Probing and Manipulating Majorana Fermions in SO Coupled Atomic Fermi Gases

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Hawthorn, July.

Outline

1. Majorana Fermion
2. Probing Majorana Fermion in SO Coupled Fermi Gases
3. Manipulating Majorana Fermion in SO Coupled Fermi Gases

Majorana Fermion

Majorana Fermion: particle is its own antiparticle

$$\gamma = \gamma^+$$

$$\gamma^2 = 1 \qquad \{\gamma_j, \gamma_k\} = 2\delta_{jk}$$

Quantum statistics of Majorana Fermion: anyon

$$|\Psi_1\Psi_2> = M_{21}|\Psi_2\Psi_1>$$

Fermion: $\left(c_j^+\right)^2 = c_j^2 = 0 \qquad \{c_j, c_k^+\} = \delta_{jk}$

Majorana Fermion

Potential System Hosts for Majorana Fermion Bound States

•Neutrinos Majorana 1937

•Supersymmetry: photino; neutral gauginos; Higgsinos

Condensed Matter System: quasiparticles

•Quasiparticles in fractional Quantum Hall effect at n=5/2 Moore Read 1991

•Unconventional superconductors -Sr2RuO4 Das Sarma, Nayak, Tewari 2006

•Proximity Effect Devices using ordinary s wave superconductors

-Topological Insulator devices Fu, Kene 2008
-Semiconductor/Magnet devices Sau, Lutchyn, Tewari, /das Sarma 2009

Current Status: Not Observed??

Current status: observed in nanowires (2012), but not unambiguously confirmed.

Majorana Fermion

Suggestion: Ultracold Fermionic Atoms near Feshbach Resonance

Das Sarma
T. Mizushima, K. Machida,
M. Sato, Y. Takahashi, S. Fujimoto,
C. Zhang, , et al..

Question: Full Microscopic Calculation ?

Signature for Majorana Fermion ?

2D trapped ultracold Fermi gas with SO coupling

Hamiltonian

$$\mathcal{H} = \int d\mathbf{r}[\mathcal{H}_0(\mathbf{r}) + \mathcal{H}_I(\mathbf{r})]$$

Single- Particle Hamiltonian

$$\mathcal{H}_0(\mathbf{r}) = \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \mathcal{H}_\sigma^S(\mathbf{r})\psi_\sigma + \left[\psi_\uparrow^\dagger V_{SO}(\mathbf{r})\psi_\downarrow + \mathrm{H.c.}\right]$$

$$V_{SO}(\mathbf{r}) = -i\lambda(\partial_y + i\partial_x)$$

$$\mathcal{H}_\sigma^S = -\hbar^2\nabla^2/(2M) + M\omega_\perp^2 r^2/2 - \mu - h\sigma_z$$

Interaction Hamiltonian

$$\mathcal{H}_I(\mathbf{r}) = U_0\psi_\uparrow^\dagger(\mathbf{r})\psi_\downarrow^\dagger(\mathbf{r})\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r})$$

Renormalization

$$1/U_0 + \sum_{\mathbf{k}} 1/(\hbar^2\mathbf{k}^2/m + E) = m/(4\pi\hbar^2)\ln(E_a/E)$$

$$E_a = (0.915/\pi)\exp(-\sqrt{2\pi\hbar/(M\omega_z)}/a_s)\hbar\omega_z$$

Rashba SO Coupling $\propto \sigma_x k_y - \sigma_y k_x$

Dresselhaus SO Coupling $\propto -\sigma_x k_y - \sigma_y k_x$

2D trapped ultracold Fermi gas with SO coupling

MF BdG Equantion

$$\mathcal{H}_{BdG}\Psi_{\eta}(\mathbf{r}) = E_{\eta}\Psi_{\eta}(\mathbf{r})$$

Bogoliubov Transformation

$$\Psi_{\eta}(\mathbf{r}) = [u_{\uparrow\eta}, u_{\downarrow\eta}, v_{\uparrow\eta}, v_{\downarrow\eta}]^T$$

BdG Hamiltonian

$$\mathcal{H}_{BdG} = \begin{bmatrix} \mathcal{H}_{\uparrow}^{S}(\mathbf{r}) & V_{SO}(\mathbf{r}) & 0 & -\Delta(\mathbf{r}) \\ V_{SO}^{\dagger}(\mathbf{r}) & \mathcal{H}_{\downarrow}^{S}(\mathbf{r}) & \Delta(\mathbf{r}) & 0 \\ 0 & \Delta^{*}(\mathbf{r}) & -\mathcal{H}_{\uparrow}^{S}(\mathbf{r}) & V_{SO}^{\dagger}(\mathbf{r}) \\ -\Delta^{*}(\mathbf{r}) & 0 & V_{SO}(\mathbf{r}) & -\mathcal{H}_{\downarrow}^{S}(\mathbf{r}) \end{bmatrix}$$

Gap Function

$$\Delta(\vec{r}) = -U_0 < \Psi_{\downarrow}\Psi_{\uparrow} >$$

$$\Delta = -(U_0/2)\sum_{\eta}[u_{\uparrow\eta}v_{\downarrow\eta}^{*}f(E_{\eta}) + u_{\downarrow\eta}v_{\uparrow\eta}^{*}f(-E_{\eta})]$$

$$n_{\sigma}(\mathbf{r}) = (1/2)\sum_{\eta}[|u_{\sigma\eta}|^2 f(E_{\eta}) + |v_{\sigma\eta}|^2 f(-E_{\eta})]$$

Single Vortex

$$\Delta(\mathbf{r}) = \Delta(r)e^{-i\varphi}$$

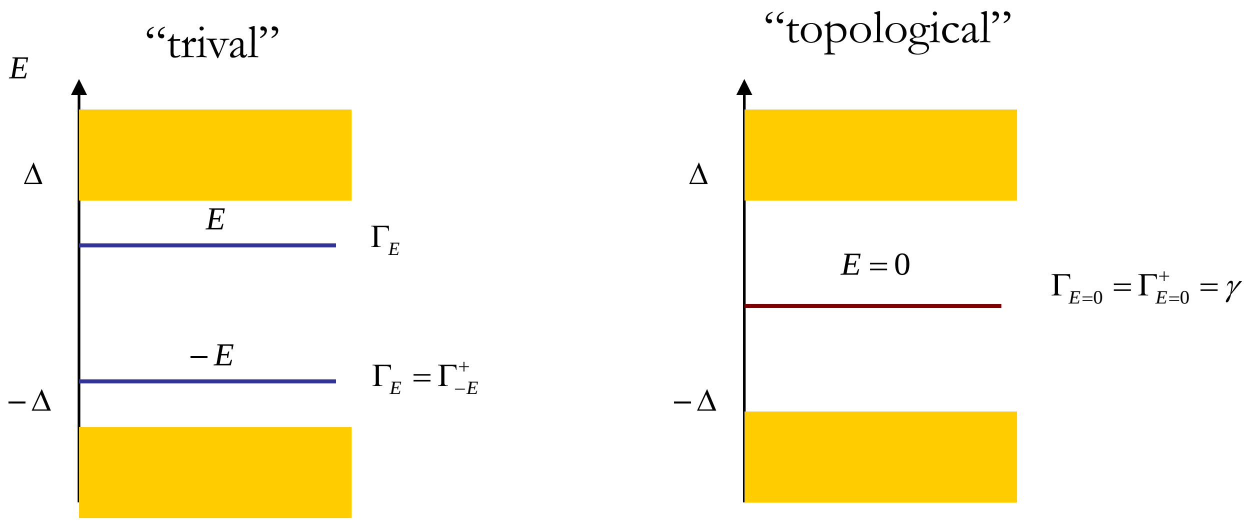
Liu, Hu and Drummond, PRA75, 023614(2007)

Quansiparticle wave function

$$[u_{\uparrow\eta}(r)e^{-i\varphi}, u_{\downarrow\eta}(r), v_{\uparrow\eta}(r)e^{i\varphi}, v_{\downarrow\eta}(r)]e^{i(m+1)\varphi}/\sqrt{2\pi}$$

2D trapped ultracold Fermi gas with SO coupling

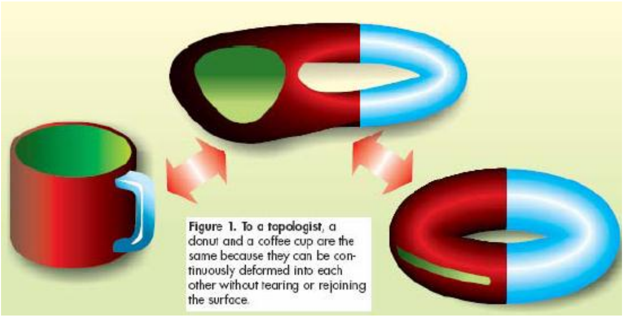
Particle-hole transformation $u_\sigma(\mathbf{r}) \rightarrow v_\sigma^*(\mathbf{r})$ $E_\eta \rightarrow -E_\eta$



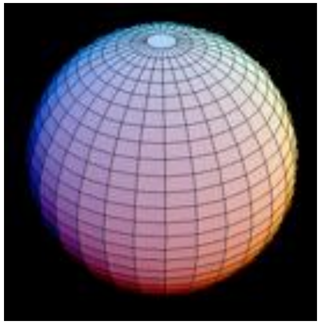
Majorana Fermion: Particle = Antiparticle $\gamma = \gamma^+$

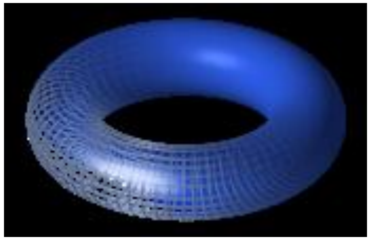
2D trapped ultracold Fermi gas with SO coupling

Topological Invariants



Topological variants





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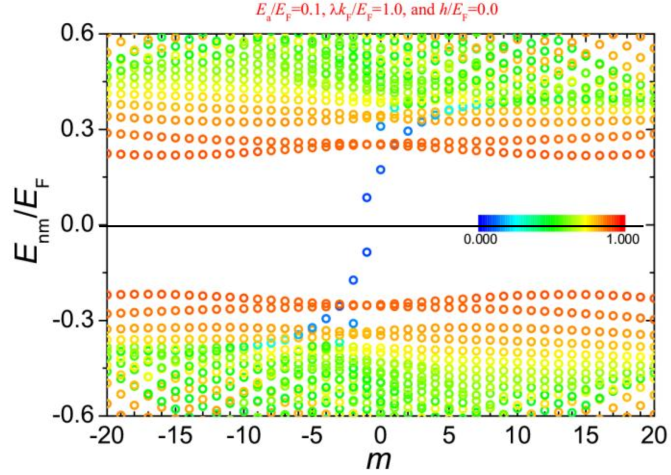
2D trapped ultracold Fermi gas with SO coupling

Question: Does the Majorana fermion bound state exist?

S-wave interaction without SO coupling $E_\eta \rightarrow -E_\eta \quad \begin{bmatrix} u_n \\ v_n \end{bmatrix} \rightarrow \begin{bmatrix} -v_n \\ u_n \end{bmatrix}$ so $u_n = v_n \equiv 0$

There is not zero mode when the Zeeman field h=0



Mixed singlet and triplet pairings ✕ $<\Psi_\uparrow \Psi_\downarrow>$ $<\Psi_\uparrow \Psi_\uparrow>$ $<\Psi_\downarrow \Psi_\downarrow>$

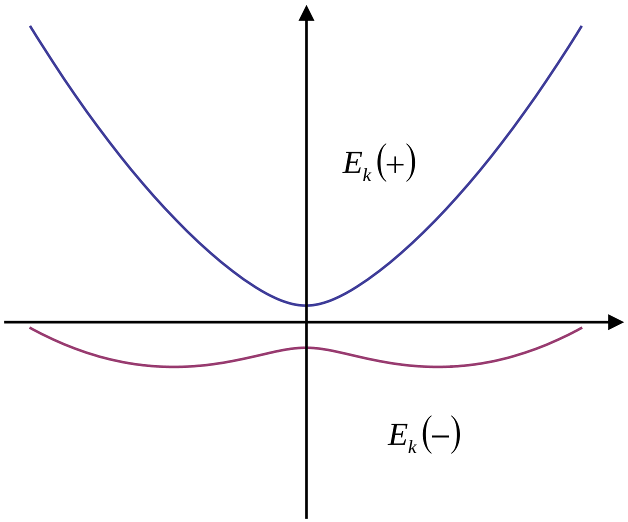
2D trapped ultracold Fermi gas with SO coupling

$$\mathcal{H}_0(\mathbf{r}) = \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \mathcal{H}_\sigma^S(\mathbf{r})\psi_\sigma + \left[\psi_\uparrow^\dagger V_{SO}(\mathbf{r})\psi_\downarrow + \mathrm{H.c.}\right]$$

helicity basis $\Psi_k(+)$ $\Psi_k(-)$

$h > h_c$ only E_k (-) is occuped

$$h_c = \sqrt{\mu^2 + \Delta^2}$$



p-wave symmetry ⇒ *zero mode bound state*

Two solutions

$$u_\sigma = v_\sigma^*$$

$$u_\sigma = -v_\sigma^*$$

$$\Gamma_0 = u_\uparrow c_\uparrow + v_\uparrow c_\uparrow^+ + u_\downarrow c_\downarrow + v_\downarrow c_\downarrow^+$$

$$\Gamma_0^+ = u_\uparrow^* c_\uparrow^+ + v_\uparrow^* c_\uparrow + u_\downarrow^* c_\downarrow^+ + v_\downarrow^* c_\downarrow$$

$$\gamma_1 = \Gamma_0 \qquad \gamma_1 = \gamma_1^+$$

$$\gamma_2 = i\Gamma_0 \qquad \gamma_2 = \gamma_2^+$$

Majorana fermion: particle is its own antiparticle

Fermion operator

$$\Psi_0 = \gamma_1 - i\gamma_2$$

Majorana fermion is a half of ordinary fermion

2D trapped ultracold Fermi gas with SO coupling

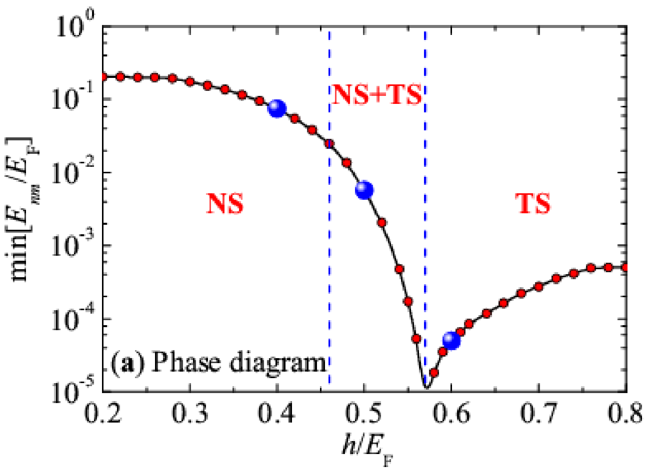
Phase diagram

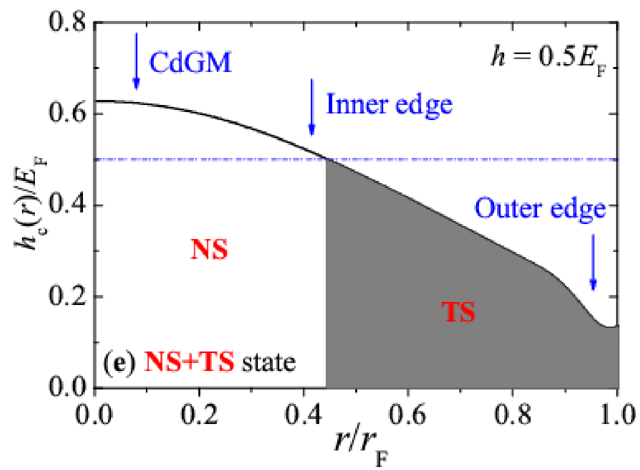
$$h_c(r) = \sqrt{\mu^2(r) + \Delta^2(r)}$$

$$\mu(r) = \mu - \frac{1}{2} m\omega_\perp^2 r^2$$

Non-topological state → *phase separation state* → *topological state*

parameters $E_a = 0.2E_F$ $\quad \lambda k_F / E_F = 1$ $\quad T/T_F = 0$





topological state $\quad h > \sqrt{\mu^2 + \Delta^2(0)}$

2D trapped ultracold Fermi gas with SO coupling

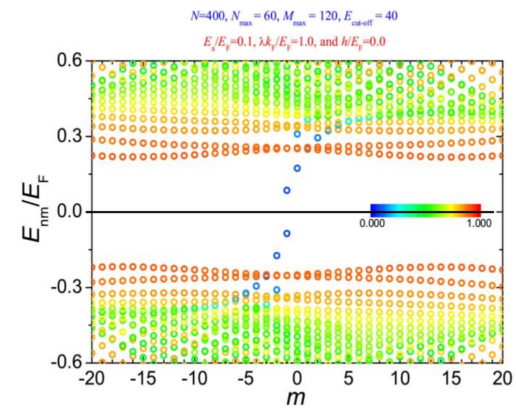
Low-Lying Quasiparticle Spectrum

Three branches with small energy spacing appear:

"Outer edge" state

"Inner edge" state

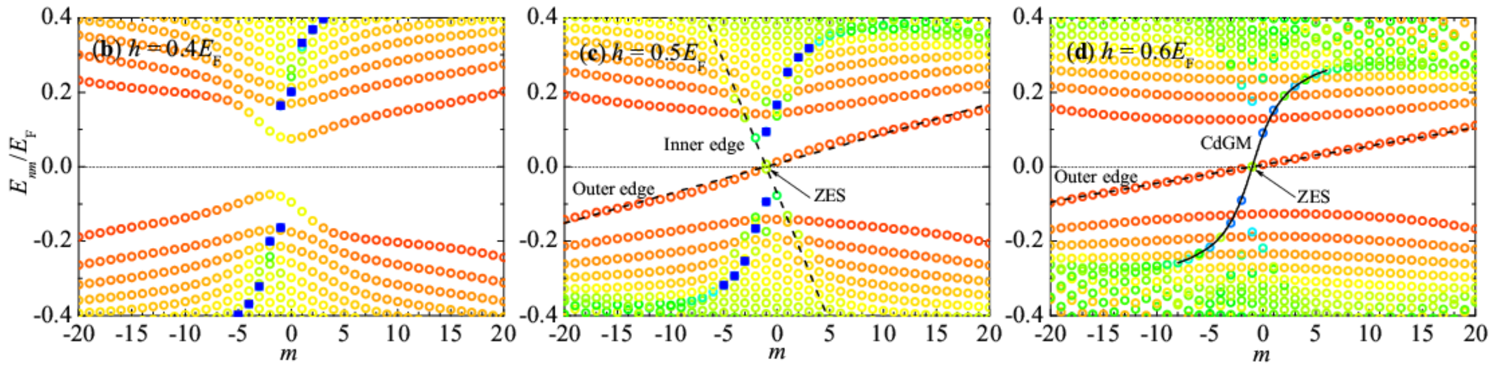
Vortex core GdGM state (garoli-de Gennes-Matricon)



Phase separation phase: "outer edge" and "inner edge"

Topological state: "outer edge" and "GdGM state"

h



2D trapped ultracold Fermi gas with SO coupling

Wave Function

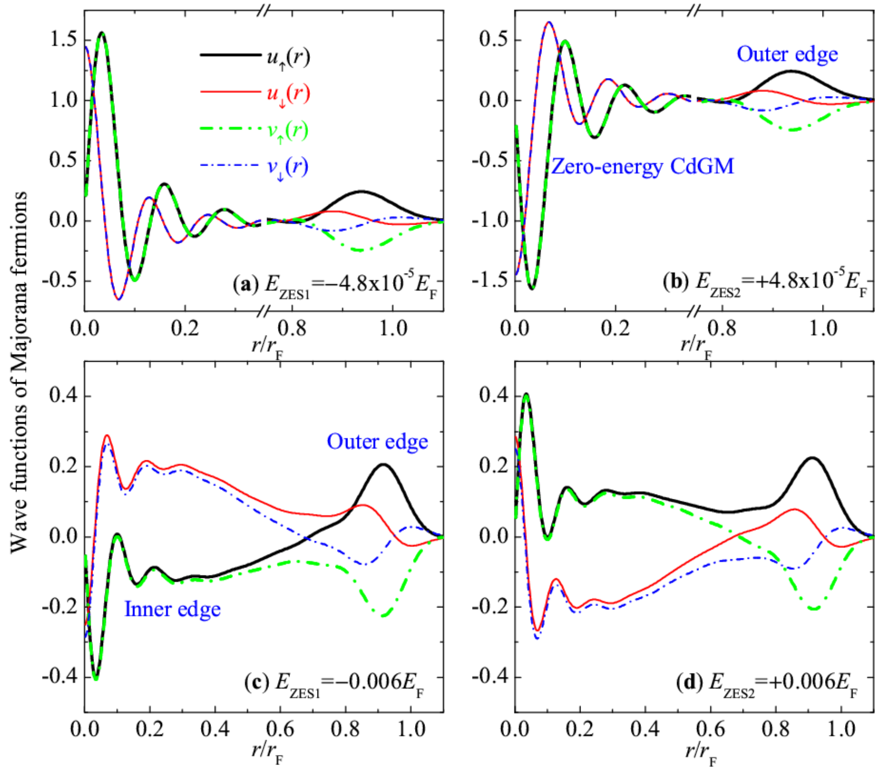
a bond and anti-bond hybridization $u_\sigma = v_\sigma^*$ *and* $u_\sigma = -v_\sigma^*$

quasiparticle tunneling → *energy splitting*

PS phase: tunneling between two edge states

T state: tunneling between outer edge state and "GdGM state"

Exponentially small energy $\xrightarrow{N_{atom}}$ *zero*



Probing Majorana Fermion

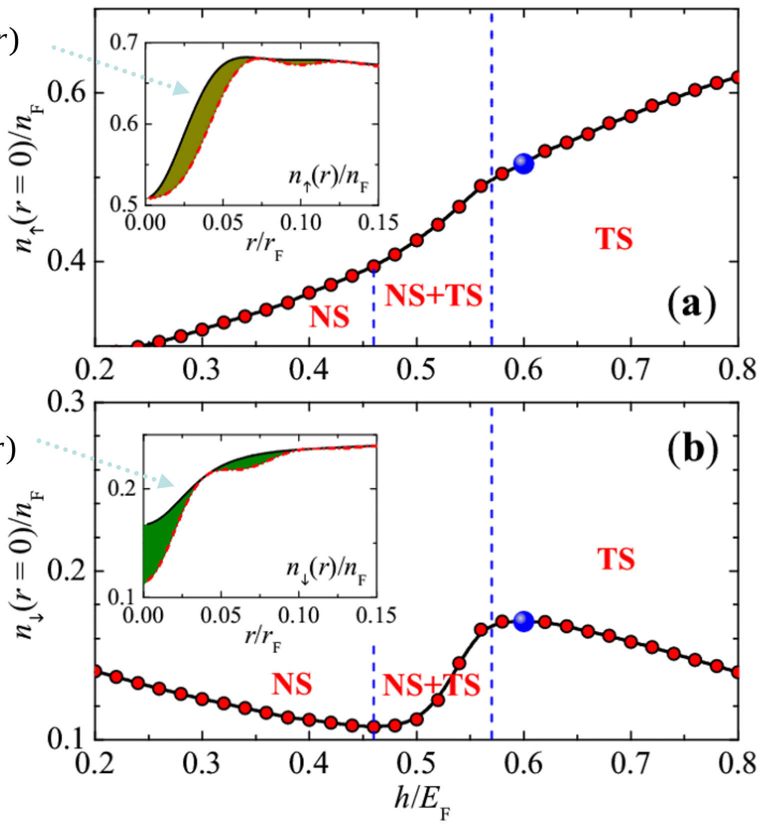
Experimental Signature

spin-up and spin-down densities at the trap center

$h = 0.6E_F$

$n_\uparrow(r) - n_{\uparrow 0}(r)$

$n_\downarrow(r) - n_{\downarrow 0}(r)$



Probing Majorana Fermion

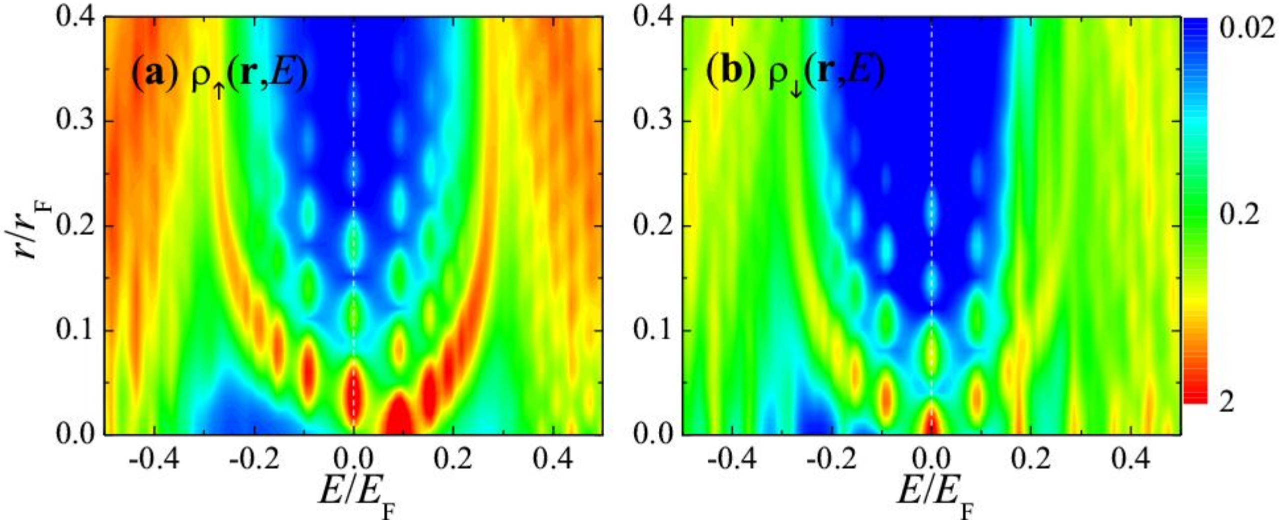
Experimental Signature

local density of state

$$\rho_\sigma(r,E)=\frac{1}{2}\sum_\eta\left[|u_{\sigma\eta}|^2\,\delta(E-E_\eta)+|v_{\sigma\eta}|^2\,\delta(E+E_\eta)\right]$$

$$\rho_\sigma(r,0)\propto|u_{\sigma\eta}(r)|^2=|v_{\sigma\eta}(r)|^2$$

rf-spectroscopy



^{40}K, B=225.3G, $Ea/E_{F=0.2}$ N=2000, $\omega_z=2\pi\times 80kHz$ $\omega_\perp=2\pi\times 125Hz$ *Kohl's group PRL 2011*

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Application:

Topological quantum computation using Majorana Fermions as qubits

1D trapped ultracold Fermi gas with SO coupling

Hamiltonian

$$\mathcal{H} = \int dx \psi^{\dagger}(x) \left[\mathcal{H}_0^S(x) - h\sigma_z + \lambda k \sigma_y\right] \psi(x) + g_{1D} \int dx \psi_{\uparrow}^{\dagger}(x) \psi_{\downarrow}^{\dagger}(x) \psi_{\downarrow}(x) \psi_{\uparrow}(x)$$

$$\mathcal{H}_0^S(x) \equiv -(\hbar^2/2m)\partial^2/\partial x^2 + m\omega^2 x^2/2 - \mu$$

$$g_{1D} = -2\hbar^2/(m a_{1D})$$

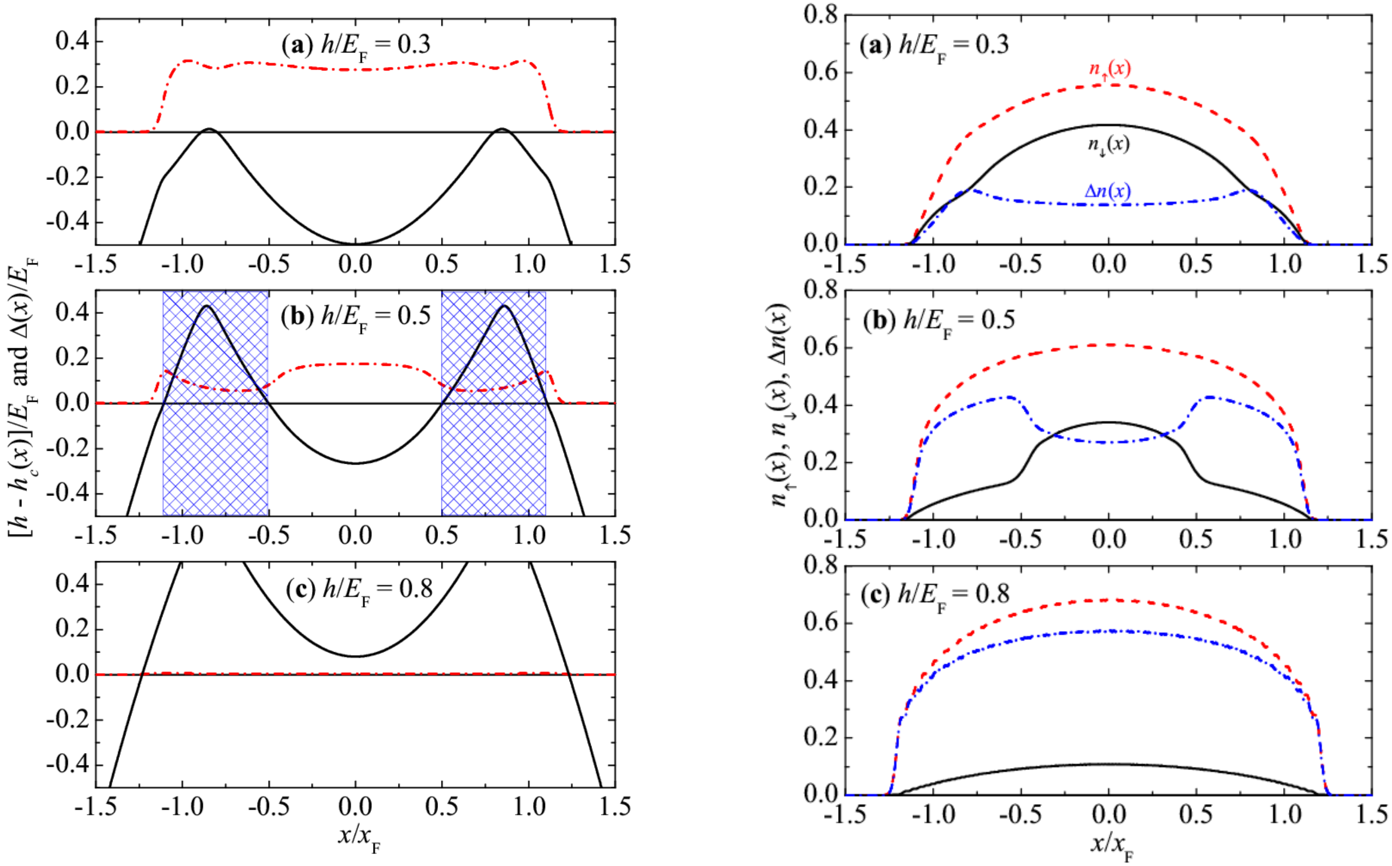
Rashba SO Coupling $\propto \sigma_x k_y - \sigma_y k_x$

Dresselhaus SO Coupling $\propto -\sigma_x k_y - \sigma_y k_x$

1D trapped ultracold Fermi gas with SO coupling

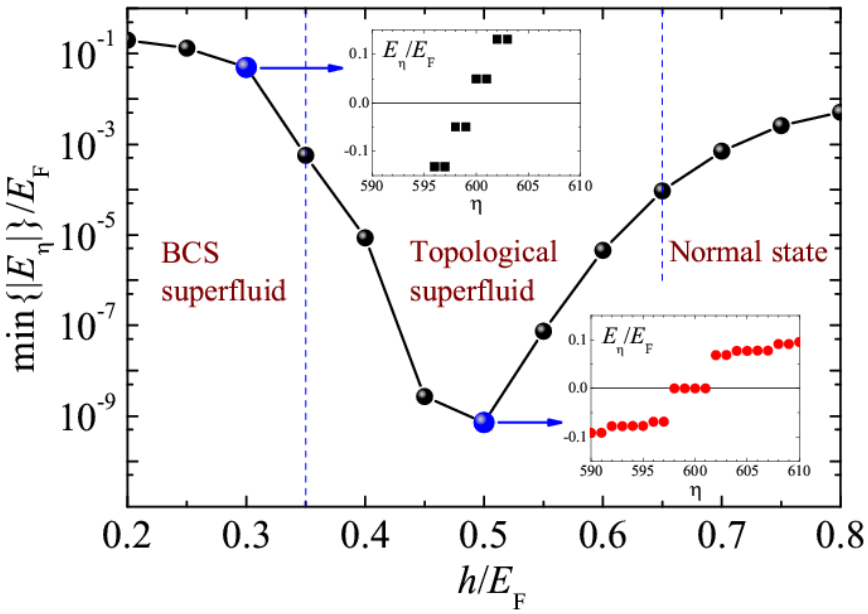
Topological superfluid

$$h_c(x) = \sqrt{\mu^2(x) + \Delta^2(x)}$$



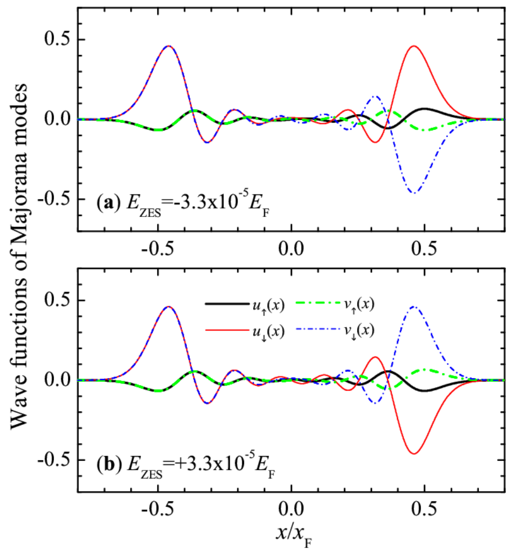
1D trapped ultracold Fermi gas with SO coupling

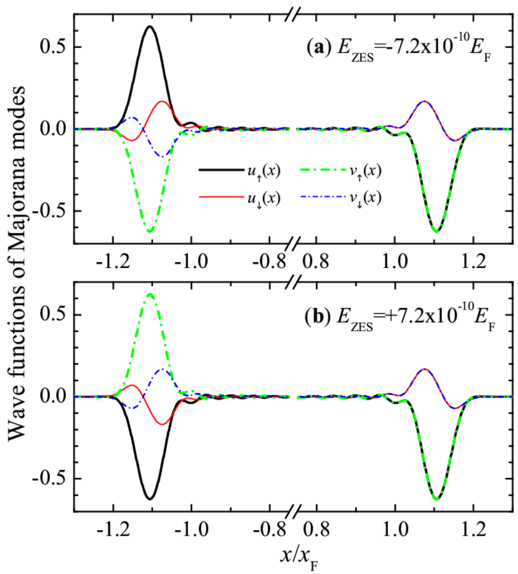
Phase diagram

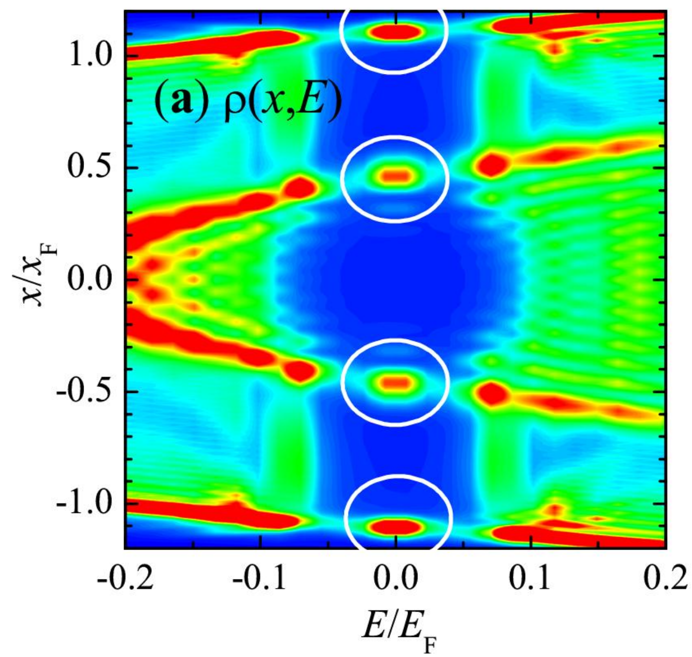


1D trapped ultracold Fermi gas with SO coupling

$h/E_F=0.5$



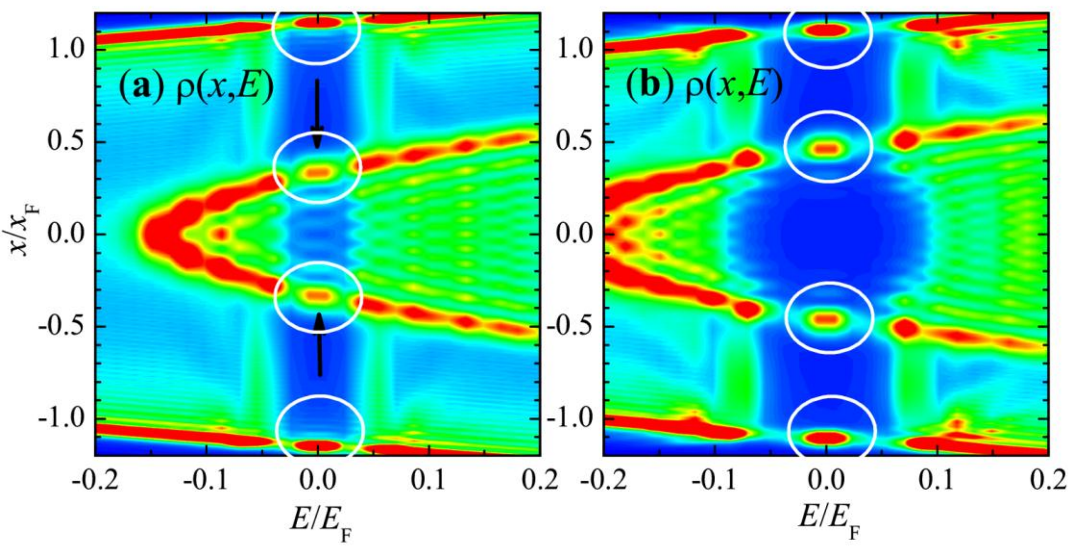


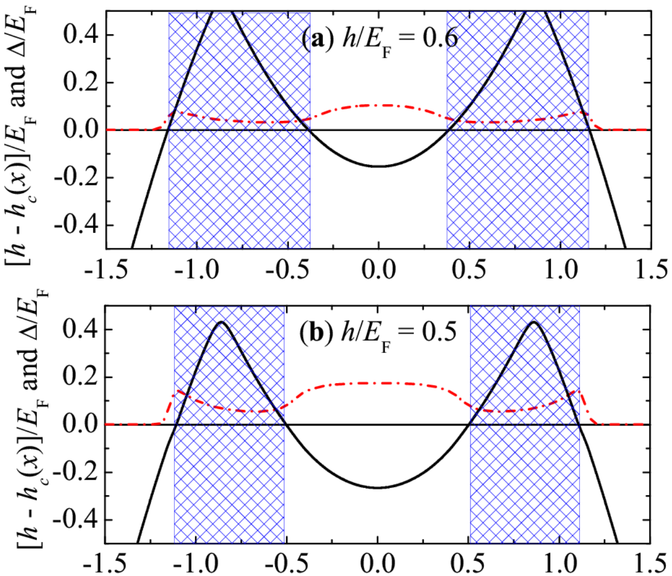


Manipulating Majorana Fermions

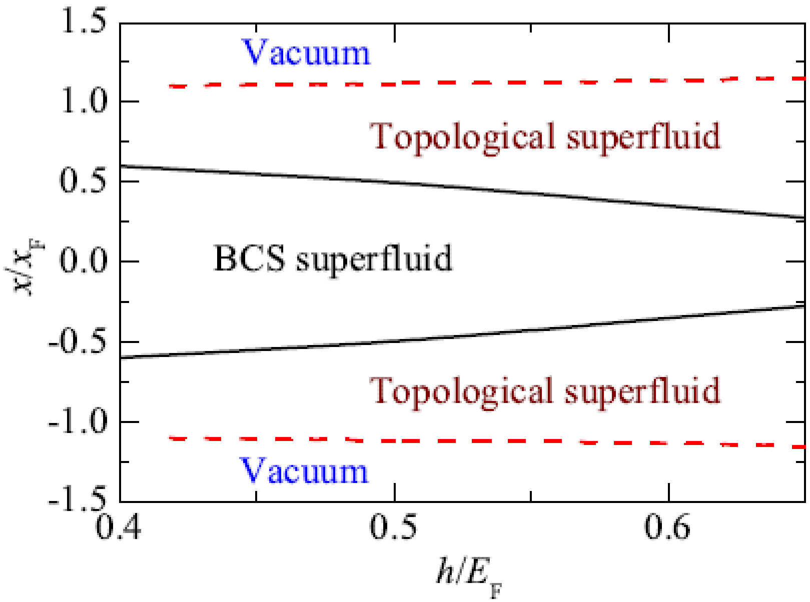
Transport Majorana Fermions: Zeeman field

$h/E_F=0.6$ $h/E_F=0.5$





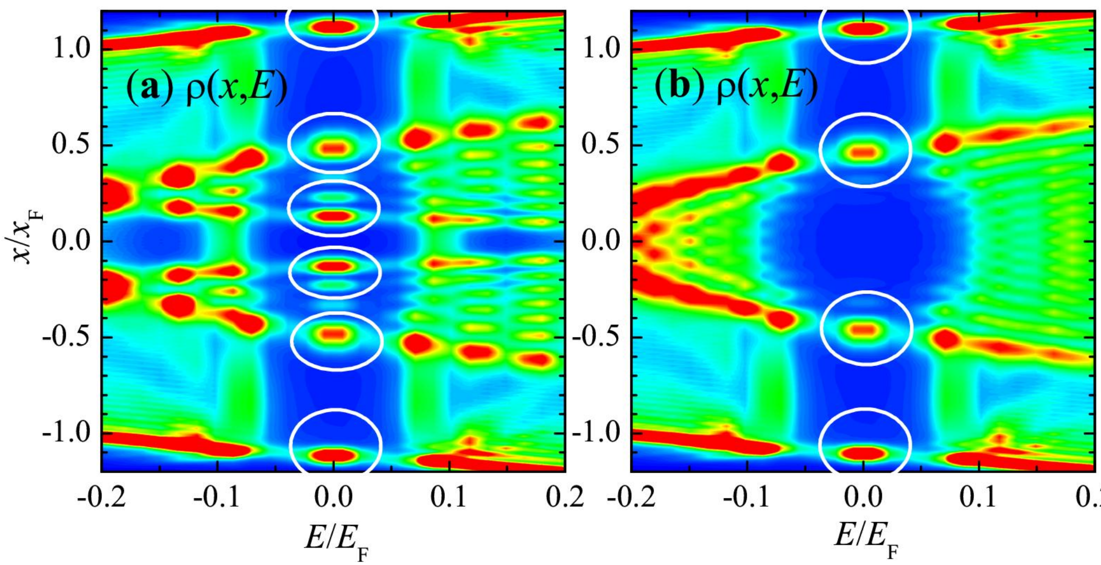
Manipulating Majorana Fermions

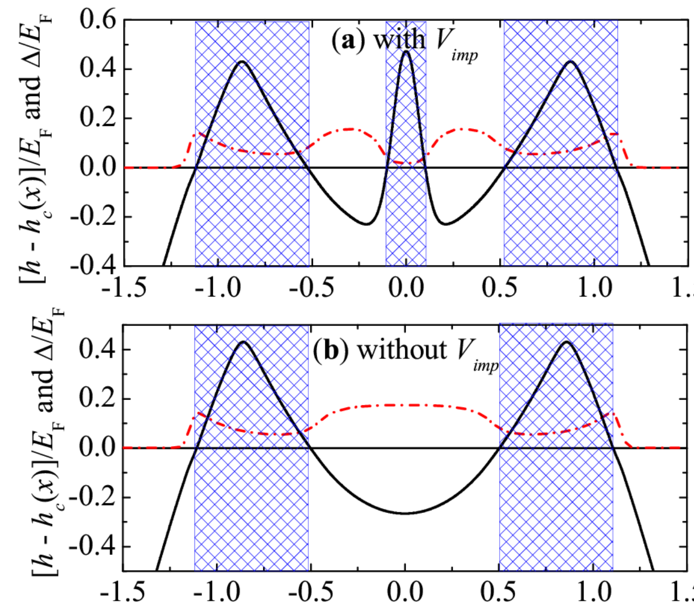


Manipulating Majorana Fermions

Create Majorana Fermions: Magnetic impurity potential $V_{imp}(x)$

$$V_{imp,\sigma}(x) = \begin{cases} -\frac{V_0}{\sqrt{2\pi}d} exp[-(x-x_0)^2/(2d^2)], \sigma = \uparrow \\ +\frac{V_0}{\sqrt{2\pi}d} exp[-(x-x_0)^2/(2d^2)], \sigma = \downarrow \end{cases}$$

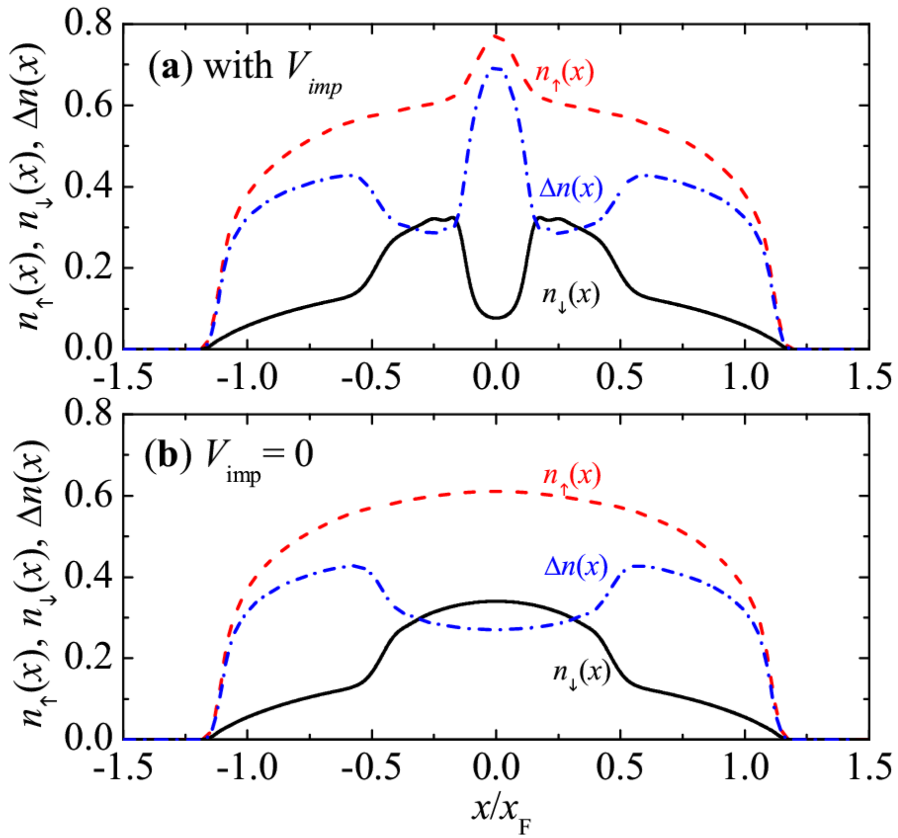




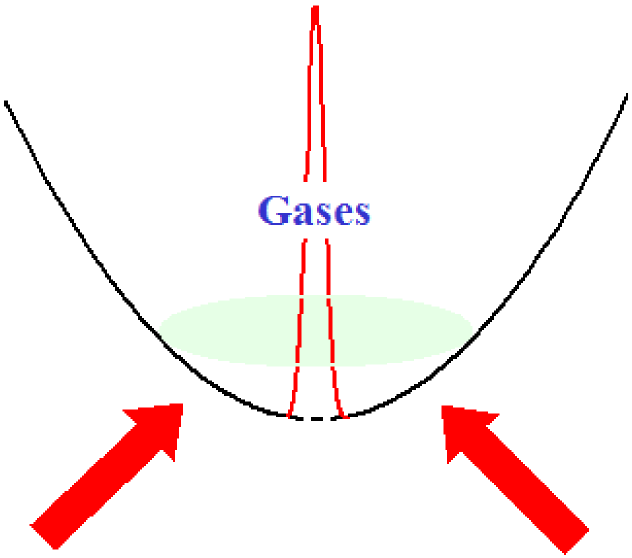
$h/E_F=0.5$

Manipulating Majorana Fermions

Magnetic impurity potential



Manipulating Majorana Fermions



^{40}K

SO Coupled ultracold atomic gases

Spielman's group Nature Vol 471, 83 (2011)

$$\mathcal{H} = \int dx \psi^{\dagger}(x)\left[\mathcal{H}_0^S(x) - h\sigma_z + \lambda k \sigma_y\right]\psi(x) + g_{1D}\int dx \psi_{\uparrow}^{\dagger}(x)\psi_{\downarrow}^{\dagger}(x)\psi_{\downarrow}(x)\psi_{\uparrow}(x)$$

Open Question:

Topological quantum computation

SO coupled bosonic system

Topological superfluidity