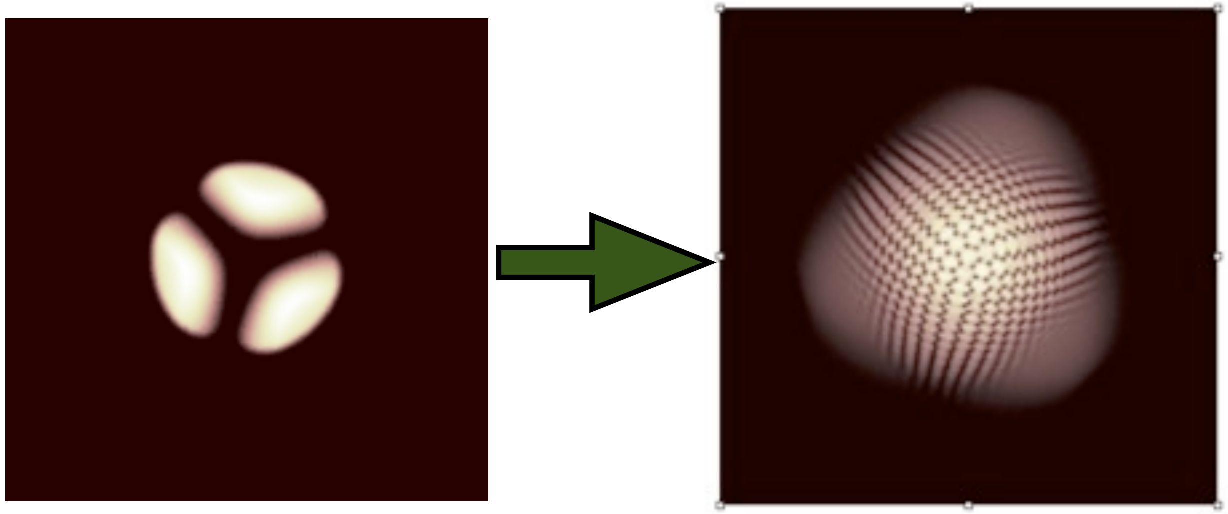
Nature **455**, 948-951 (2008)

examples: **matterwave interference**

interfering **three or more** waves

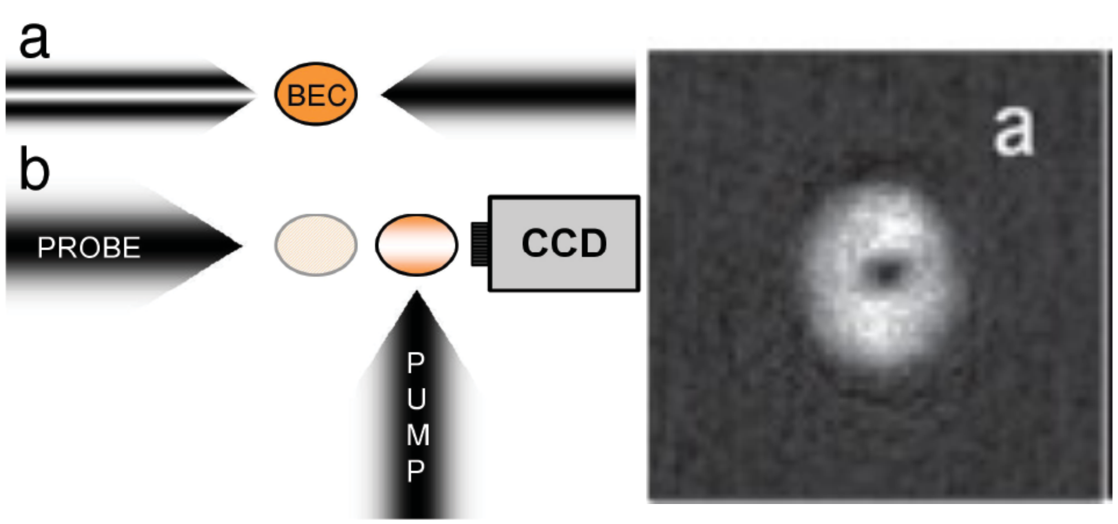


PRL **98**, 110402 (2007)

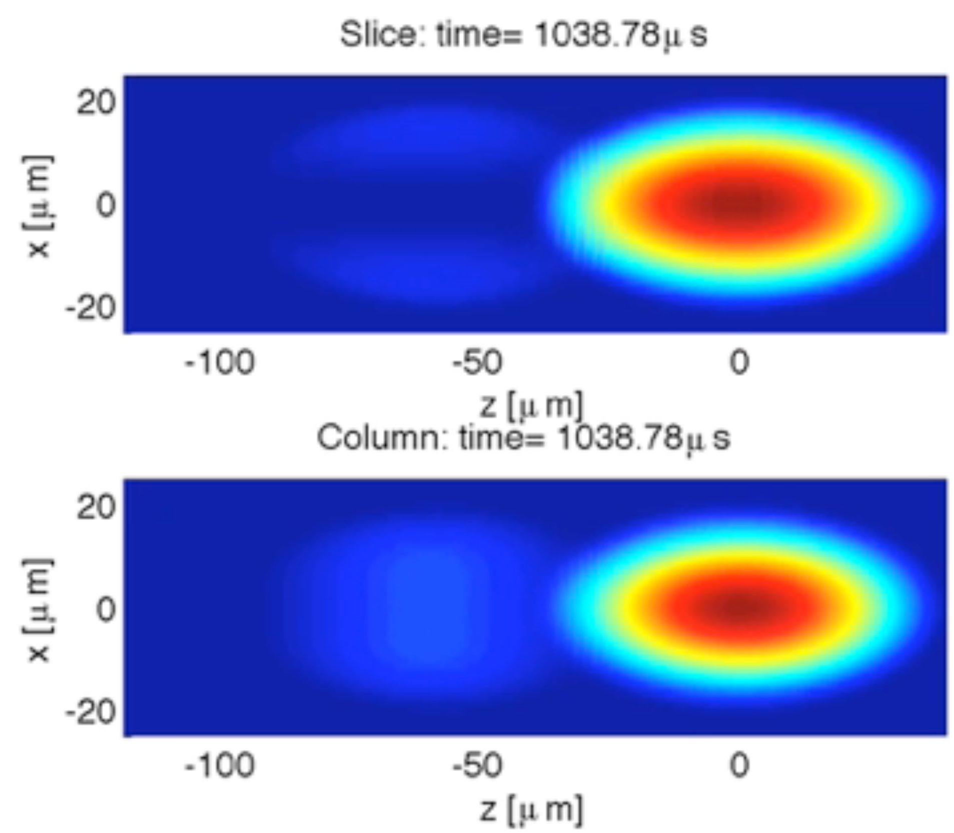


PRA **78**, 013631 (2008)

examples: **Bragg-scattering using Laguerre-Gauss beams**



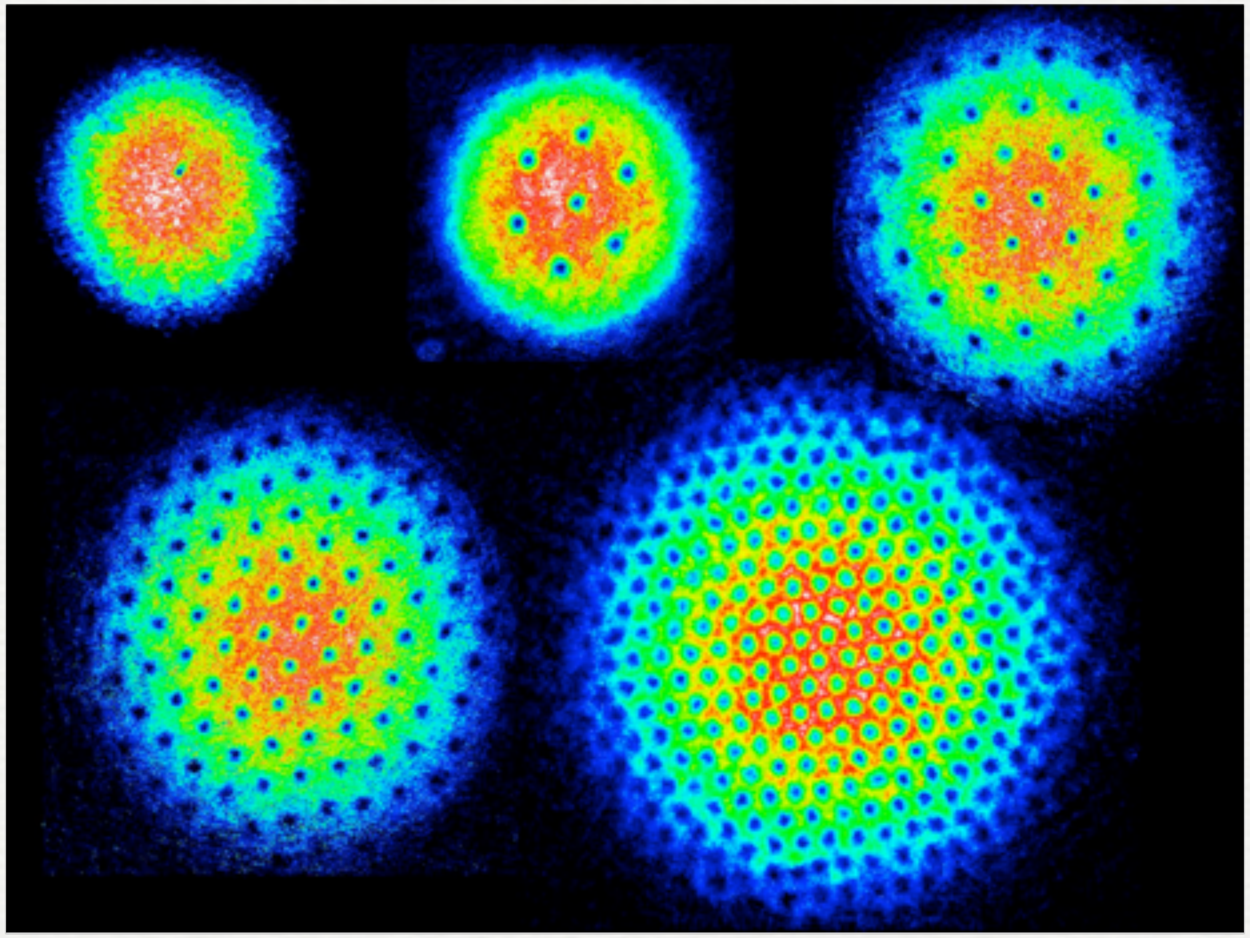
Phys. Rev. Lett. **97**, 170406 (2006)



Phys. Rev. A **77**, 015401 (2008)

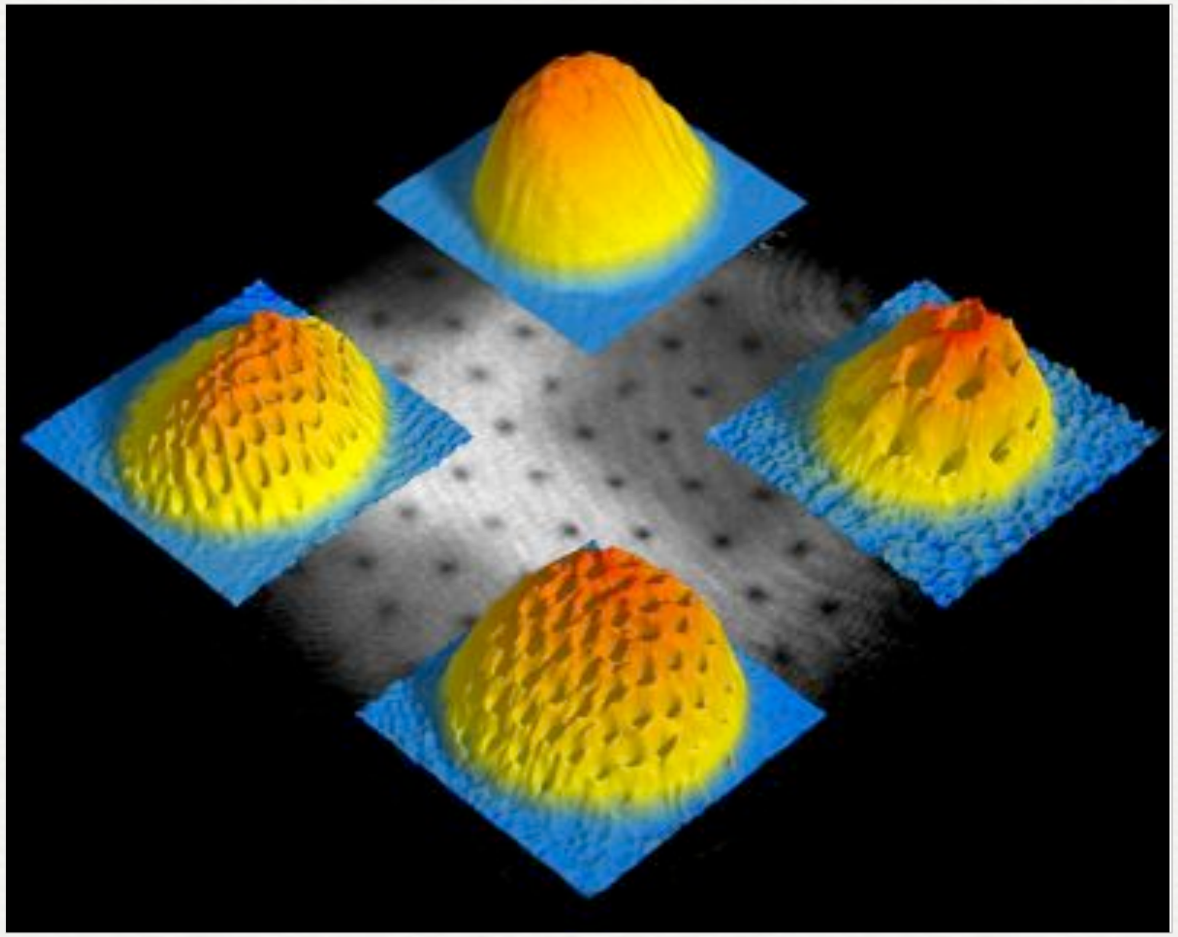
examples: **rotating bucket**

JILA



Phys.Rev.Lett. **87,** 210403 (2001)

MIT



Science **292**, 476 (2001)

Gross-Pitaevskii equation in the rotating frame

$$i\hbar\frac{\partial}{\partial t}\phi(\mathbf{r},t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\mathbf{r},t) + g|\phi(\mathbf{r},t)|^2 - \Omega L_z\right)\phi(\mathbf{r},t)$$

Landau critical angular frequency

$$\Omega_c = \min\left(\frac{\omega_l}{l}\right)$$

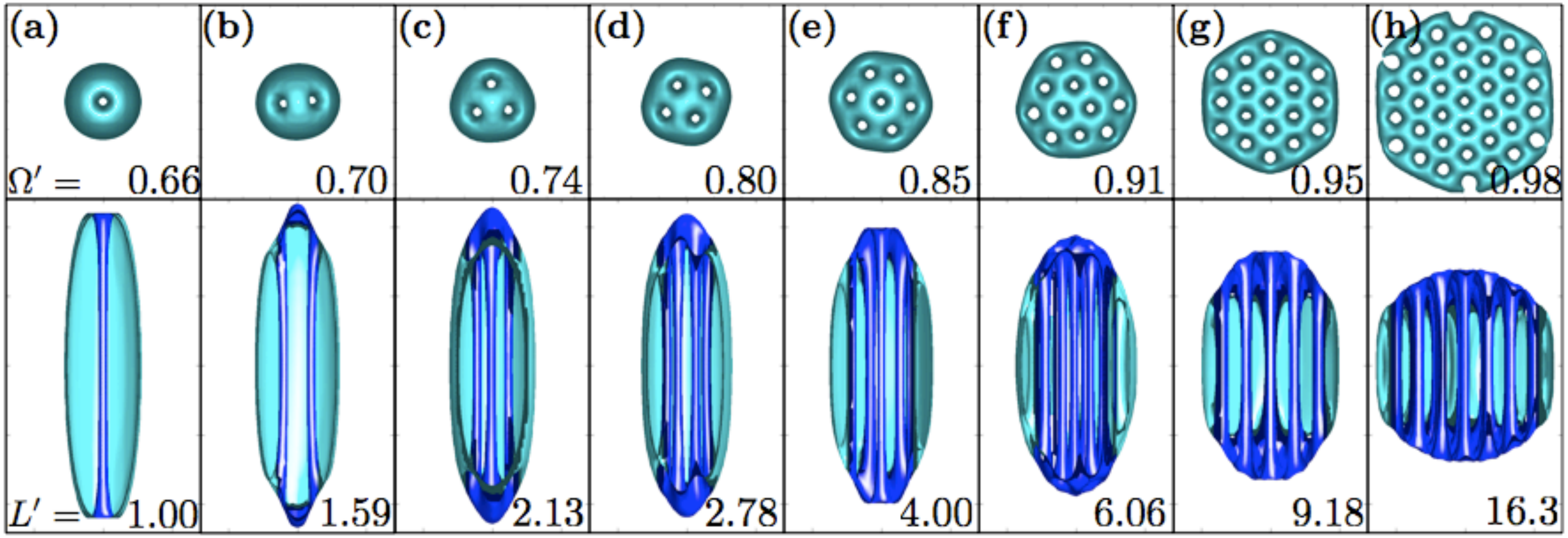
Feynman's rule

$$\kappa n_v = 2\Omega$$

$$\omega = \nabla \times \mathbf{v} = \nabla \times \boldsymbol{\Omega} \times \mathbf{r} = 2\Omega$$

classical rigid body rotation

Abrikosov vortex lattice minimizes free energy
(c.f. closest packing of spherical objects)



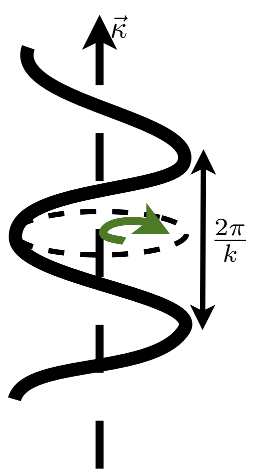
Phys. Rev. A **82**, 063627 (2010)

What happens to a condensate if you hit it gently? *Bogoliubov modes !*

- dipole mode (sloshing, Kohn, centre-of-mass)
- breathing mode
- quadrupole mode
- scissors mode
- etc.

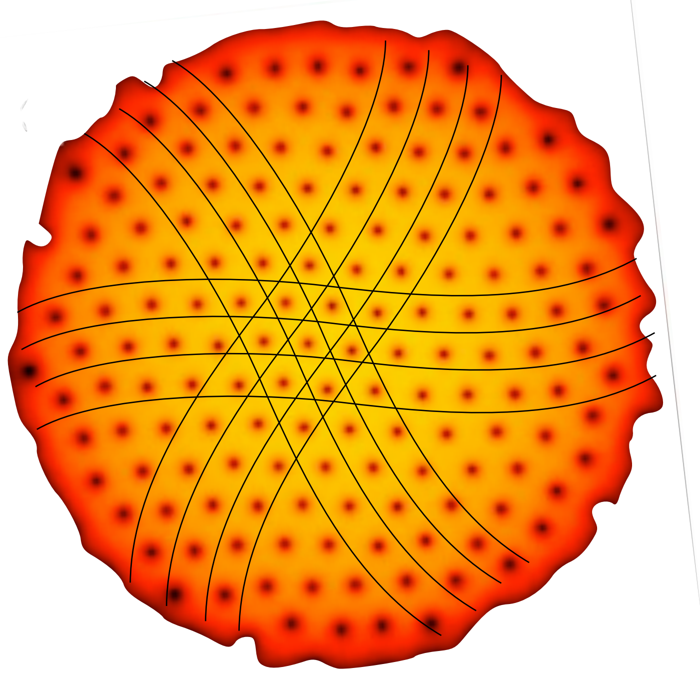
in the presence of vortices Kelvin-Tkachenko collective modes emerge

superfluid Kelvin waves



Pitaevskii 1961

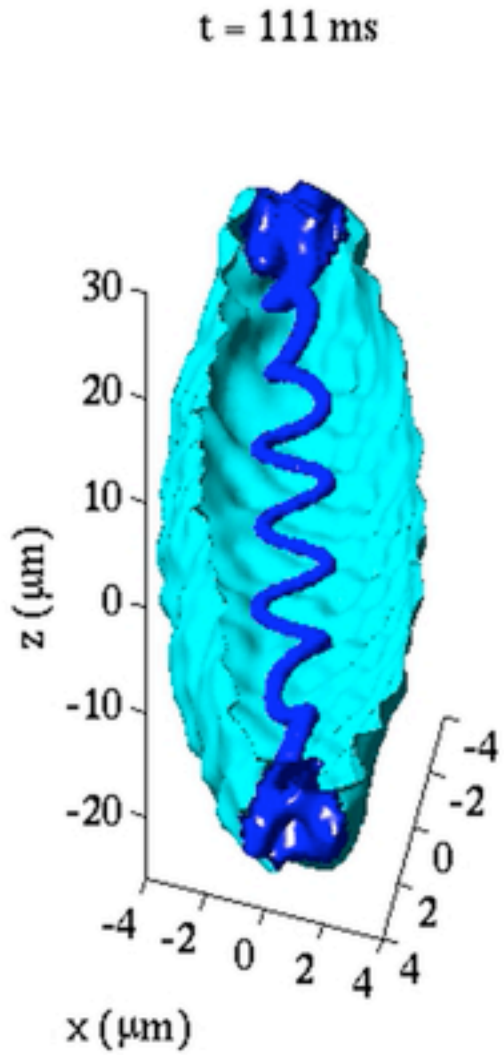
Tkachenko waves



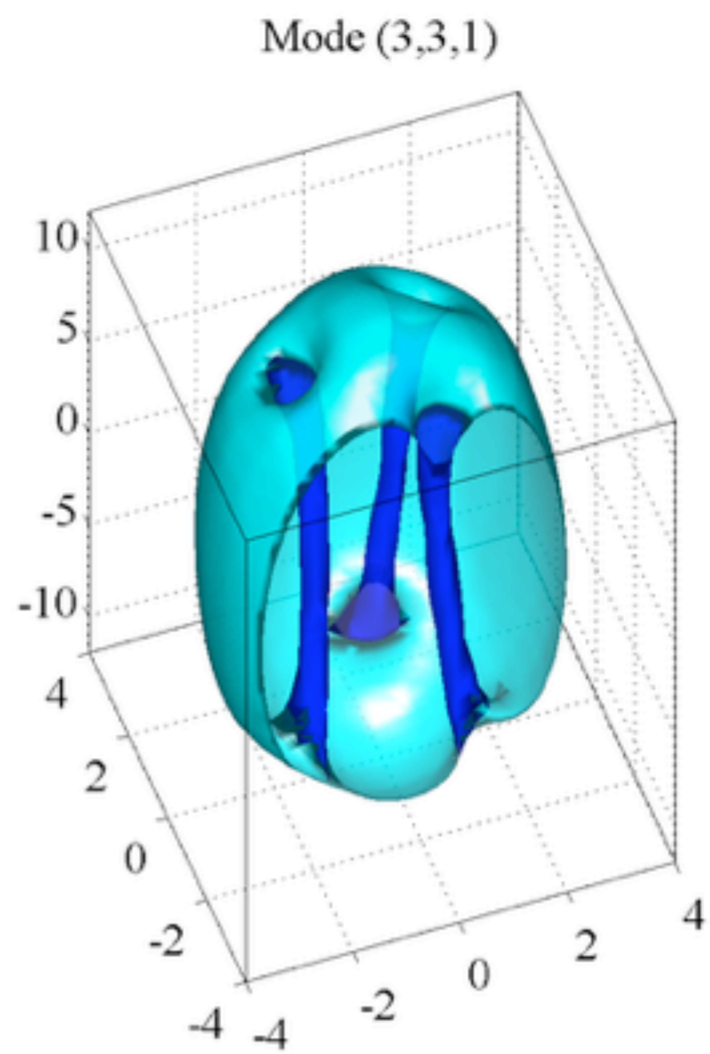
Tkachenko 1966

What happens to a condensate if you hit it gently? *Bogoliubov modes !*

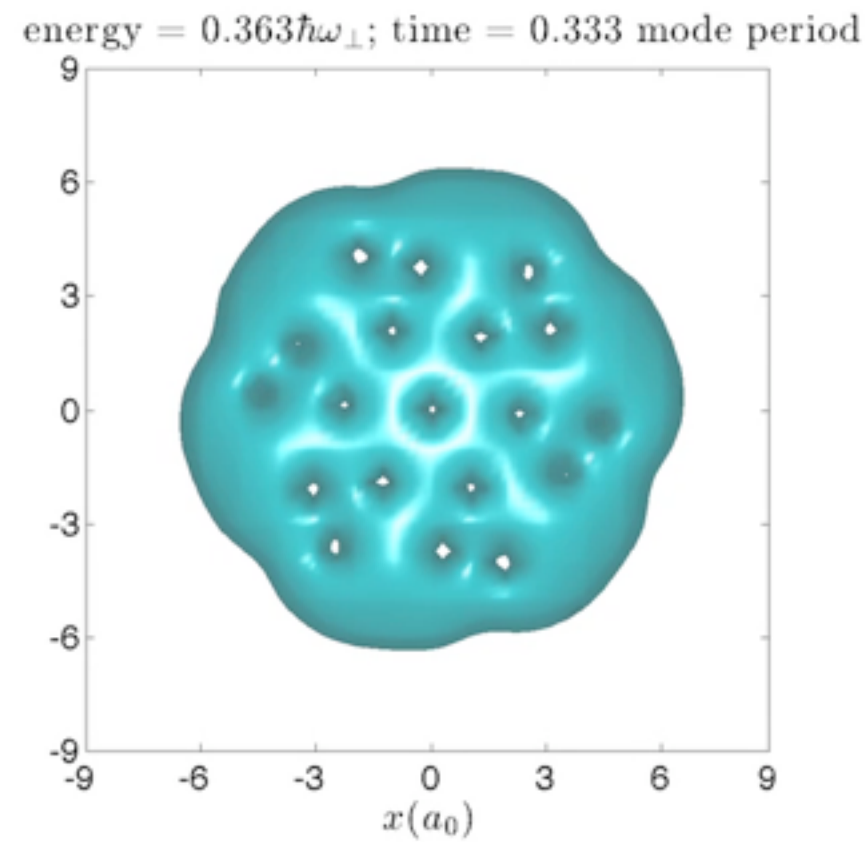
in the presence of vortices Kelvin-Tkachenko collective modes emerge



Phys. Rev. Lett. **101**, 020402 (2008)

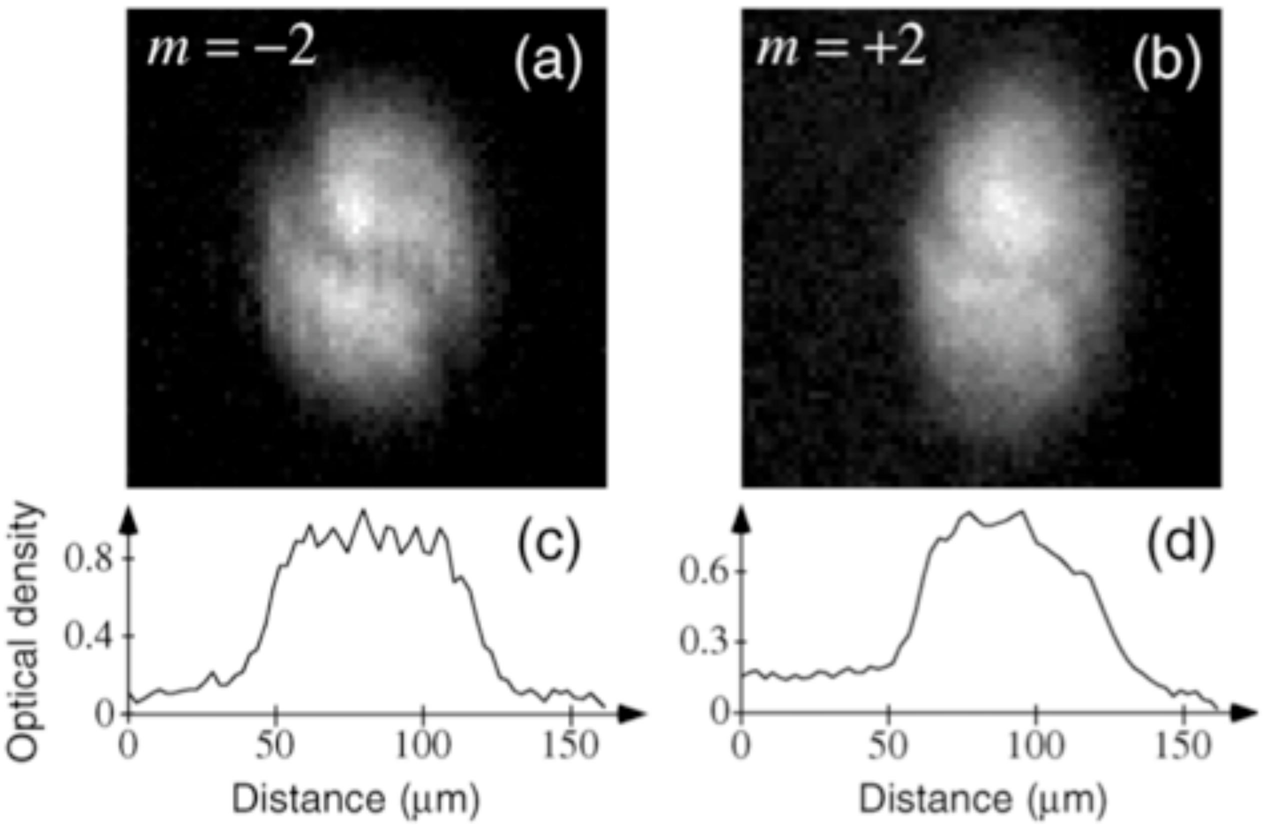


Phys. Rev. A **82**, 063627 (2010)



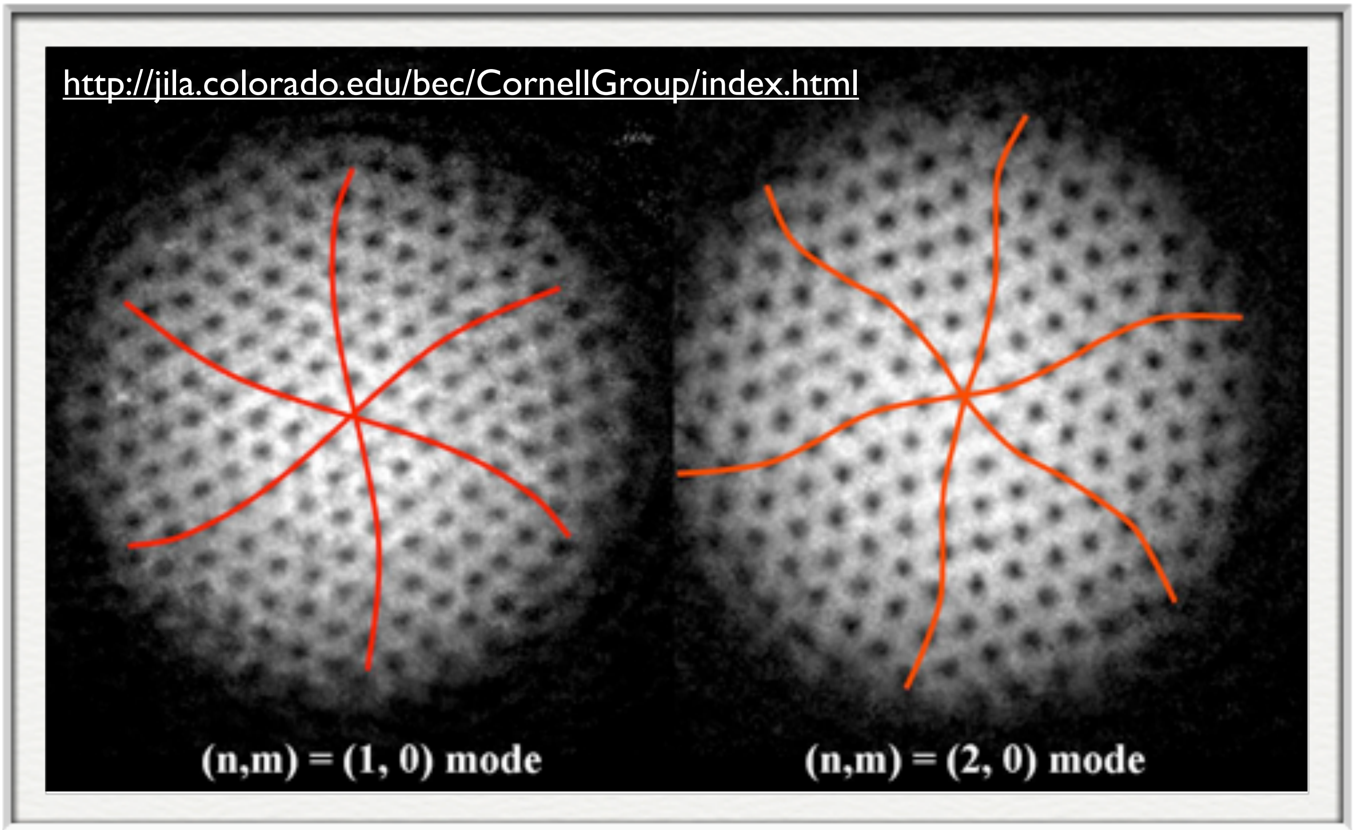
experimental observations of vortex waves

Kelvin wave @ ENS



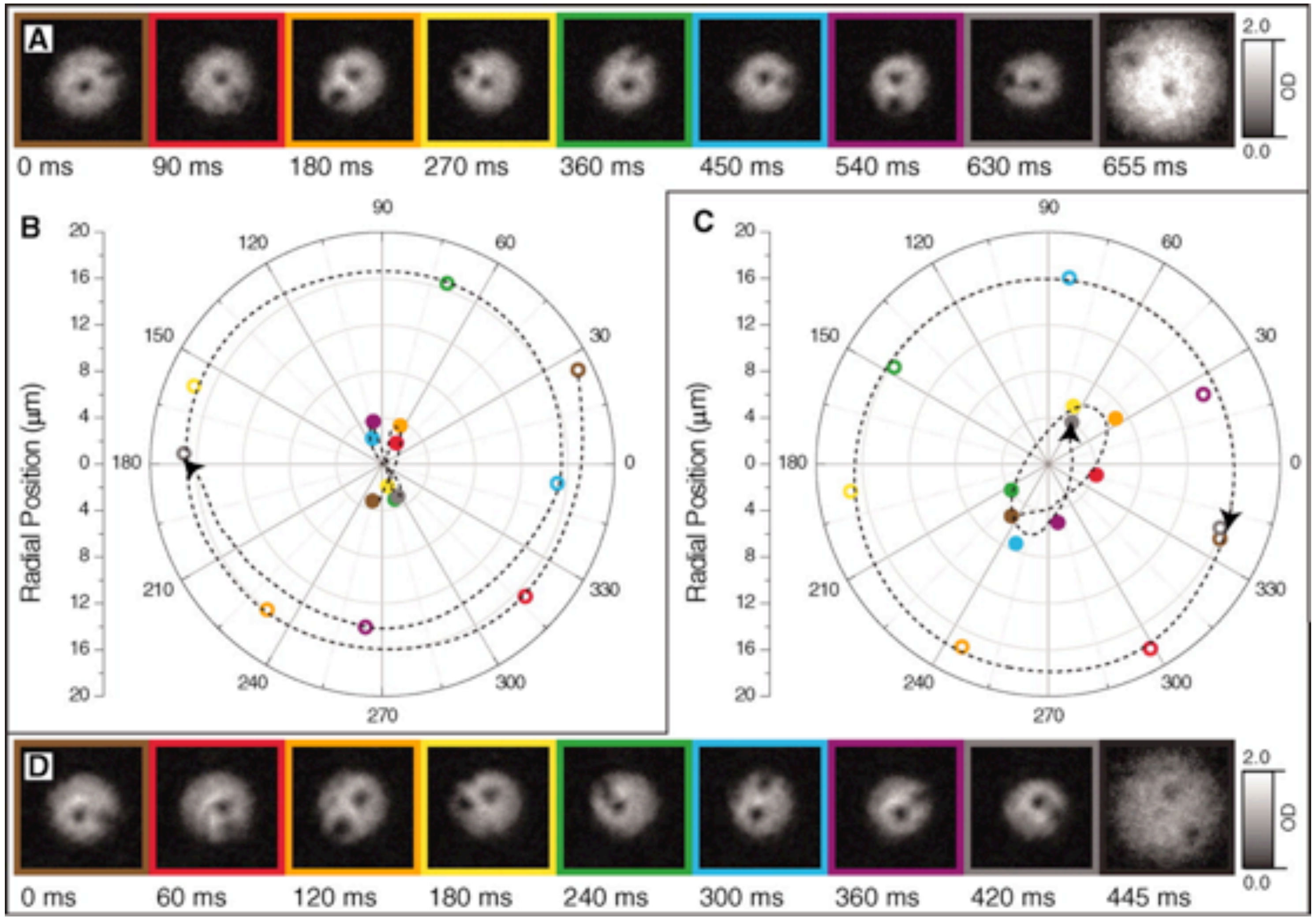
Phys. Rev. Lett. **90**, 100403 (2003)

Tkachenko wave @ JILA



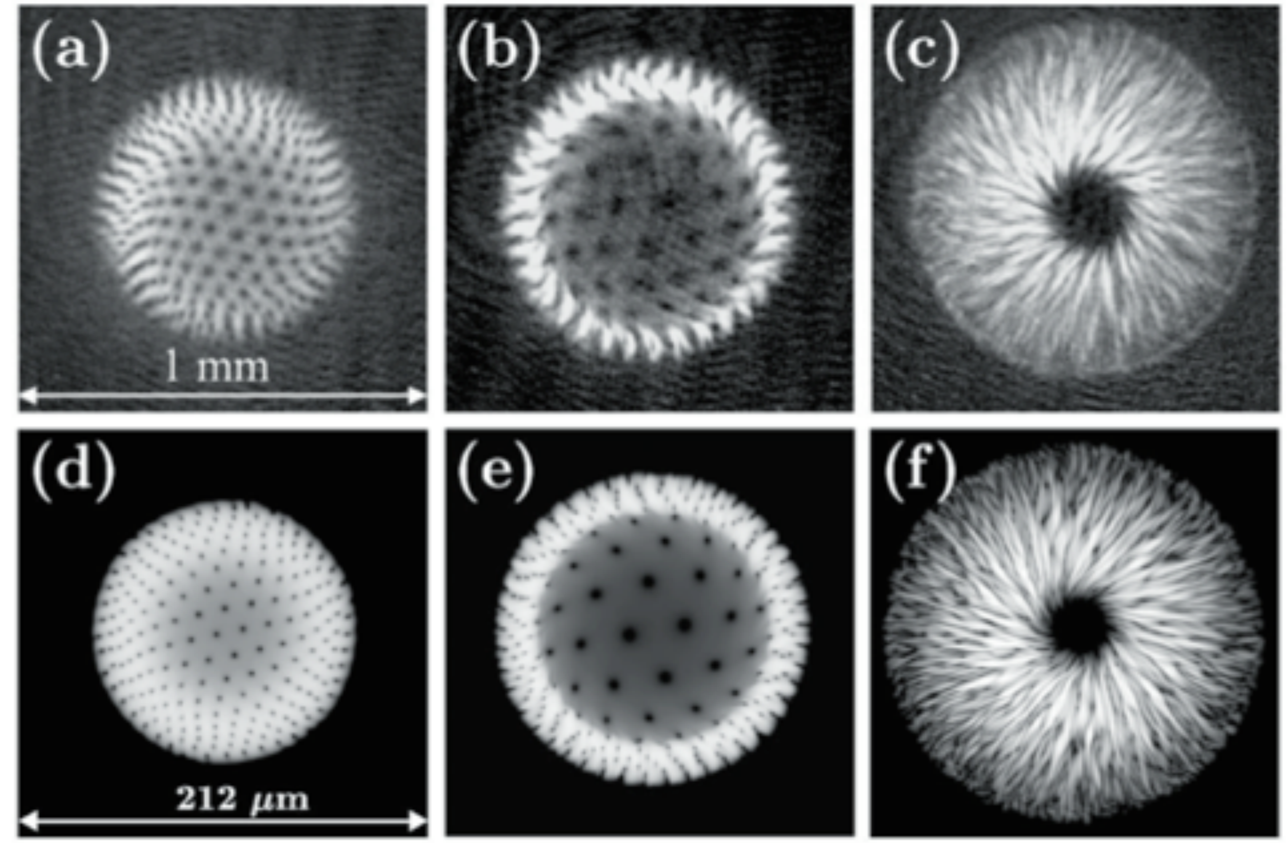
Phys. Rev. Lett. **91**, 100402 (2003)

collective dynamics of a vortex-antivortex pair



Real-Time Dynamics of Single Vortex Lines and Vortex Dipoles in a Bose-Einstein Condensate
D. V. Freilich *et al. Science* **329**, 1182 (2010)

What happens to a condensate if you hit it hard? *shock waves and turbulence!*



Phys. Rev. Lett. **94**, 080404 (2005)